Part 2: Modeling and Control of Robots with Variable Stiffness Actuation for Safety and Performance

Alessandro De Luca

DIPARTIMENTO DI INFORMATICA
E SISTEMISTICA ANTONIO RUBERTI

SAPIENZA
UNIVERSITÀ DI ROMA

Summary

- collision detection and reaction
  - for rigid manipulators
  - in the presence of joint elasticity
- control of robots with Variable Stiffness Actuation (VSA)
  - dynamic modeling of antagonistic VSA (single joint)
  - simultaneous tracking of smooth motion/stiffness trajectories
  - collision detection and reaction for VSA-based robots
  - perfect gravity cancellation (rigid, elastic, or VSA joints)
  - on-line stiffness estimation for feedback control

Handling of robot collisions

- safety in physical Human-Robot Interaction (pHRI)
  - mechanics: lightweight construction and inclusion of compliance
  - elastic joints and/or variable (nonlinear) stiffness actuation
  - additional exteroceptive sensing and monitoring may be needed
  - learning and understanding human motion
  - human-aware motion planning ("legible" robot trajectories)
  - reactive control strategies with safety objectives/constraints
  - intentional interaction vs. accidental collisions
- prevent, avoid, detect and react to collisions
  - possibly, using only robot proprioceptive sensors

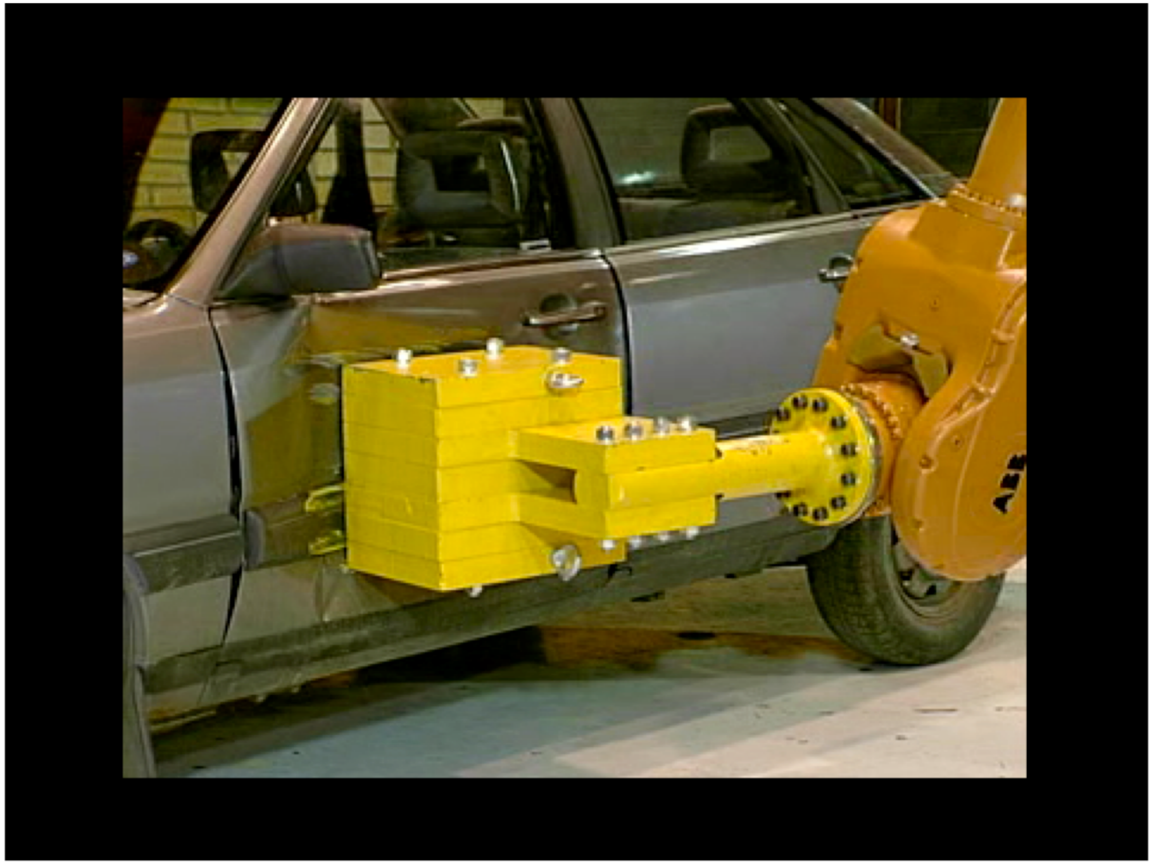
EC FP-6 STREP
(2006-09)

PHRIENDS → SAPHARI

EC FP-7 IP
(2011-14 ??)

ABB collision detection

- ABB IRB 7600



video

- the only feasible robot reaction is to stop!

Collisions as system faults

- (rigid) robot model, with possible collisions

control torque

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + \tau_K = \tau_{\text{tot}}$$

inertia matrix — Coriolis/centrifugal (with factorization) — gravity

joint torque caused by link collision

$$\tau_K = J_K^T(q) F_K$$

transpose of the Jacobian associated to the contact point/area

- collisions may occur at any (unknown) place along the whole robotic structure
- simplifying assumptions (not strictly needed)
  - single contact/collision
  - manipulator as an open kinematic chain

Relevant dynamic properties

- total energy and its variation

$$E = T + U = \frac{1}{2}\dot{q}^T M(q)\dot{q} + U_g(q) \qquad \boxed{\dot{E} = \dot{q}^T \tau_{\text{tot}}}$$

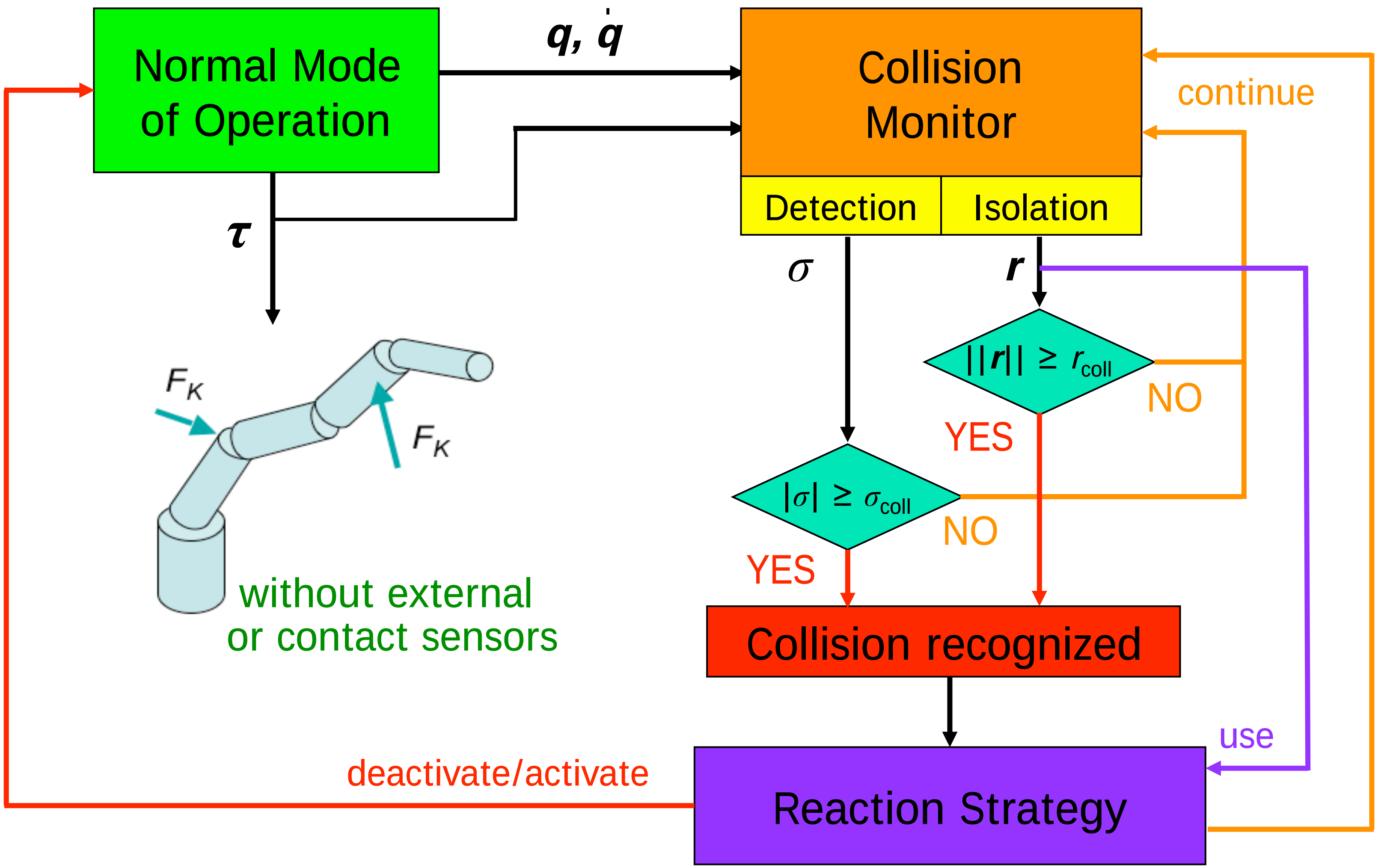
- generalized momentum and its decoupled dynamics

$$p = M(q)\dot{q}$$

$$\dot{p} = \tau_{\text{tot}} + C^T(q,\dot{q})\dot{q} - g(q)$$

using the skew-symmetric property $\dot{M}(q) = C(q,\dot{q}) + C^T(q,\dot{q})$

Monitoring collisions using residuals



Energy-based detection of collisions

- scalar residual (computable, e.g., by N-E algorithm)

$$\sigma(t) = k_D \left[E(t) - \int_0^t (\dot{\boldsymbol{q}}^T \boldsymbol{\tau} + \sigma) ds - E(0) \right]$$

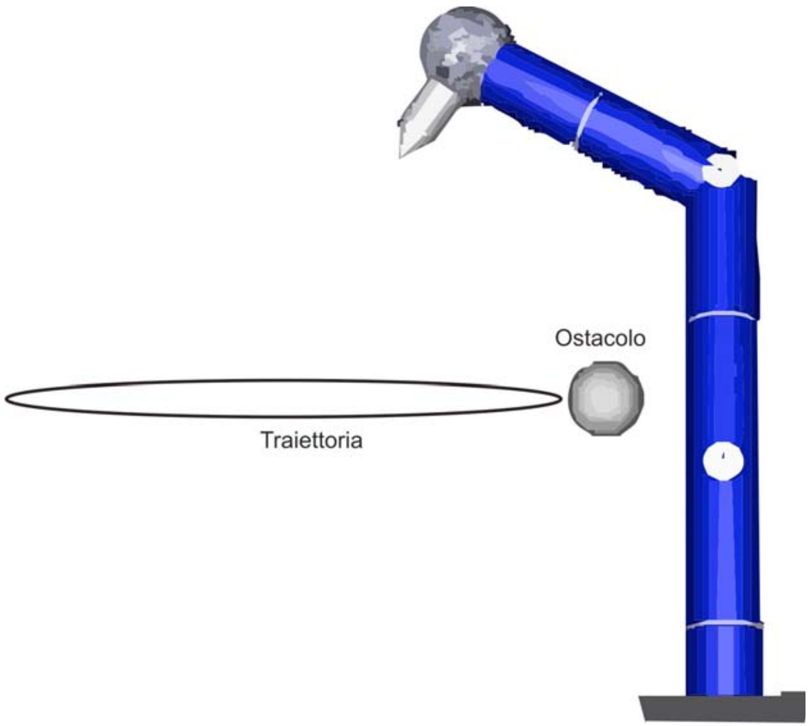
$$\sigma(0) = 0 \qquad\qquad k_D > 0$$

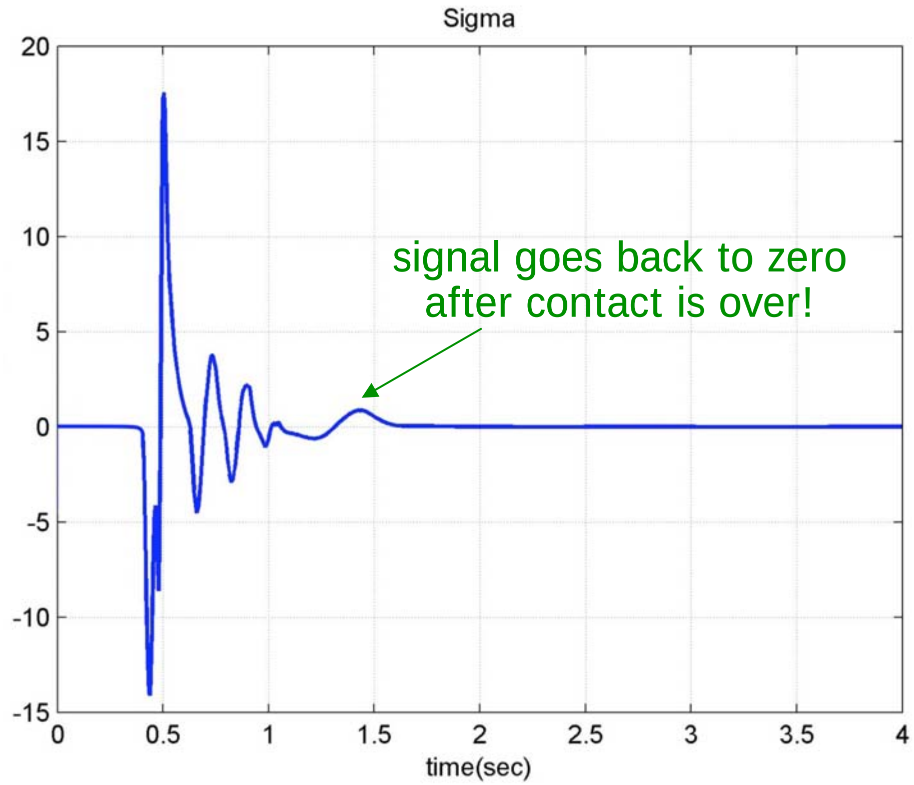
- ... and its dynamics (needed only for analysis)

$$\dot{\sigma} = -k_D\,\sigma + k_D \dot{\boldsymbol{q}}^T \boldsymbol{\tau}_K$$

a stable first-order linear filter, excited by a collision!

Simulation for a 7R robot





detection of a collision with a fixed obstacle in the work space during the execution of a Cartesian trajectory (redundant robot)

Momentum-based isolation of collisions

- residual vector (computable...)

$$\boldsymbol{r}(t) = \boldsymbol{K}_I \left[\boldsymbol{p}(t) - \int_0^t \left(\boldsymbol{\tau} + \boldsymbol{C}^T(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{r} \right) ds - \boldsymbol{p}(0) \right]$$

$$\boldsymbol{r}(0) = \boldsymbol{0} \qquad \boldsymbol{K}_I > \boldsymbol{0} \text{ (diagonal)}$$

- ... and its decoupled dynamics

$$\dot{\boldsymbol{r}} = -\boldsymbol{K}_I \boldsymbol{r} + \boldsymbol{K}_I \boldsymbol{\tau}_K \qquad \frac{r_j(s)}{\tau_{K,j}(s)} = \frac{K_{I,j}}{s + K_{I,j}} \quad j = 1, \ldots, N$$

N independent stable first-order linear filters, excited by a collision!
(all residuals go back to zero if there is no longer contact = post-impact phase)

Analysis of the momentum method

- ideal situation (no noise/uncertainties)

$$\boldsymbol{K}_I \to \infty \quad \Rightarrow \quad \boxed{\boldsymbol{r} \approx \boldsymbol{\tau}_K}$$

- isolation property: collision has occurred in an area located up to the i-th link if

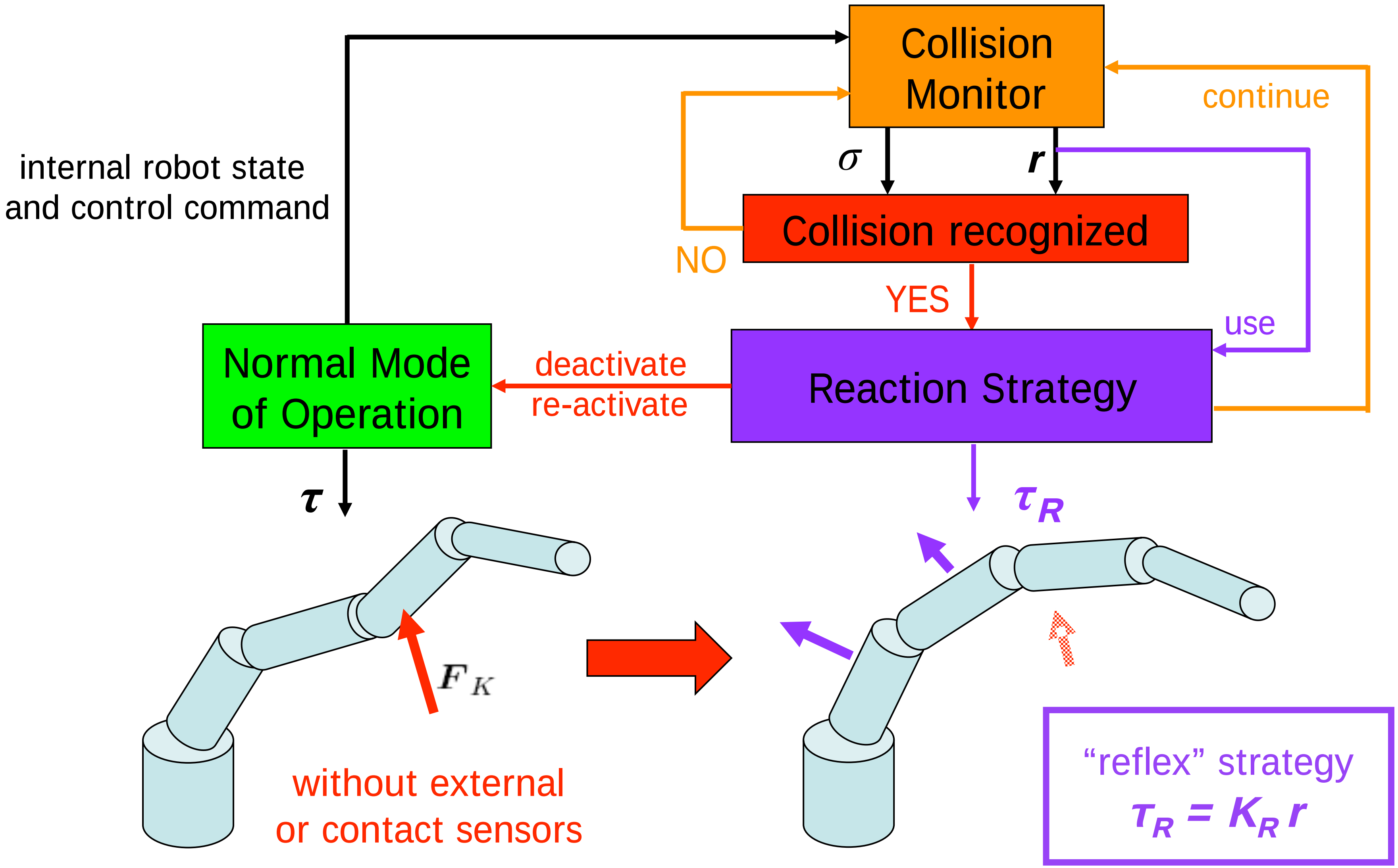
$$\boldsymbol{r} = \begin{bmatrix} * & \dots & * & * & \boxed{0 \quad \dots \quad 0} \end{bmatrix}^T$$

$$\qquad\qquad\qquad\qquad\qquad\qquad \uparrow \qquad\qquad \uparrow$$

$$\qquad\qquad\qquad\qquad\qquad\qquad \circled{i}+1 \quad \dots \quad N$$

- residual vector contains directional information on the torque at the robot joints resulting from the link collision (useful for robot reaction in post-impact phase)

Safe reaction to collisions



Robot reaction strategy

- "zero-gravity" control in any operational mode

$$\boldsymbol{\tau} = \boldsymbol{\tau}' + \boldsymbol{g}(\boldsymbol{q})$$

- upon detection of a collision ($\boldsymbol{r}$ is over some threshold)
  - no reaction (strategy 0): robot continues its planned motion...
  - stop robot motion (strategy 1): either by braking or by stopping the motion reference generator and switching to a high-gain position control law
  - reflex* strategy: switch to a residual-based control law

$$\boldsymbol{\tau}' = \boldsymbol{K}_R \boldsymbol{r} \qquad \boldsymbol{K}_R > 0 \quad \text{(diagonal)}$$

"*joint torque command in the same direction of collision torque*"

* = in robots with joint elasticity, the reflex strategy
can be implemented in different ways (strategies 2,3,4)

Inclusion of joint elasticity

DLR LWR-III

- lightweight (14 kg) 7R antropomorfic robot with harmonic drives (elastic joints) and joint torque sensors

motor torques commands

$$
\begin{aligned}
M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) &= \tau_J + \tau_K \\
B\ddot{\theta} + \tau_J &= \tau \\
\tau_J &= K(\theta - q)
\end{aligned}
$$

joint torques due to link collision

elastic torques at the joints

- proprioceptive sensing: motor positions and joint elastic torques

$$\theta \quad \tau_J \quad \Longrightarrow \quad q = \theta - K^{-1}\tau_J$$

Collision isolation for LWR-III robot
elastic joint case

- two alternatives for extending the rigid case results
- the simplest one takes advantage of the presence of joint torque sensors, e.g. for collision isolation

$$\boldsymbol{\tau} \rightarrow \boldsymbol{\tau}_J$$

"replace the commanded torque to the motors with the elastic torque measured at the joints"

$$\boldsymbol{r}_{\mathrm{EJ}}(t) = \boldsymbol{K}_I \left[\boldsymbol{p}(t) - \int_0^t \left(\boldsymbol{\tau}_J + \boldsymbol{C}^T(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{r}_{\mathrm{EJ}} \right) ds - \boldsymbol{p}(0) \right]$$

$$\dot{\boldsymbol{r}}_{\mathrm{EJ}} = -\boldsymbol{K}_I \boldsymbol{r}_{\mathrm{EJ}} + \boldsymbol{K}_I \boldsymbol{\tau}_K$$

- the other alternative uses joint position and velocity measures at the motor and link sides and again the commanded torque
- with joint elasticity, more complex motion control laws needed
- different active strategies of reaction to collisions are possible

Control of DLR LWR-III robot
elastic joint case

- general control law using full state feedback
  (motor position and velocity, joint elastic torque and its derivative)

$$\boldsymbol{\tau} = \boldsymbol{K}_P(\boldsymbol{\theta}_d - \boldsymbol{\theta}) - \boldsymbol{K}_D\dot{\boldsymbol{\theta}} + \boldsymbol{K}_{P\tau}(\boldsymbol{\tau}_{J,d} - \boldsymbol{\tau}_J) - \boldsymbol{K}_{D\tau}\dot{\boldsymbol{\tau}}_J + \boldsymbol{\tau}_{J,d}$$

motor position error

elastic joint torque error

elastic joint torque ffw command

- a "zero-gravity" condition is realized in an approximate (quasi-static) way, using only motor position measures

$$\bar{g}(\boldsymbol{\theta}) = \boldsymbol{g}(\boldsymbol{q}), \quad \forall(\boldsymbol{\theta}, \boldsymbol{q}) \in \Omega := \{(\boldsymbol{\theta}, \boldsymbol{q}) | \, \boldsymbol{K}(\boldsymbol{\theta} - \boldsymbol{q}) = \boldsymbol{g}(\boldsymbol{q})\}$$

motor position

link position

(diagonal) matrix of joint stiffness

Reaction strategies
specific for elastic joint robots

- strategy 2: floating reaction (robot ≈ in "zero-gravity")

$$\boldsymbol{\tau}_{J,d} = \bar{\boldsymbol{g}}(\boldsymbol{\theta}) \qquad \boldsymbol{K}_P = \mathbf{0}$$

- strategy 3: reflex torque reaction (closest to the rigid case)

$$\boldsymbol{\tau}_{J,d} = \boldsymbol{K}_R \boldsymbol{r}_{\mathrm{EJ}} + \bar{\boldsymbol{g}}(\boldsymbol{\theta}) \qquad \boldsymbol{K}_P = \mathbf{0}$$

- strategy 4: admittance mode reaction (residual is used as the new reference for the motor velocity)

$$\boldsymbol{\tau}_{J,d} = \bar{\boldsymbol{g}}(\boldsymbol{\theta}) \qquad \dot{\boldsymbol{\theta}}_d = \boldsymbol{K}_R \boldsymbol{r}_{\mathrm{EJ}}$$

- further possible reaction strategies (rigid or elastic case)
  - based on impedance control
  - sequence of strategies (e.g., 4+2)
  - time scaling: stop/reprise of reference trajectory, keeping the path
  - Cartesian task preservation (exploits robot redundancy by projecting reaction torque in a task-related dynamic null space)

Dummy head impact

video

strategy 0: no reaction

planned trajectory ends just after the position of the dummy head

video

strategy 2: floating reaction

impact velocity is here rather low and the robot stops quite immediately

Balloon impact

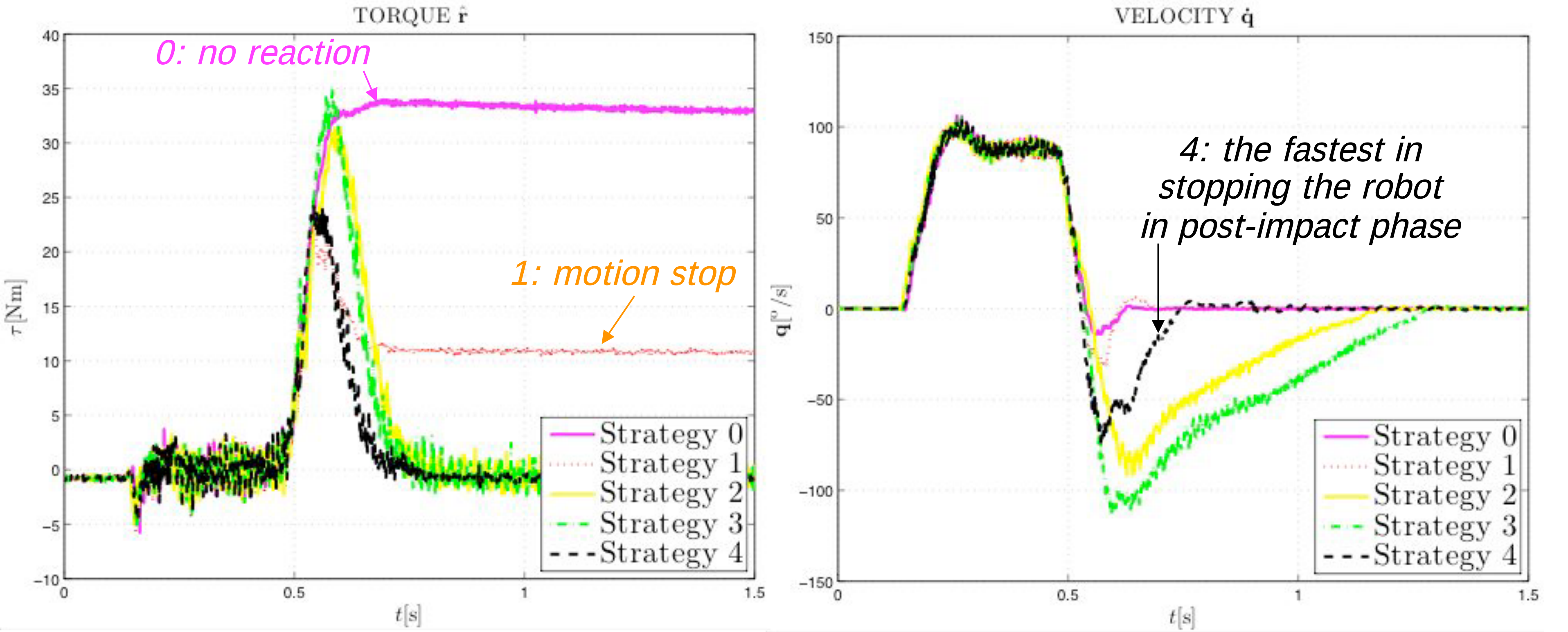
video

coordinated
joint motion
@100°/sec

strategy 4: admittance mode reaction

Comparison of reaction strategies
balloon impact

- residual and velocity at joint 4 with various reaction strategies



impact at 100°/sec with coordinated joint motion

Human-Robot Interaction (1)

- first impact @60°/sec

video

strategy 4: admittance mode

video

strategy 3: reflex torque

Human-Robot Interaction (2)

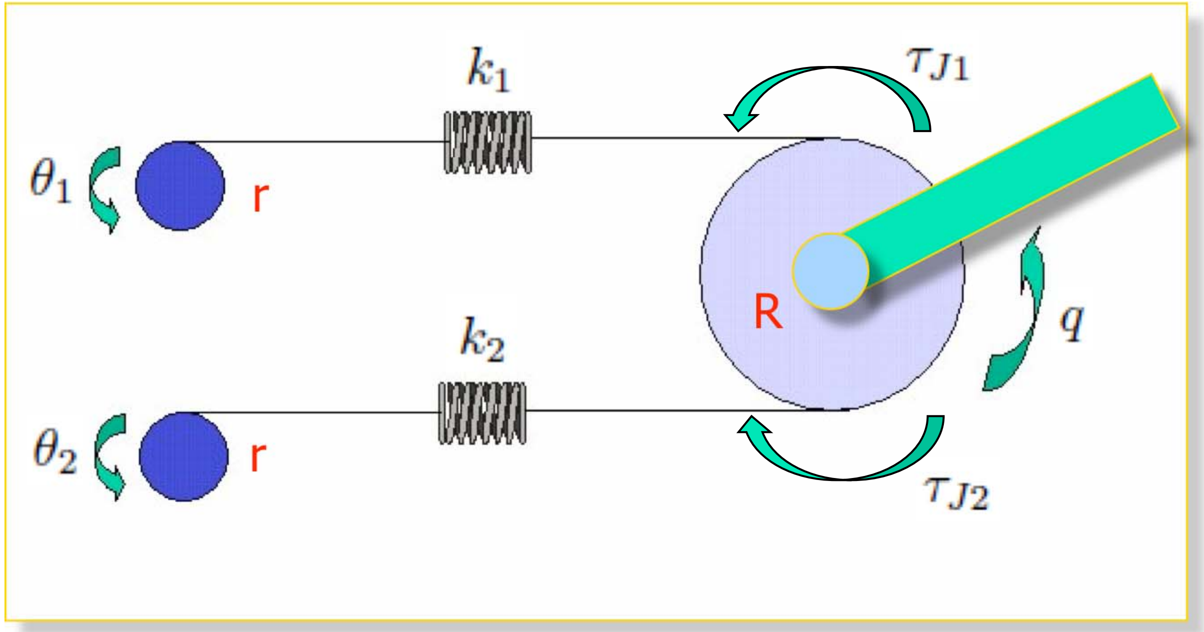
- first impact @90°/sec

video

strategy 3: reflex torque

Double actuation of a joint

example of agonistic/antagonistic behavior



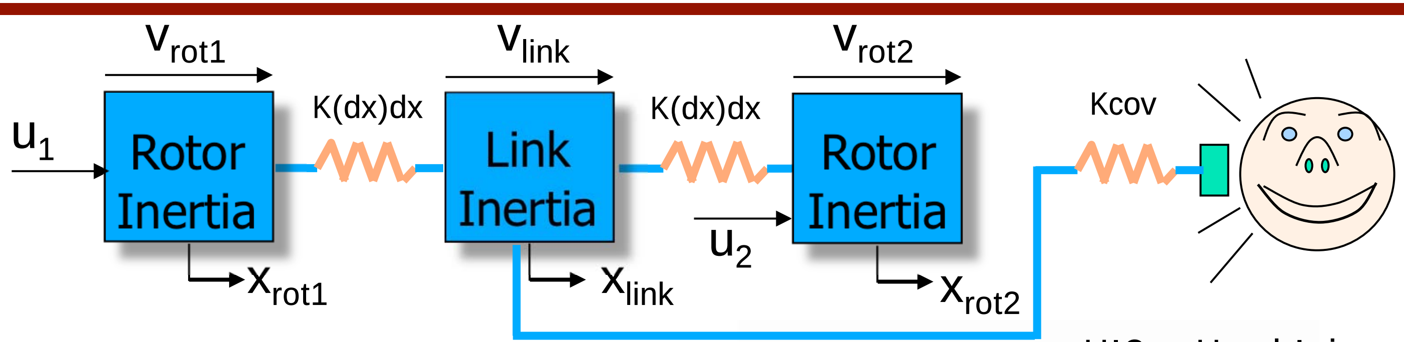
$$\tau_J = \tau_{J1} - \tau_{J2} = R\left[k_1(r\theta_1 - Rq) - k_2(r\theta_2 + Rq)\right]$$

$$\sigma = \frac{\partial \tau_J}{\partial q} = -R^2(k_1 + k_2) + R\left[\frac{\partial k_1}{\partial q}(r\theta_1 - Rq) - \frac{\partial k_2}{\partial q}(r\theta_2 + Rq)\right]$$

to achieve controllable variable mechanical stiffness,
it is necessary to have nonlinear characteristics for the $k_i(q)$

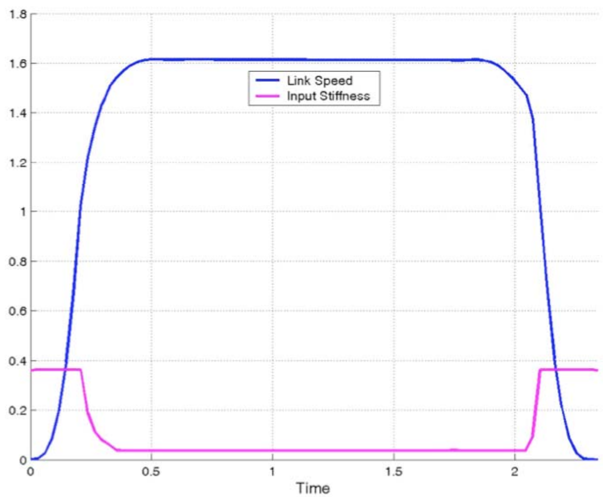
Variable stiffness actuation

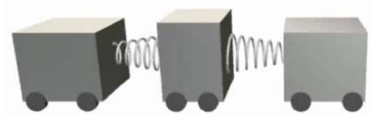
performance and safety



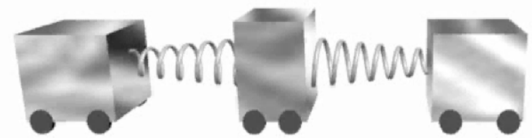
HIC = Head Injury Criterion

safe "brachistochrone"= fastest rest-to-rest motion
with bounded inputs and injury index HIC (≈ link speed$^{2.5}$)





with high initial & final stiffness



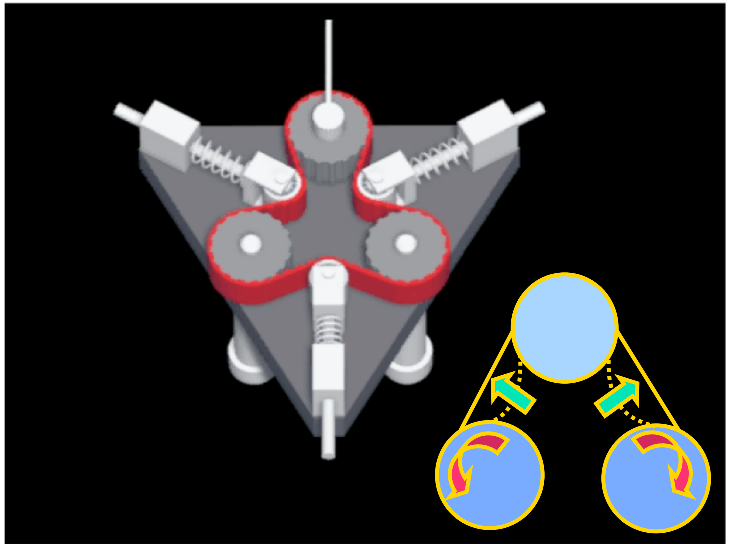
with low initial & final stiffness

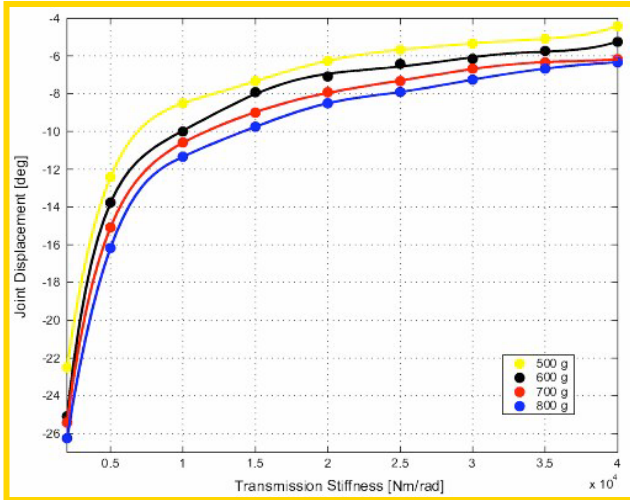
two videos Pisa (A. Bicchi)

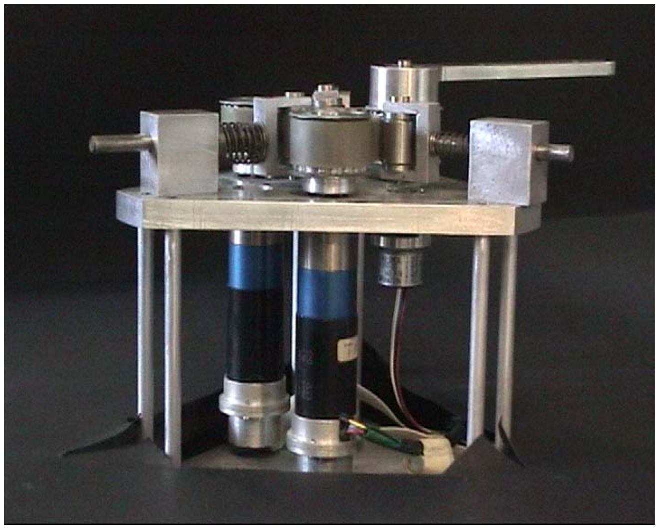
A first prototype: VSA-I

University of Pisa

video
Pisa
(A. Bicchi)





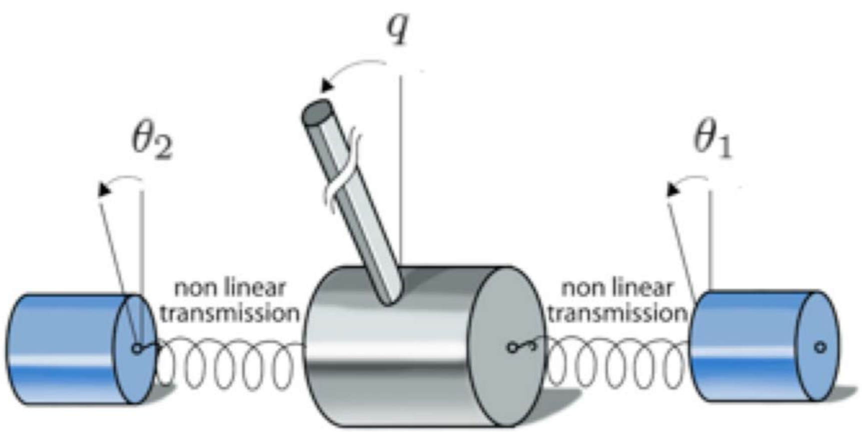


Control of robots with VSA

- simultaneous control of motion and stiffness
  - use of model-based nonlinear feedback
    - feedback linearization and input-output decoupling
  - tracking of smooth reference trajectories
    - planned for safety: “slow/stiff & fast/soft” (brachistochrone)
- extension of collision detection method
  - avoiding use of variable stiffness/elastic torque information
- reaction strategies to collisions
  - stop, reflex motion, softening the joints while reacting, ...
- applicability to 1-dof and multi-dof devices
- analysis for antagonistic case (can be easily extended to *separately/directly* controlled stiffness devices)

VSA: Antagonistic case

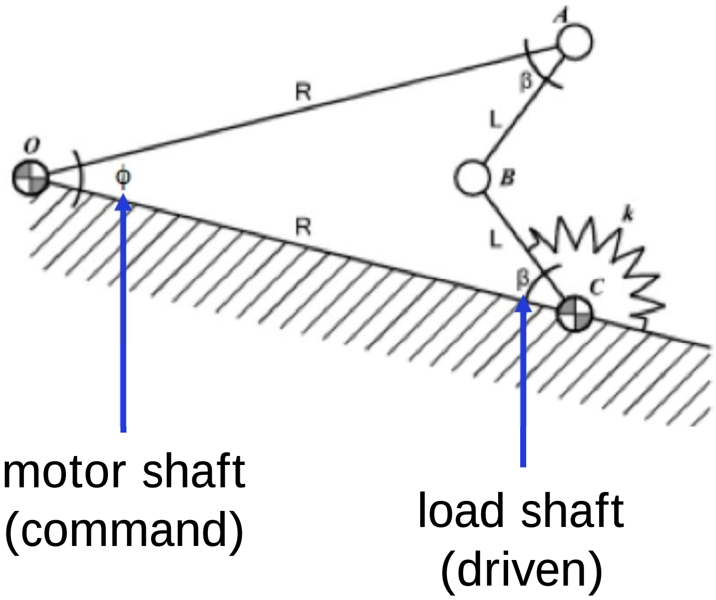
- developments for the VSA-II (University of Pisa)



- bi-directional, symmetric arrangement of two motors in antagonistic mode
- nonlinear flexible transmission
  - four-bar linkage + linear spring

Nonlinear transmission of VSA-II

- for each motor there is a pair of Grashof 4-bar linkages



$$\phi \in [0, \phi_{\max}] \qquad \phi_{\max} = 2 \arcsin\left(\frac{L}{R}\right)$$

from sine theorem on triangle OBC

$$\frac{L}{\sin\frac{\phi}{2}} = \frac{R}{\sin\left(\pi - \left(\beta + \frac{\phi}{2}\right)\right)}$$

$$\frac{R}{L}\sin\frac{\phi}{2} = \sin\left(\pi - \left(\beta + \frac{\phi}{2}\right)\right) = \sin\left(\beta + \frac{\phi}{2}\right)$$

$$\beta(\phi) = \arcsin\left(\frac{R}{L}\sin\left(\frac{\phi}{2}\right)\right) - \frac{\phi}{2}$$

the map is also invertible (in I quadrant)

$$\phi(\beta) = 2\arctan\left(\frac{\frac{L}{R}\sin\beta}{1 - \frac{L}{R}\cos\beta}\right)$$

Nonlinear transmission of VSA-II

- for each linkage

motor shaft (command)

$$\beta(\phi) = \arcsin\left(\frac{R}{L}\sin\left(\frac{\phi}{2}\right)\right) - \frac{\phi}{2}$$

load shaft (driven)

- potential energy $P(\phi) = \frac{1}{2}k\beta^2(\phi)$

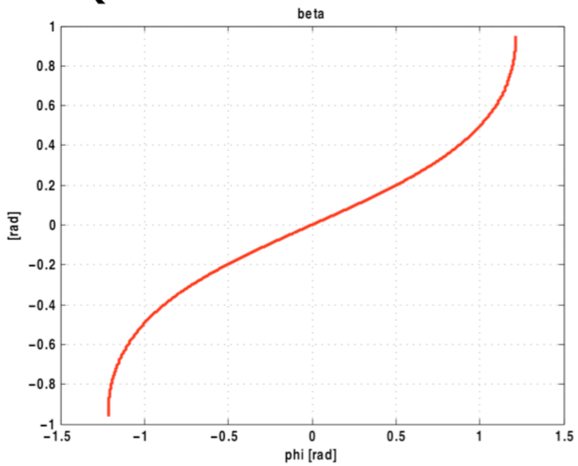
- torque $$T(\phi) = \frac{\partial P(\phi)}{\partial \phi} = k\,\beta(\phi)\,\frac{\partial \beta(\phi)}{\partial \phi} \geq 0$$

- stiffness $$\sigma(\phi) = \frac{\partial T(\phi)}{\partial \phi} = k\left(\left(\frac{\partial \beta(\phi)}{\partial \phi}\right)^2 + \beta(\phi)\,\frac{\partial^2 \beta(\phi)}{\partial \phi^2}\right)$$

same passages for any form of function $\beta(\phi)$!!

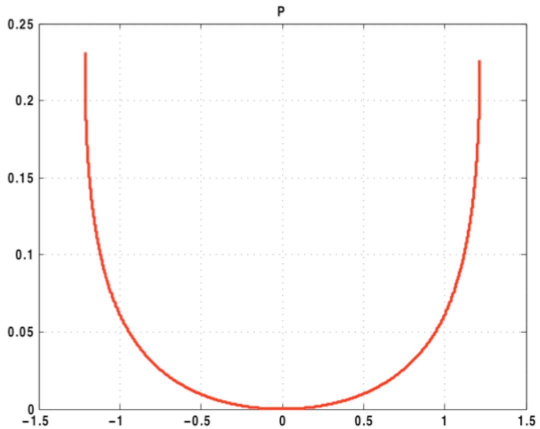
Plots of flexibility quantities

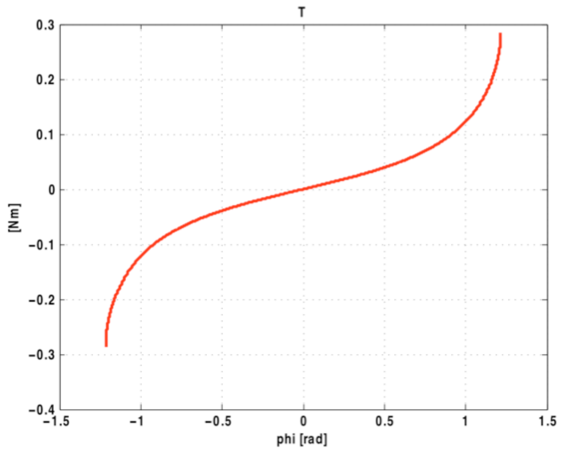
- nonlinear deflection (as a function of ϕ)

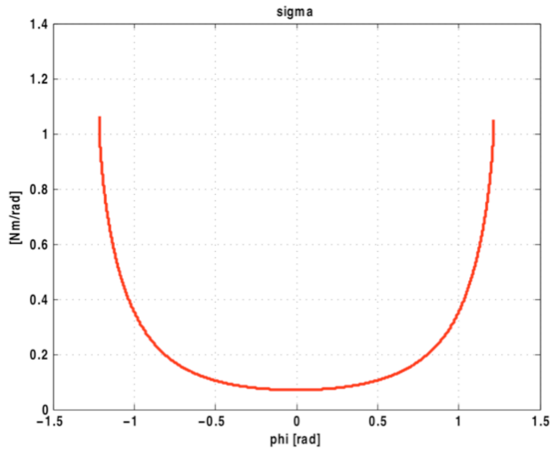


Note:
- $P(0)=0$, symmetric
- $T(0)=0$, anti-symmetric
- $\sigma(0)>0$, symmetric
- however, we are interested only in operating region $\phi \geq 0$

- potential energy $\Rightarrow$ torque $\Rightarrow$ stiffness







Dynamic model

- replace ϕ by $\theta_1 - q$ and $\theta_2 - q$, respectively, for the two actuation sides
- total joint torque $\tau_J = 2\left(T_1(\theta_1 - q) + T_2(\theta_2 - q)\right) = 2\left(\tau_{J1} + \tau_{J2}\right)$
- total (device) stiffness $\sigma = \dfrac{\partial \tau_J}{\partial q} = -2\left(\sigma_1(\theta_1 - q) + \sigma_2(\theta_2 - q)\right)$

two linkages for each motor/side

- dynamic equations (under gravity)

$$
\begin{aligned}
B\,\ddot{\theta}_1 + D\,\dot{\theta}_1 + 2\,\tau_{J1} &= \tau_1 \\
B\,\ddot{\theta}_2 + D\,\dot{\theta}_2 + 2\,\tau_{J2} &= \tau_2 \\
M\ddot{q} + D_q\dot{q} + mgd\sin q &= 2\left(\tau_{J1} + \tau_{J2}\right) + \tau_K
\end{aligned}
$$

- six-dimensional state $x = (\theta_1, \theta_2, q, \dot{\theta}_1, \dot{\theta}_2, \dot{q})$

external (collision) torque
[set to zero for control design]

Motion/stiffness control of VSA

- include gravity, and go beyond PD-type feedback laws
- choose the output vector to be controlled as

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} q \\ \sigma \end{pmatrix}$$

- apply the input-output decoupling algorithm
  - differentiate outputs until the input torques $\boldsymbol{\tau} = (\tau_1, \tau_2)$ appear and then try to invert ...
  - if the sum of relative degrees equals the state dimension (= 6), the system will also be exactly linearized by the decoupling feedback

Decoupling algorithm

- after four derivatives of the position and two of the stiffness

$$
\begin{aligned}
y_1 &= q \\
\dot{y}_1 &= \dot{q} \\
\ddot{y}_1 &= \ddot{q} = \frac{1}{M}\left(\tau_J - D_q\dot{q} - mgd\sin q\right) \\
y_1^{[3]} &= \frac{d^3 q}{dt^3} = \frac{1}{M}\Big(2\left(\sigma_1\dot{\theta}_1 + \sigma_2\dot{\theta}_2\right) + \sigma\,\dot{q} \\
&\qquad\qquad - D_q\ddot{q} - mgd\cos q\,\dot{q}\Big) \\
y_1^{[4]} &= b_1(x) + \frac{2}{MB}\left(\sigma_1\tau_1 + \sigma_2\tau_2\right),
\end{aligned}
$$

$$
\begin{aligned}
y_2 &= \sigma \\
\dot{y}_2 &= \dot{\sigma} = -2\left(\frac{\partial\sigma_1}{\partial\theta_1}\dot{\theta}_1 + \frac{\partial\sigma_2}{\partial\theta_2}\dot{\theta}_2\right) + \frac{\partial\sigma}{\partial q}\dot{q} \\
\ddot{y}_2 &= b_2(x) - \frac{2}{B}\left(\frac{\partial\sigma_1}{\partial\theta_1}\tau_1 + \frac{\partial\sigma_2}{\partial\theta_2}\tau_2\right),
\end{aligned}
$$

4 + 2 = 6 (= n!!)

$$
\Rightarrow \begin{pmatrix} y_1^{[4]} \\ \ddot{y}_2 \end{pmatrix} = b(x) + A(x)\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix}
$$

- if the decoupling matrix is nonsingular, then the solution is

$$
\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = A^{-1}(x)\left(\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} - b(x)\right)
$$

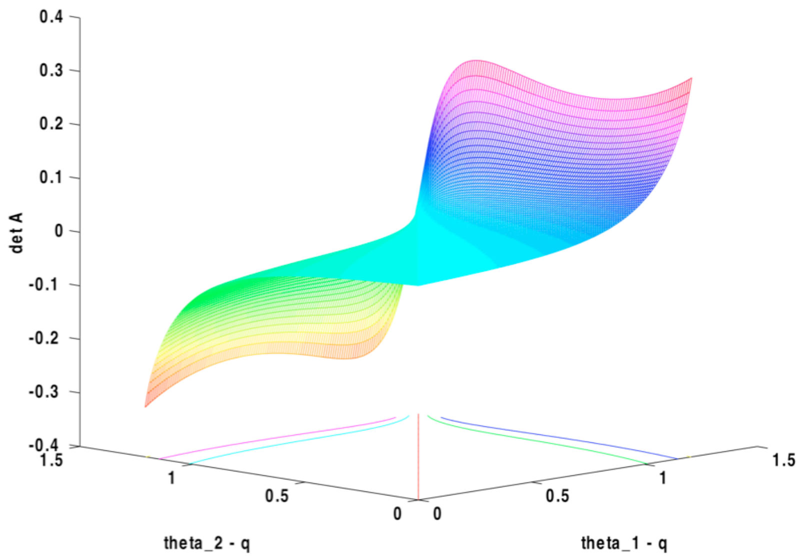
Decoupling matrix

- analysis of determinant of the decoupling matrix

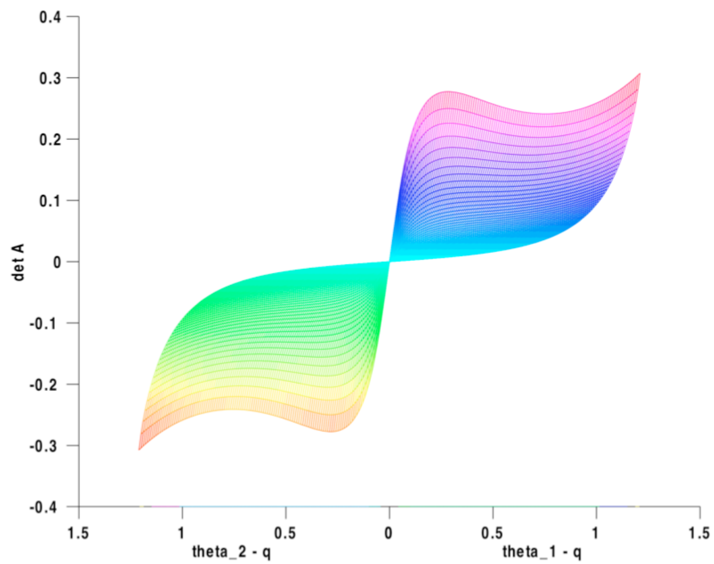
$$\mathcal{A}(\boldsymbol{x}) = \boldsymbol{\Gamma}\begin{pmatrix} \sigma_1 & \sigma_2 \\ \dfrac{\partial \sigma_1}{\partial \theta_1} & \dfrac{\partial \sigma_2}{\partial \theta_2} \end{pmatrix}$$

a function of
$\theta_1 - q$ and $\theta_2 - q$
only

standard view



side view



singular if and only if $\theta_1 = \theta_2$!

Trajectory tracking

- on the transformed (linear and decoupled) system, control design is completed by standard stabilization techniques
  - a PD on the double integrator of the stiffness channel and a PDDD on the chain of four integrators of the position channel

$$v_1 = q_d^{[4]} + k_{q,3}(q_d^{[3]} - q^{[3]}) + k_{q,2}\,(\ddot{q}_d - \ddot{q}) + k_{q,1}\,(\dot{q}_d - \dot{q}) + k_{q,0}\,(q_d - q)$$

$$v_2 = \ddot{\sigma}_d + k_{\sigma,1}\,(\dot{\sigma}_d - \dot{\sigma}) + k_{\sigma,0}\,(\sigma_d - \sigma)$$

  - pole placement is arbitrary, but should consider motor saturation
  - exact reproduction is obtained only for sufficiently smooth position and stiffness reference trajectories (and matched initial conditions)
- a pre-loading is applied at start so as to avoid control singularities during the whole motion task

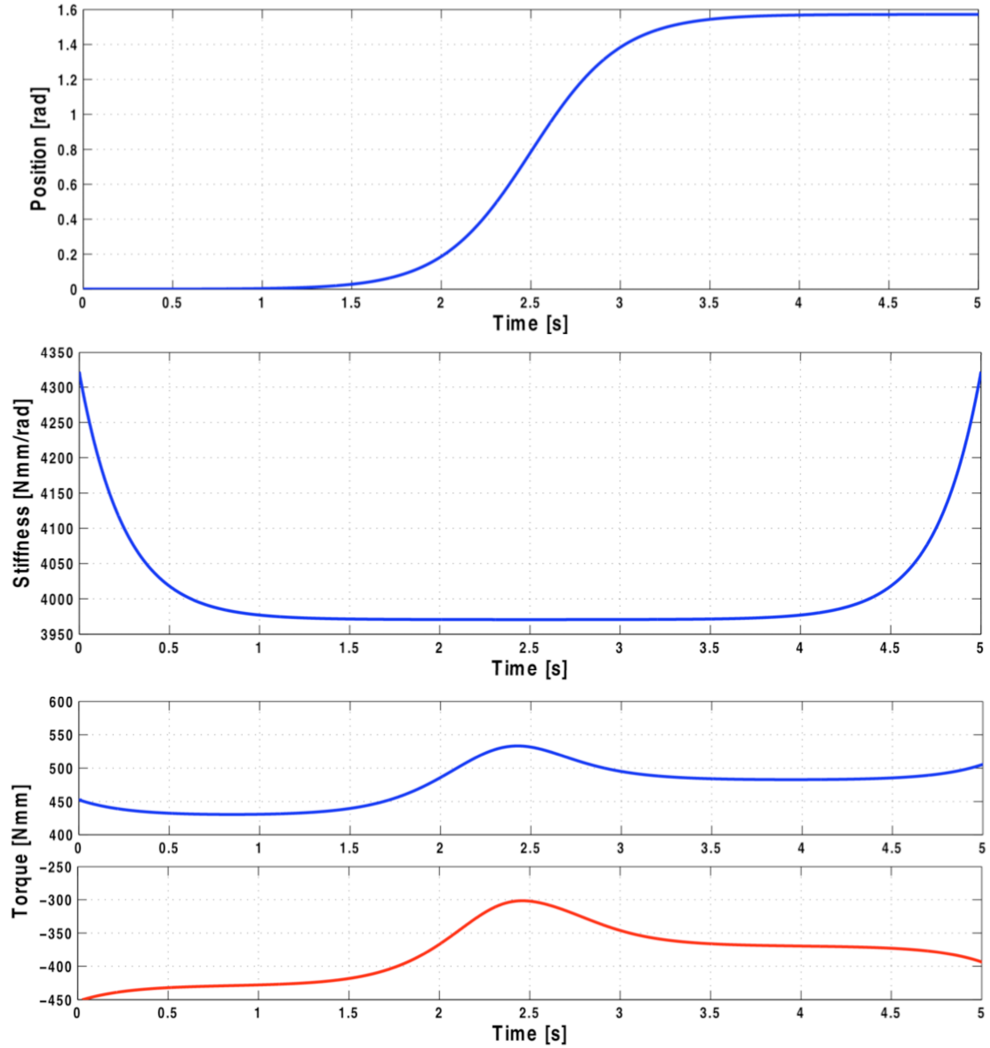
$$\theta_1(0) - q(0) \neq \theta_2(0) - q(0) \quad \Rightarrow \quad \theta_1 \neq \theta_2$$

Inverse dynamics in nominal conditions

rest-to-rest 90° link motion (under gravity)

safety-type stiffness trajectory

left/right command torques



with matched initial conditions: smooth references are exactly reproduced

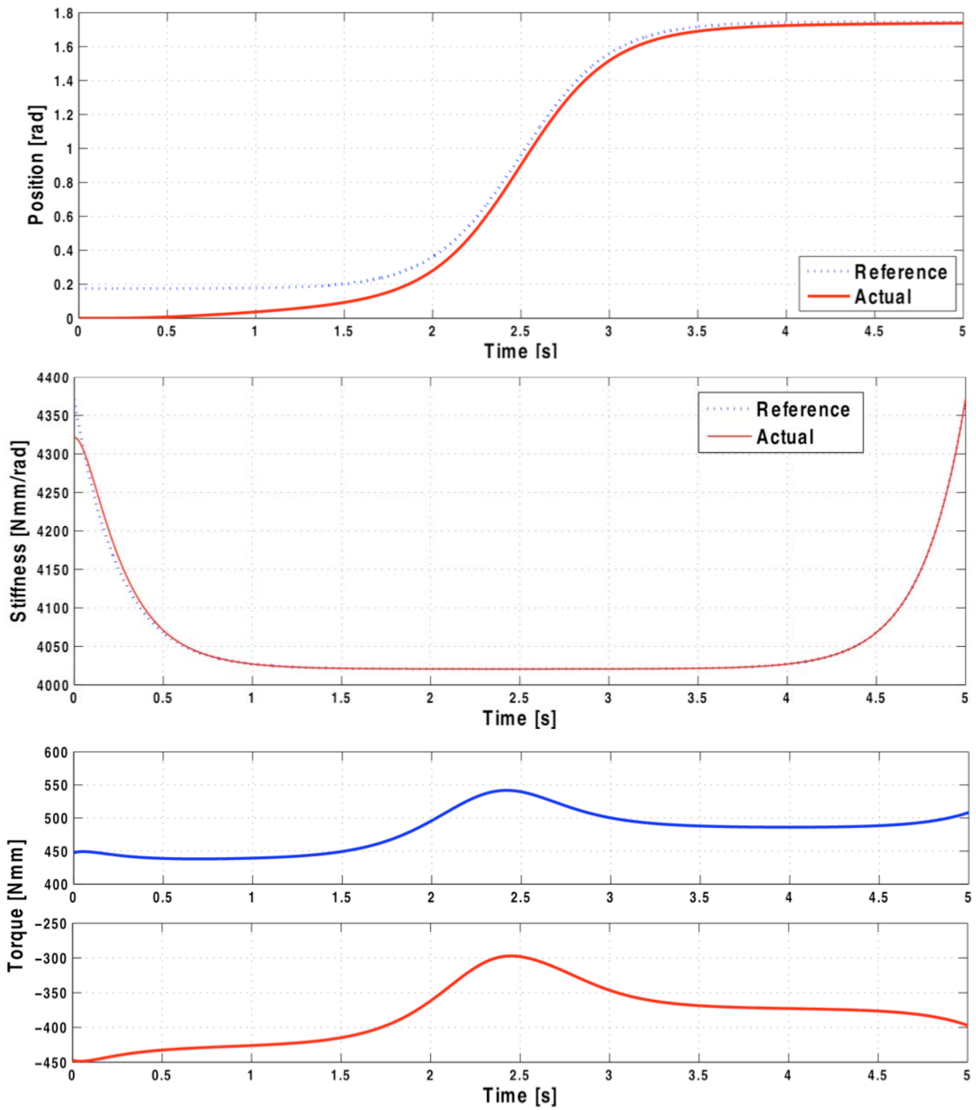
feedback law collapses into a simple feedforward!

Trajectory tracking with initial error

rest-to-rest
90° link motion
(under gravity)

safety-type
stiffness
trajectory

left/right
command
torques



initial mismatch:
10° ≅ 0.2 rad
on link position;
50 N·mm/rad
on stiffness

errors are
recovered
exponentially

Collision detection in VSA

- to detect a collision τ_K, a first option would be to use the momentum of the link (i.e., the third model equation)
- a better solution is considering the sum of the momentum of the whole device (and thus all three model equations)

$$p_{\text{sum}} = B(\dot{\theta}_1 + \dot{\theta}_2) + M\dot{q}$$

  - residual

$$r = k_I \left(p_{\text{sum}} - \int_0^t (r + \tau_1 + \tau_2 - \tau_D - mgd \sin q)\, ds \right)$$

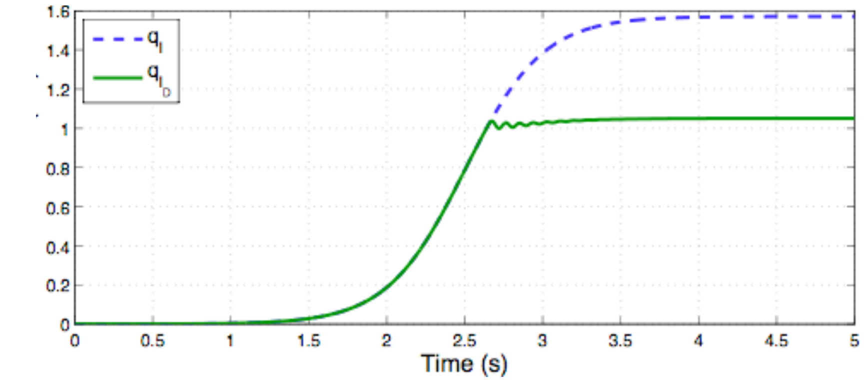
    is evaluated without any knowledge of the joint torque or stiffness

  - its dynamics shows that the residual is a filtered version of the (unknown/unmeasured) collision torque

$$\dot{r} = k_I (\tau_K - r) \qquad k_I > 0$$

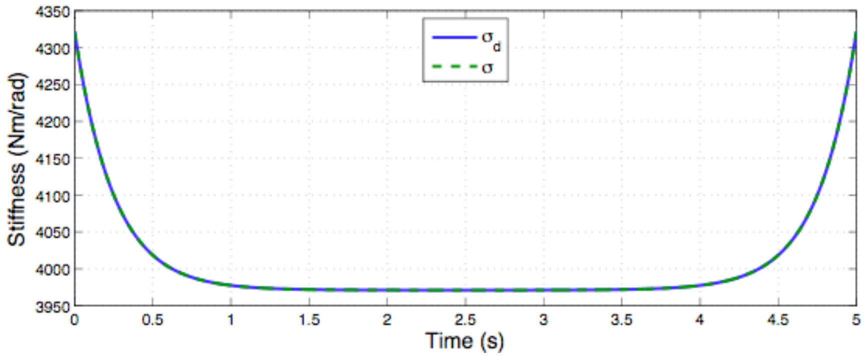
Tracking with collision detection but no reaction

link motion
hits a fixed
(elastic) obstacle



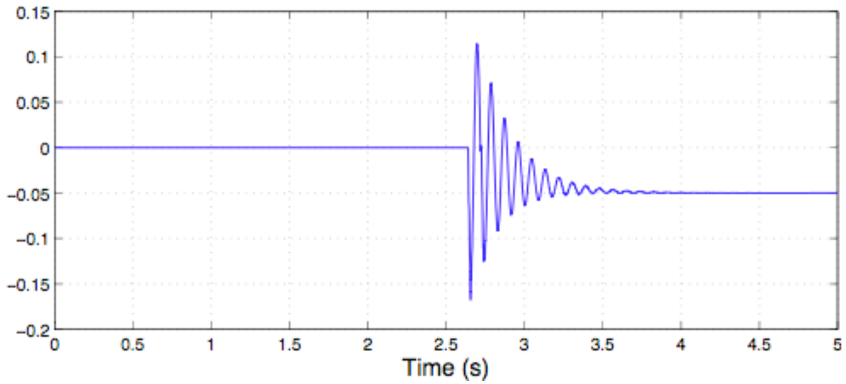
chattering at
contact state
indicates
a need for
reaction!

execution of
stiffness
trajectory
is unaffected



this shows the
properties of
decoupling
control

residual
(without reaction)



at the final
equilibrium
residual =
contact
torque

Collision reaction in VSA

- robot reaction is activated once the residual exceeds a suitable (small) threshold, i.e.

$$|r| > r_{\text{coll}}$$

- a simple choice is to keep the decoupling/linearizing controller and modify just the linear design
  - amplify the robot "reflex" to collision, by letting the residual drive the arm back/away from the contact area, so as to stabilize it to a neutral safe position with $\dot{q}_d = 0$

$$v_1 = -k_{q,3}\, q^{[3]} - k_{q,2}\, \ddot{q} - k_{q,1}\, \dot{q} + k_R\, r \qquad k_R > 0$$

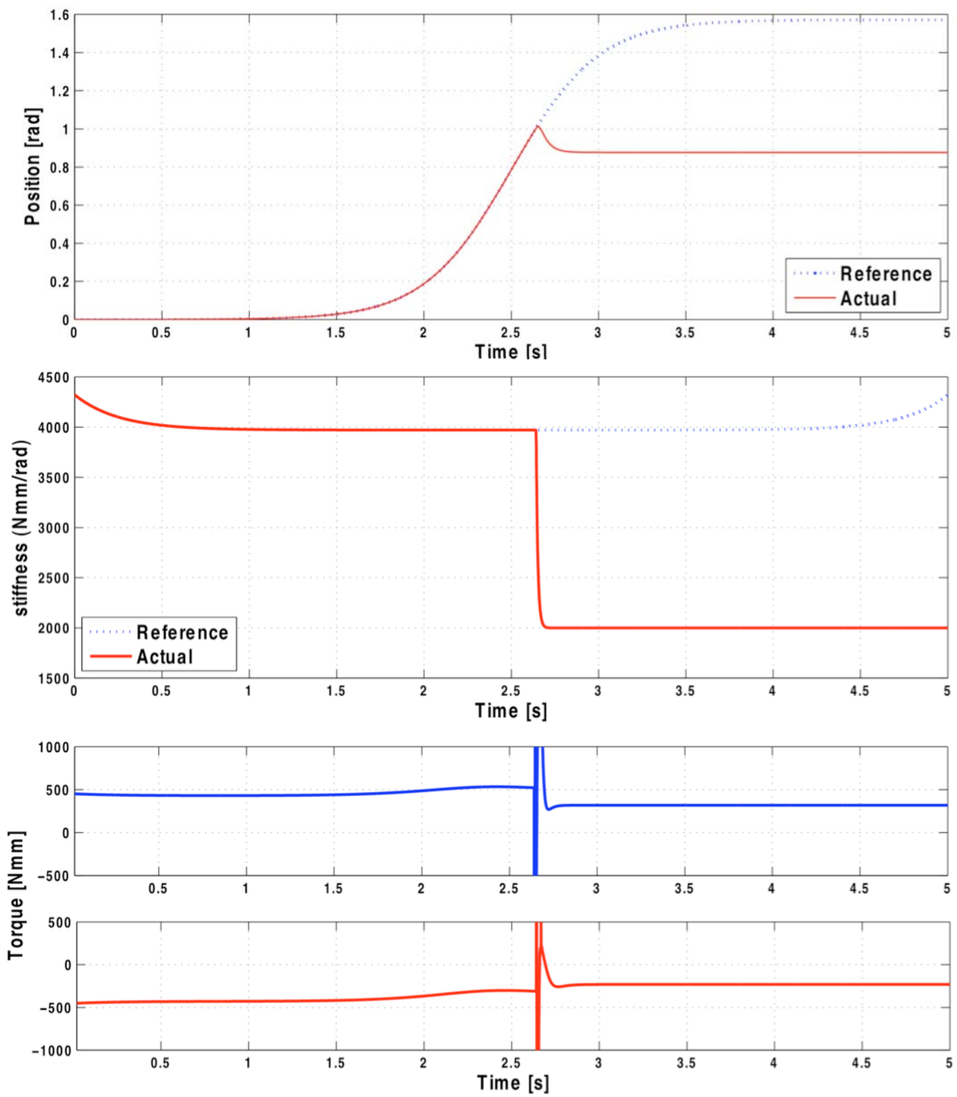
  - the value of the stiffness reference may be modified as well
- however, we do not obtain in this way a physically meaning torque reaction

Collision detection and reaction

link motion
bounces back and smoothly stops after impact

stiffness
reference is dropped down
upon detection

left/right
command torques

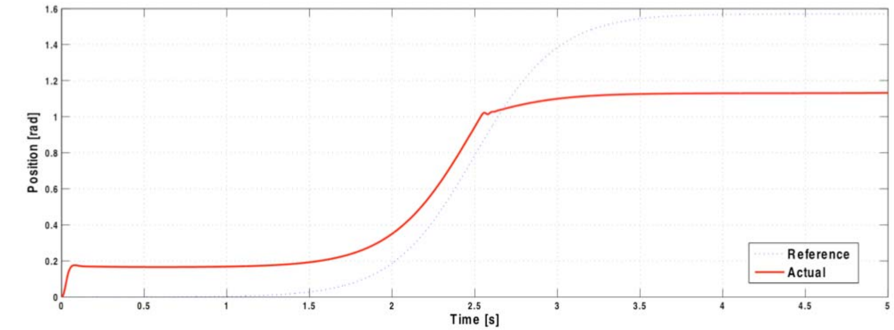


... one out of a set of possible
post-impact
strategies

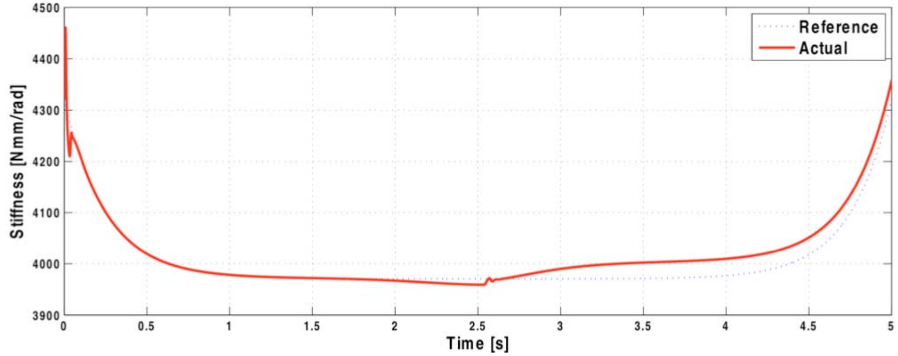
at the final equilibrium
sum of torques
= gravity load

Perturbed conditions:
tracking, collision, and no reaction

30% variation
of inertias
+5% and -5%
in spring stiffness

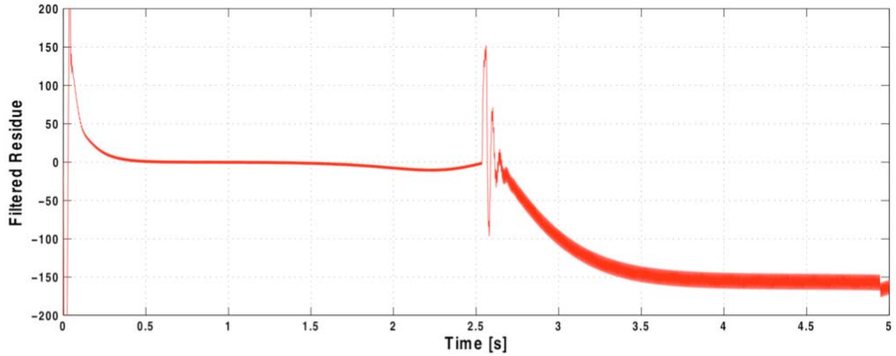


link motion
ahead of reference
(add integral action!)
before impact



execution of
stiffness
trajectory
still reasonable

residual
(without reaction)



collision can
still be
detected
(modifying
threshold)

Multi-dof robots with VSA

- all previous developments apply "verbatim" also to N-dof VSA robots having a dynamic model *of the form*

$$B\ddot{\theta}_1 + D\dot{\theta}_1 + 2\tau_{J1} = \tau_1$$
$$B\ddot{\theta}_2 + D\dot{\theta}_2 + 2\tau_{J2} = \tau_2$$
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = 2(\tau_{J1} + \tau_{J1}) + \tau_K$$

  where all variables are N-dimensional vectors
- e.g., the collision isolation method is defined as

$$p_{\text{sum}} = B(\dot{\theta}_1 + \dot{\theta}_2) + M(q)\dot{q}$$

$$r = K_I\left(p_{\text{sum}} - \int_0^t \left(r + C^T(q,\dot{q})\dot{q} - g(q) + \tau_1 + \tau_2 - D(\dot{\theta}_1 + \dot{\theta}_2)\right) ds\right)$$

$$\Rightarrow \quad \dot{r} = K_I(\tau_K - r) \qquad K_I > 0$$

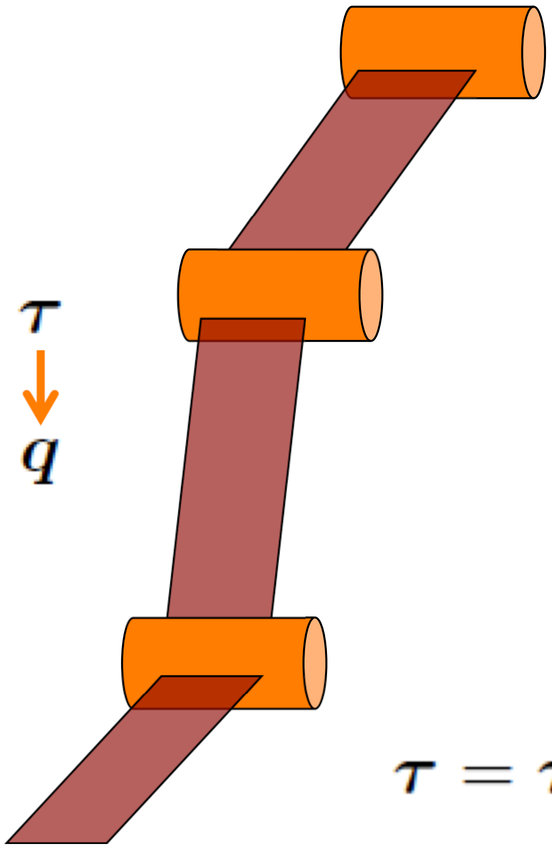
Gravity cancellation

- perfect cancellation of gravity from the dynamics of a robot with flexible transmissions by feedback
  - the robot should behave as in the absence of gravity
- at least, some relevant output variables should match their behavior under no gravity
  - both in static and dynamic conditions
  - applicability to 1-dof and multi-dof devices

- zero-gravity field for unbiased robot reaction to collisions
  - useful for the design of torque-based reflex laws
- controllers for regulation tasks that get rid of gravity
  - easier tuning of PD control gains
  - no strictly positive lower bound on gains and joint stiffness

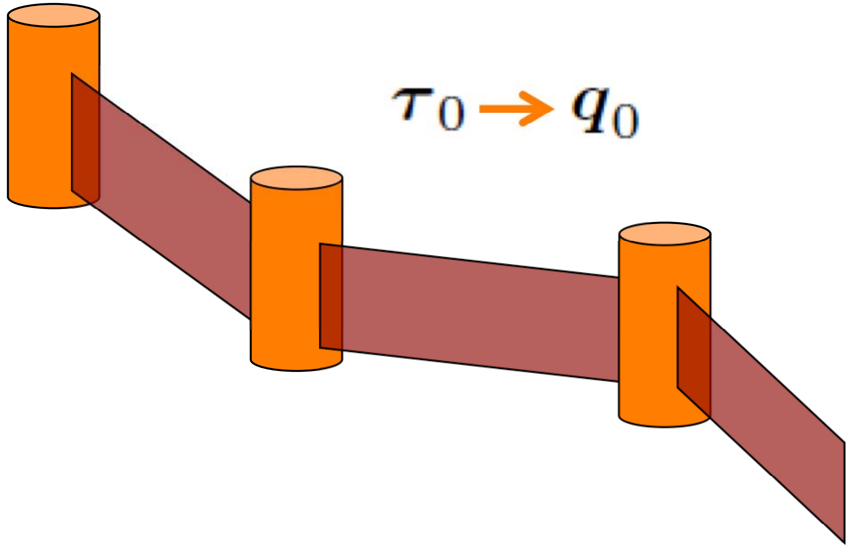
Gravity cancellation in
Rigid robots

trivial, due to collocation and full actuation



$\tau \downarrow q$

$\vec{g}_0$



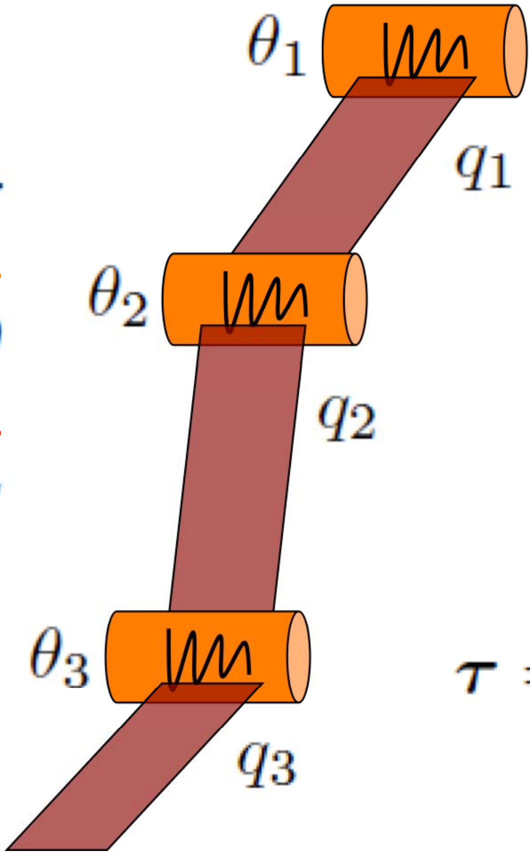
$\tau_0 \rightarrow q_0$

$$\tau = \tau_g + \tau_0 \qquad \boxed{\tau_g = g(q)} \qquad \boxed{q \equiv q_0}$$

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) = \tau \quad \Rightarrow \quad M(q)\ddot{q} + c(q,\dot{q}) = \tau_0$$

Gravity cancellation in

Flexible joint robots

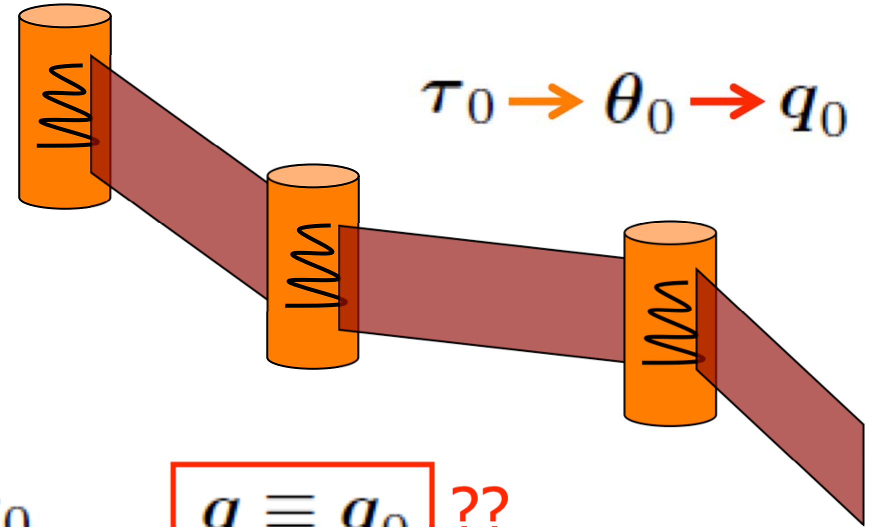


$\boldsymbol{\tau}$ ↓ $\boldsymbol{\theta}$ ↓ $\boldsymbol{q}$

constant
(but also nonlinear)
joint stiffness

$\vec{g}_0$

non-collocation
of input torque and output



$\boldsymbol{\tau}_0 \rightarrow \boldsymbol{\theta}_0 \rightarrow \boldsymbol{q}_0$

??

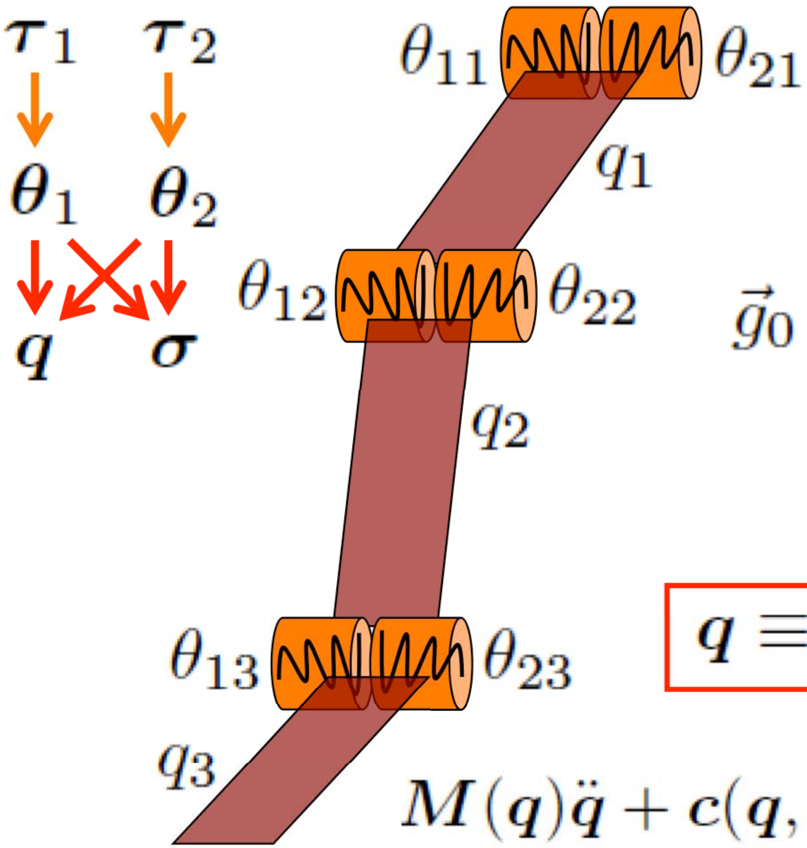
$$\boldsymbol{\tau} = \boldsymbol{\tau}_g + \boldsymbol{\tau}_0$$

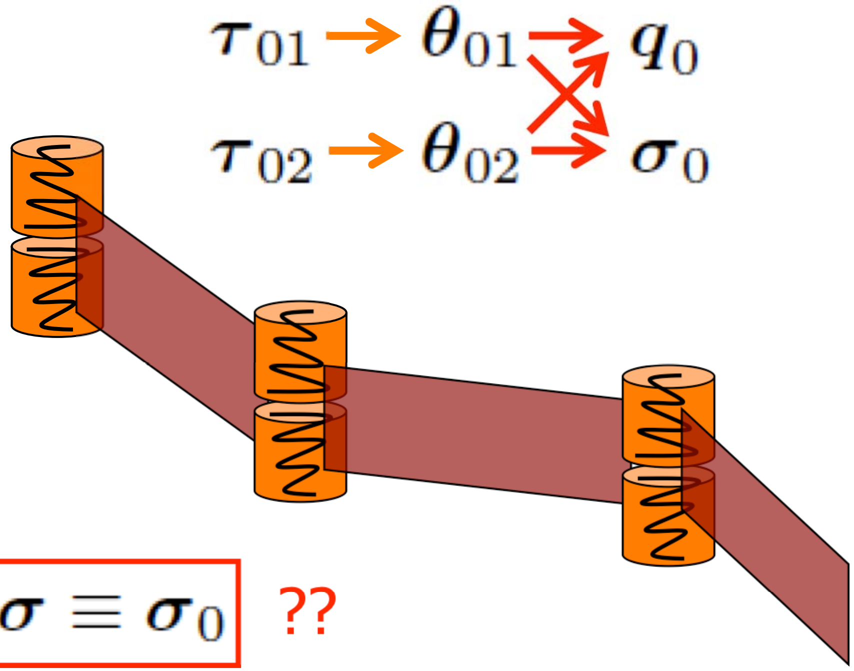
$$\boldsymbol{q} \equiv \boldsymbol{q}_0$$??

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}(\boldsymbol{q} - \boldsymbol{\theta}) = \boldsymbol{0}$$

$$\boldsymbol{B}\ddot{\boldsymbol{\theta}} + \boldsymbol{K}(\boldsymbol{\theta} - \boldsymbol{q}) = \boldsymbol{\tau}$$

Gravity cancellation in
Variable Stiffness Actuation robots





$q \equiv q_0$ $\sigma \equiv \sigma_0$??

$$\phi_i = q - \theta_i \quad i = 1,2$$

$$\begin{aligned} M(q)\ddot{q} + c(q,\dot{q}) + g(q) + \tau_{e1}(\phi_1) + \tau_{e2}(\phi_2) &= 0 \\ B_1\ddot{\theta}_1 - \tau_{e1}(\phi_1) &= \tau_1 \\ B_2\ddot{\theta}_2 - \tau_{e2}(\phi_2) &= \tau_2 \end{aligned}$$

Feedback equivalence

$$u = \tilde{\alpha}(x) + \tilde{\beta}(x) u_0$$
$$x_0 = T_0^{-1}(T(x)) = \tilde{T}(x)$$

$$\dot{x} = f(x) + G(x)u$$

gravity-loaded system

$$\dot{x}_0 = f_0(x_0) + G_0(x_0)u_0$$

gravity-free system

feedback transformations
static state feedback
+
change of coordinates
(both invertible)

$$u = \alpha(x) + \beta(x)v$$
$$z = T(x)$$

$$u_0 = \alpha_0(x_0) + \beta_0(x_0)v$$
$$z = T_0(x_0)$$

$$\dot{z} = Az + Bv$$

linear, controllable system

z ≈ linearizing outputs

Flexible robots are feedback linearizable!

- robots with elastic joints

DLR LWR-III
(Harmonic Drives)

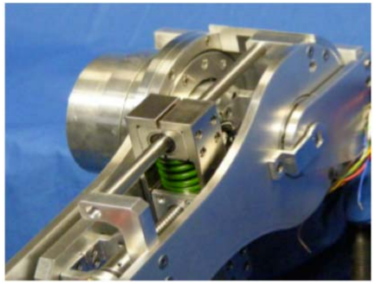
Dexter 8R arm
(cables and pulleys)

linearizing outputs =
link position (relative degree 4)

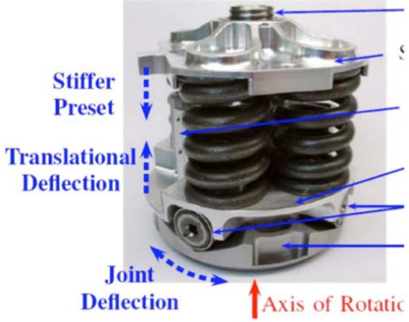
- robots with VSA

VSA-II
antagonistic

VSA-HD



IIT AwAS



DLR-VS joint

linearizing outputs =
link position (relative degree 4) +
device stiffness (relative degree 2)

Gravity cancellation
in robots with elastic joints

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) + D_q\dot{q} + K(q-\theta) = 0$$
$$B\ddot{\theta} + D_\theta\dot{\theta} + K(\theta - q) = \tau$$

$$q(t) \equiv q_0(t) \quad \forall t \geq 0 \qquad \tau = \tau_g + \tau_0$$

$$\Rightarrow \quad \boxed{\tau_g = g(q) + D_\theta K^{-1}\dot{g}(q) + BK^{-1}\ddot{g}(q)}$$

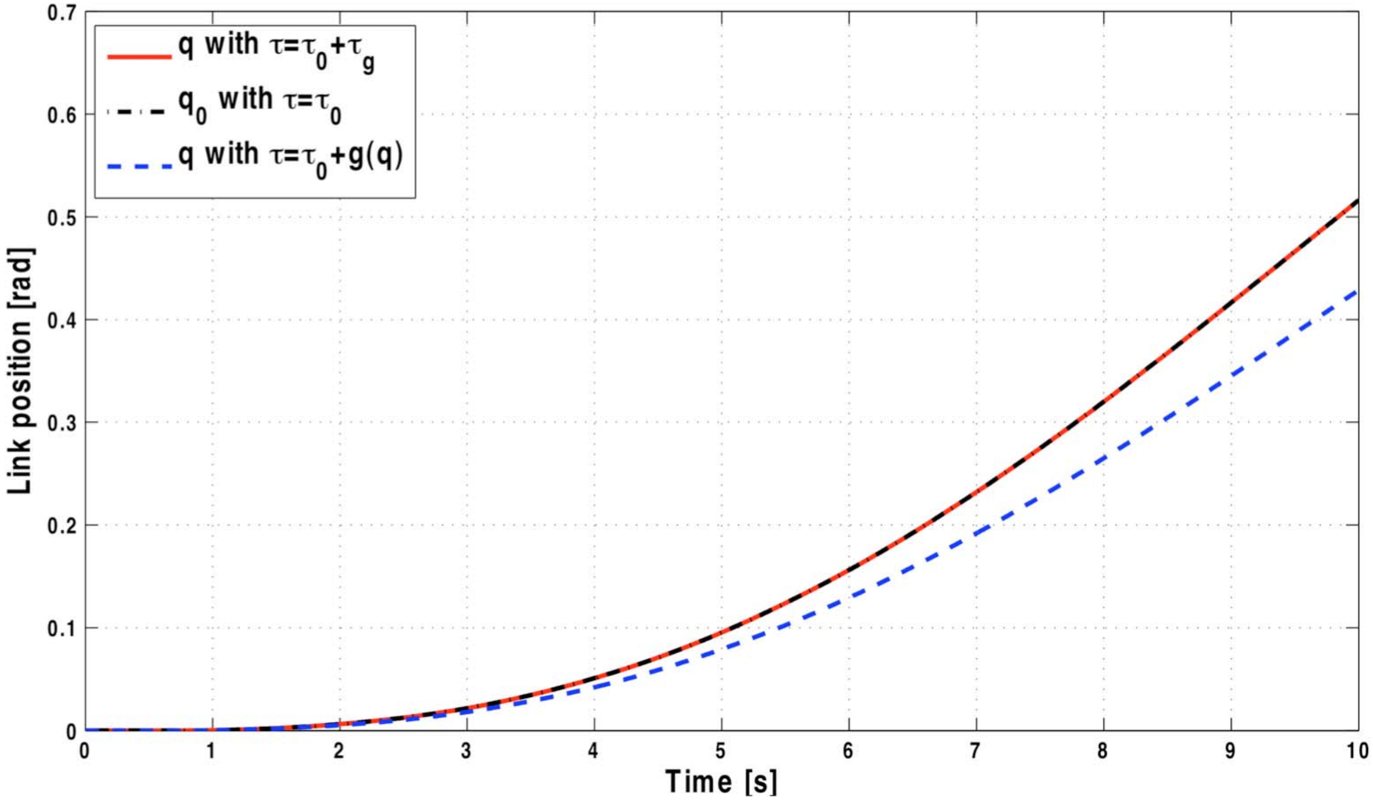
$$\dot{g}(q) = \frac{\partial g(q)}{\partial q}\,\dot{q}$$
$$\ddot{g}(q) = \frac{\partial g(q)}{\partial q} M^{-1}(q)\big(K(\theta-q) - c(q,\dot{q}) - g(q) - D_q\dot{q}\big) + \sum_{i=1}^{n} \frac{\partial^2 g(q)}{\partial q\,\partial q_i}\,\dot{q}\,\dot{q}_i$$

requires full state feedback

Numerical results
gravity cancellation for 1-dof elastic joint

$$\tau_g = mdg_0 \left\{ \left(1 - \frac{B}{K}\dot{q}^2\right)\sin q - \frac{B}{M}\frac{mdg_0}{K}\sin q \cos q + \frac{MD_\theta - BD_q}{KM}\dot{q}\cos q + \frac{B}{M}(\theta - q)\cos q \right\}$$

$$\boxed{\tau_0 = \sin 0.1\pi t} \qquad g(q) = mdg_0 \sin q$$



exact reproduction of same link behavior with and without gravity

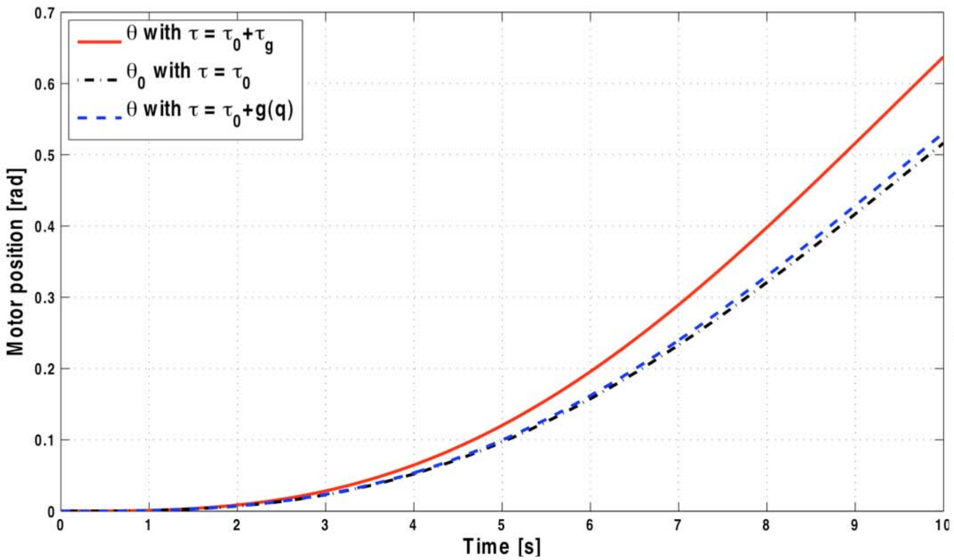
Numerical results

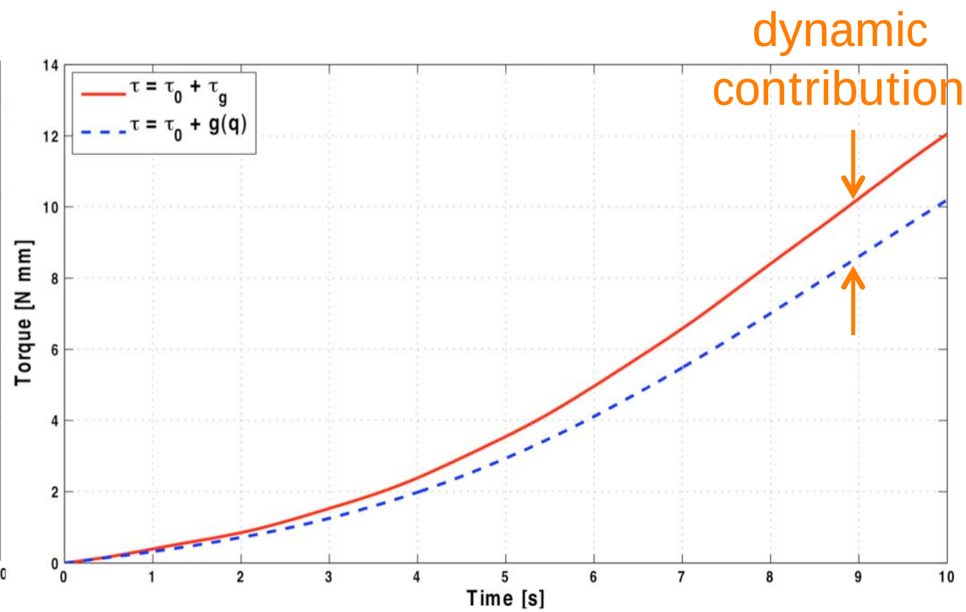
gravity cancellation for 1-dof elastic joint

$$\tau_g = mdg_0 \left\{ \left(1 - \frac{B}{K}\dot{q}^2\right)\sin q - \frac{B}{M}\frac{mdg_0}{K}\sin q \cos q + \frac{MD_\theta - BD_q}{KM}\dot{q}\cos q + \frac{B}{M}(\theta - q)\cos q \right\}$$

$$\tau_0 = \sin 0.1\pi t \qquad g(q) = mdg_0 \sin q$$

$$\boldsymbol{\theta = \theta_0 + K^{-1} g(q)}$$





different motor behavior with and without gravity

torque comparison w.r.t. static gravity compensation

Gravity cancellation

in robots with variable stiffness actuation – 1-dof case

symmetric, antagonistic arrangement

$$M\ddot{q} + D_q\dot{q} + g(q) + \tau_e(\phi_1) + \tau_e(\phi_2) = 0$$
$$B\ddot{\theta}_1 + D_\theta\dot{\theta}_1 - \tau_e(\phi_1) = \tau_1$$
$$B\ddot{\theta}_2 + D_\theta\dot{\theta}_2 - \tau_e(\phi_2) = \tau_2$$

$$\phi_i = q - \theta_i \quad i = 1, 2$$

total device stiffness

$$\sigma_t(\phi_1, \phi_2) = \frac{\partial(\tau_e(\phi_1) + \tau_e(\phi_2))}{\partial q} = \sigma(\phi_1) + \sigma(\phi_2)$$

$q(t) \equiv q_0(t)$

AND

$\sigma_t(t) \equiv \sigma_{t0}(t)$

$\forall t \geq 0$

⇒

$$\mathcal{A}(\phi_1, \phi_2) = \begin{pmatrix} \sigma(\phi_1) & \sigma(\phi_2) \\ \dfrac{\partial\sigma(\phi_1)}{\partial\phi_1} & \dfrac{\partial\sigma(\phi_2)}{\partial\phi_2} \end{pmatrix}$$

⇒ generically non-singular for $\theta_1 \neq \theta_2$

Gravity cancellation

in robots with variable stiffness joints – 1-dof case

$$\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} D_\theta \dot{\theta}_1 - \tau_e(\phi_1) \\ D_\theta \dot{\theta}_2 - \tau_e(\phi_2) \end{pmatrix} + \boldsymbol{\mathcal{A}}^{-1}(\phi_1, \phi_2) \cdot$$

$$\Bigg\{ \boldsymbol{\mathcal{A}}(\phi_{10}, \phi_{20}) \left(\begin{pmatrix} \tau_{10} \\ \tau_{20} \end{pmatrix} + \begin{pmatrix} \tau_e(\phi_{10}) - D_\theta \dot{\theta}_{10} \\ \tau_e(\phi_{20}) - D_\theta \dot{\theta}_{20} \end{pmatrix} \right)$$

$$+ B \begin{pmatrix} \ddot{g}(q) + \sum_{i=1}^{2} \left(\frac{\partial \sigma(\phi_i)}{\partial \phi_i} \dot{\phi}_i^2 - \frac{\partial \sigma(\phi_{i0})}{\partial \phi_{i0}} \dot{\phi}_{i0}^2 \right) \\ \sum_{i=1}^{2} \left(\frac{\partial \sigma(\phi_i)}{\partial \phi_i} - \frac{\partial \sigma(\phi_{i0})}{\partial \phi_{i0}} \right) \ddot{q} \\ + \sum_{i=1}^{2} \left(\frac{\partial^2 \sigma(\phi_i)}{\partial \phi_i^2} \dot{\phi}_i^2 - \frac{\partial^2 \sigma(\phi_{i0})}{\partial \phi_{i0}^2} \dot{\phi}_{i0}^2 \right) \end{pmatrix} \Bigg\}$$

ϕ_{10} ϕ_{20}

numerically solve
(except for special cases)

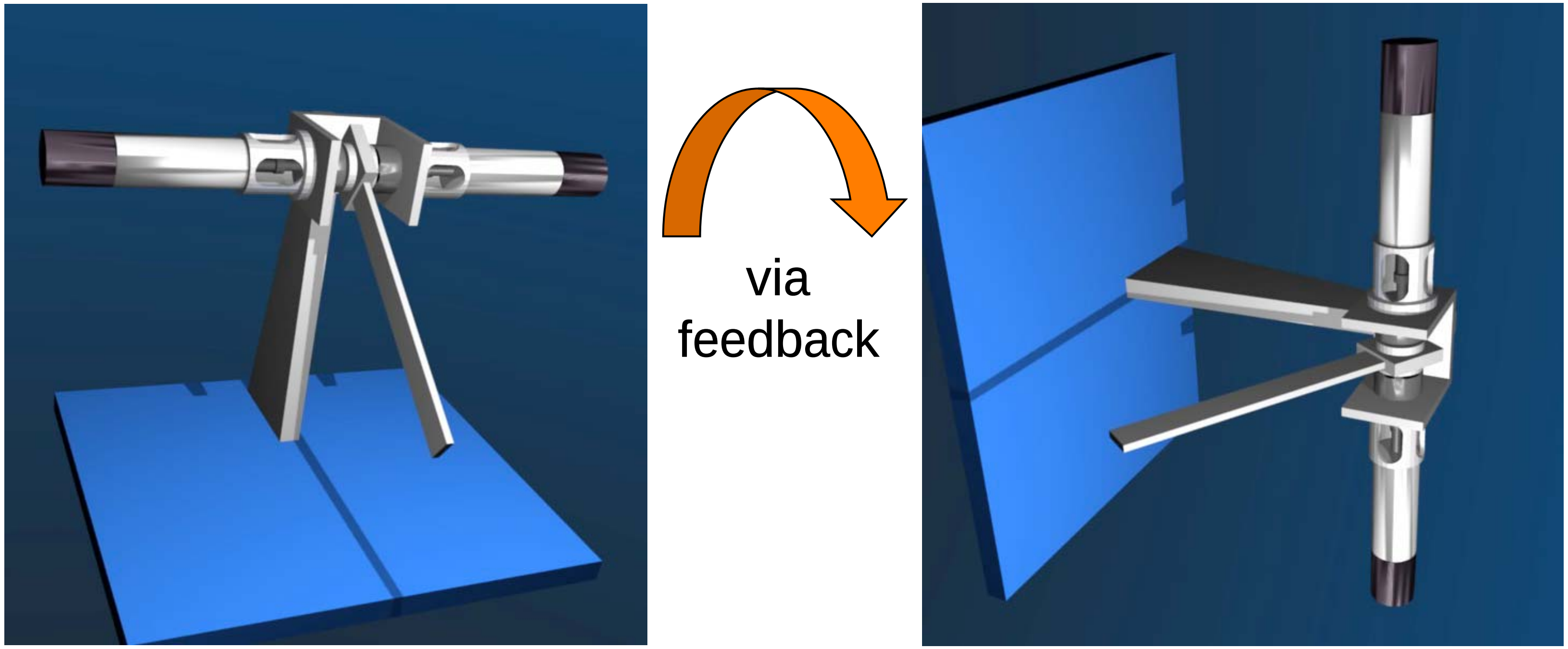
$$\begin{aligned} \tau_e(\phi_{10}) + \tau_e(\phi_{20}) &= -M\ddot{q} - D_q \dot{q} = a_1(q, \theta_1, \theta_2) \\ \sigma(\phi_{10}) + \sigma(\phi_{20}) &= \sigma_t(q, \theta_1, \theta_2) \end{aligned}$$

Gravity cancellation

for VSA-II driving a single link

- bi-directional antagonistic VSA

$$\tau_e(\phi_i) = 2K\,\beta(\phi_i)\,\frac{\partial \beta(\phi_i)}{\partial \phi_i} \qquad \beta(\phi_i) = \arcsin\left(C\,\sin\left(\frac{\phi_i}{2}\right)\right) - \frac{\phi_i}{2}$$

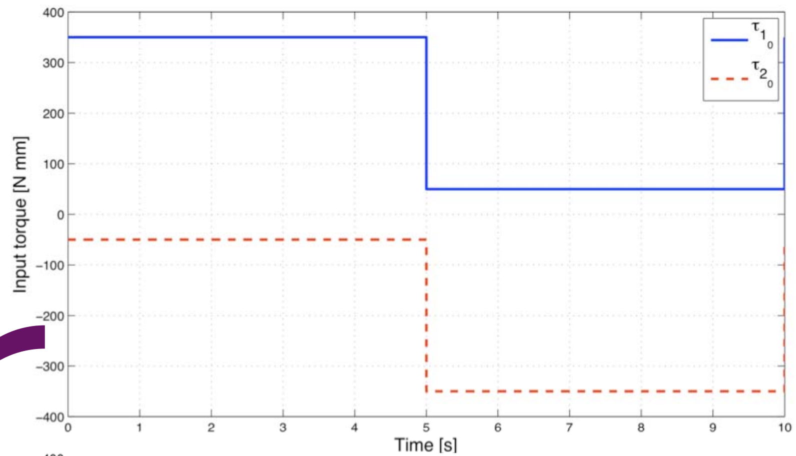


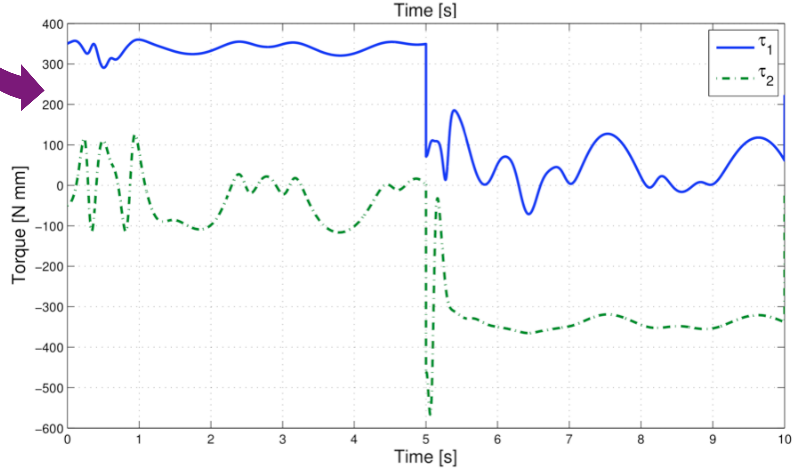
under gravity ... ... gravity cancelled

Numerical results

gravity cancellation on the VSA-II joint

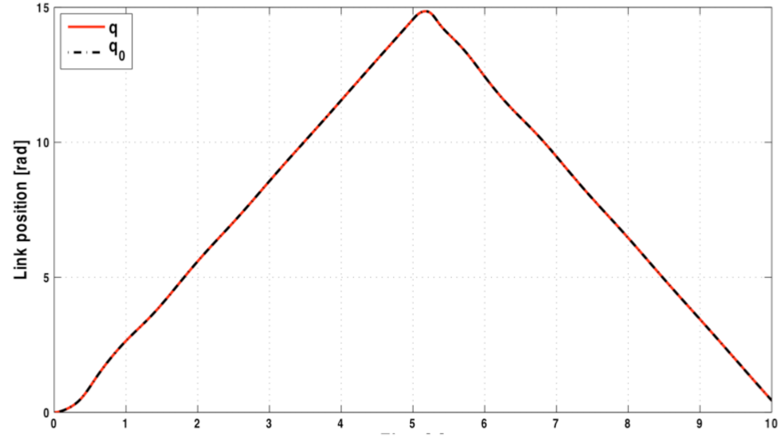
open-loop torques in absence of gravity

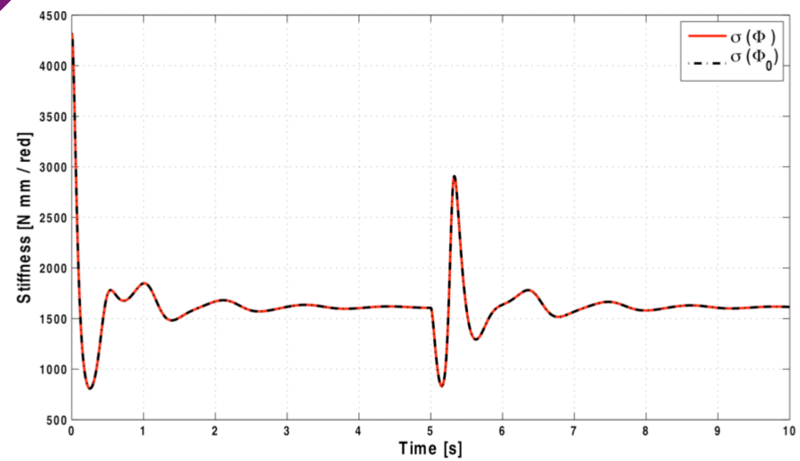




applied torques for gravity cancellation

exact reproduction of link behavior





exact reproduction of stiffness behavior

A global PD-type regulator
for robots with elastic joints

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) + K(q-\theta) = 0$$

$$B\ddot{\theta} + K(\theta - q) = \tau$$

$$\tau = \tau_g + \tau_0$$

$$\tau_g = g(q) + BK^{-1}\ddot{g}(q)$$

$$\boxed{\tau_0 =} K_P(\theta_{d0} - \theta_0) - K_D\dot{\theta}_0$$

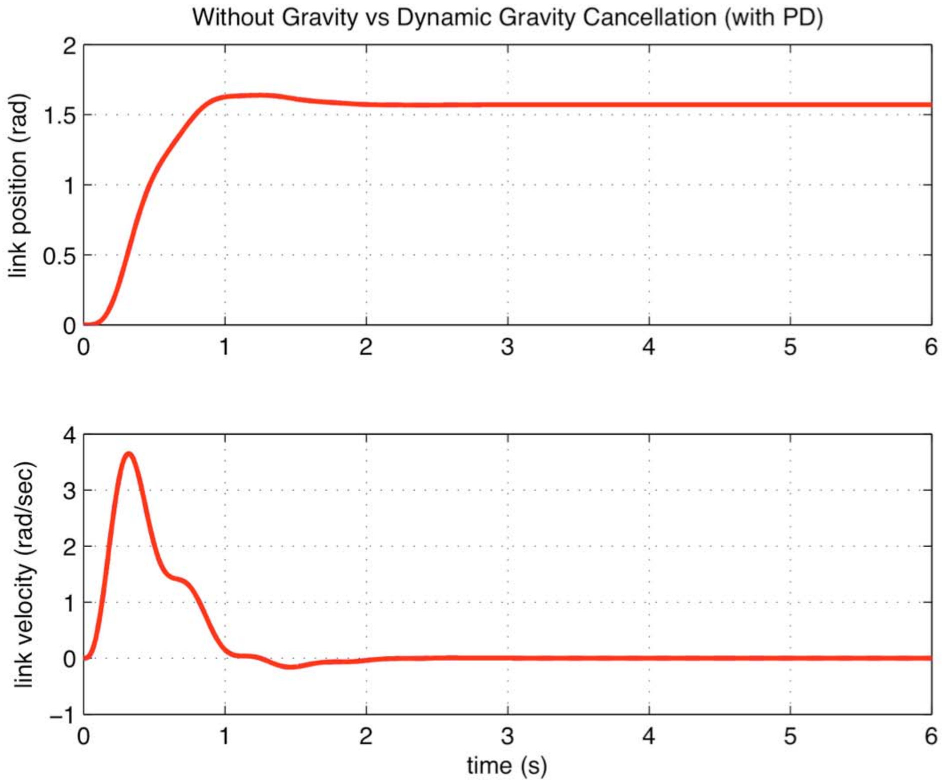
$$= \boxed{K_P(q_d - \theta + K^{-1}g(q)) - K_D(\dot{\theta} - K^{-1}\dot{g}(q))}$$

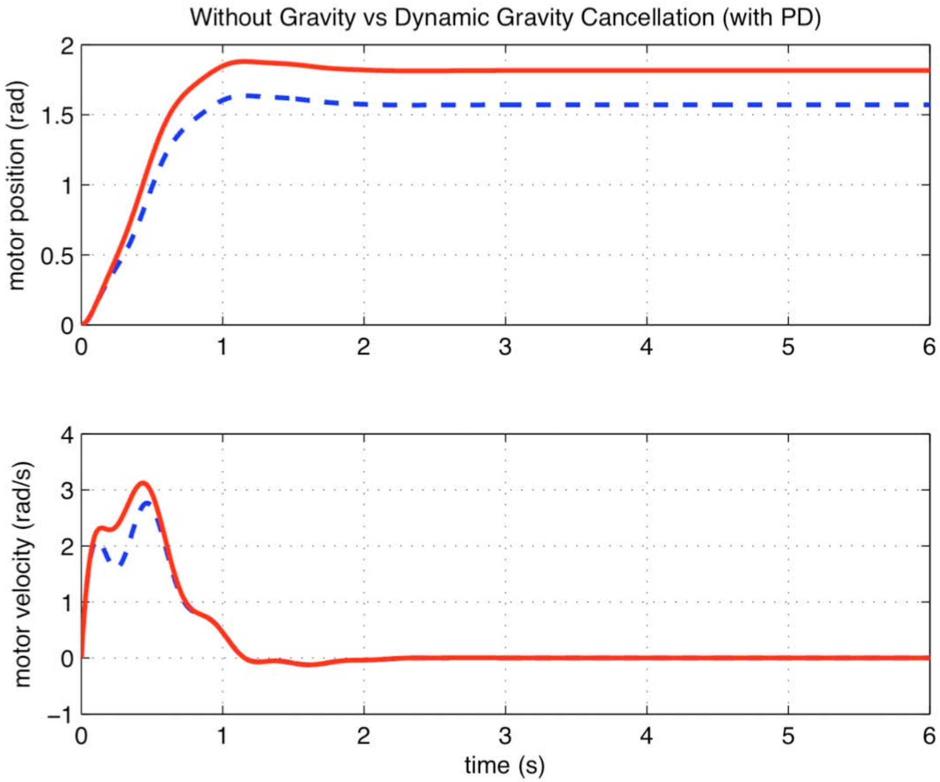
motor PD law on the equivalent system

Global asymptotic stability can be shown using a Lyapunov analysis under “minimal” sufficient conditions

$\boxed{K_P > 0}$ $\boxed{K > 0}$ i.e., **no** strictly positive lower bounds and $K_D > 0$

Numerical results
regulation of a one-link arm with EJ under gravity





identical dynamic behavior of link
in gravity-loaded system
under PD + gravity cancellation
and in gravity-free system under same PD

still a different motor behavior

Stiffness estimation problem

- in VSA/VIA robots, stiffness is intrinsically nonlinear and possibly time-varying
- advanced control laws are based on a stiffness model, i.e. a complex and uncertain function of joint deformations
  - fundamental robustness issue: stiffness output to be controlled is not directly measurable!
  - need for on-line estimation of stiffness
- multiple approaches
  - with or without joint/external torque sensing
  - assuming or not that other dynamic parameters are known
    - variable impedance estimation
  - estimation of total device stiffness (external: what we really want) or of stiffness of single transmissions (internal: needs then the "kinematics" of couplings, but is decentralized to the motors)

Stiffness estimation - 1

- consider a "single" nonlinear flexible transmission

$$M\ddot{q} + D_q\dot{q} + \tau_e(\phi) + g(q) = \tau_k$$
$$B\ddot{\theta} + D_\theta\dot{\theta} - \tau_e(\phi) = \tau$$

$\Rightarrow$

$$\sigma(\phi) = \frac{\partial \tau_e(\phi)}{\partial q} = \frac{\partial \tau_e(\phi)}{\partial \phi} > 0$$
$$\phi = q - \theta$$

- define a residual as

$$r_{\tau_e} = K_{\tau_e}\left(p_\theta + D_\theta\theta - \int_0^t (\tau + r_{\tau_e})\, dt_1\right) \quad \text{with} \begin{cases} p_\theta = B\dot{\theta} \\ K_{\tau_e} > 0 \\ r_{\tau_e}(0) = 0 \end{cases}$$

$\Rightarrow$ $$\dot{r}_{\tau_e} = K_{\tau_e}(\tau_e - r_{\tau_e})$$

a (first-order) filtered estimate of the transmission torque!

- time-differentiation of this estimate is critical (especially at low deformation speed) --just as differentiating a joint torque measure!

Stiffness estimation - 2

- idea: use the residual to find a n-dim parameterized approximation of the transmission torque

$$\tau_e(\phi) \simeq f(\phi, \boldsymbol{\alpha}) \qquad \boldsymbol{\alpha} = (\alpha_1 \ \ldots \ \alpha_n)^T$$

typically (but not necessarily) in the linear format

$$f(\phi, \boldsymbol{\alpha}, n) = \sum_{h=1}^{n} f_h(\phi)\alpha_h = \boldsymbol{F}^T(\phi)\boldsymbol{\alpha}$$

- polynomial basis functions are chosen only of odd powers

$$\left.\begin{array}{r} \tau_e(0) = 0 \\ \tau_e(-\phi) = -\tau_e(\phi), \quad \forall \phi \end{array}\right\} \Rightarrow \quad f_h(\phi) = \phi^{2h-1}, \quad h = 1, \ldots, n$$

- a recursive on-line estimation $\widehat{\boldsymbol{\alpha}}$ of vector $\boldsymbol{\alpha}$ is set up

Stiffness estimation - 3

- Recursive Least Squares (RLS) residual-based solution

$$\hat{\alpha}(k) = \hat{\alpha}(k-1) + \Delta\hat{\alpha}(k)$$

$$\Delta\hat{\alpha}(k) = L(k)\left(r_{\tau_e}(k) - F^T(k)\hat{\alpha}(k-1)\right)$$

$$L(k) = \frac{P(k-1)F(k)}{1 + F^T(k)P(k-1)F(k)} \qquad F^T(k) = \left(\phi(k) \quad \phi^3(k) \quad \dots \quad \phi^{2n-1}(k)\right)$$

covariance matrix

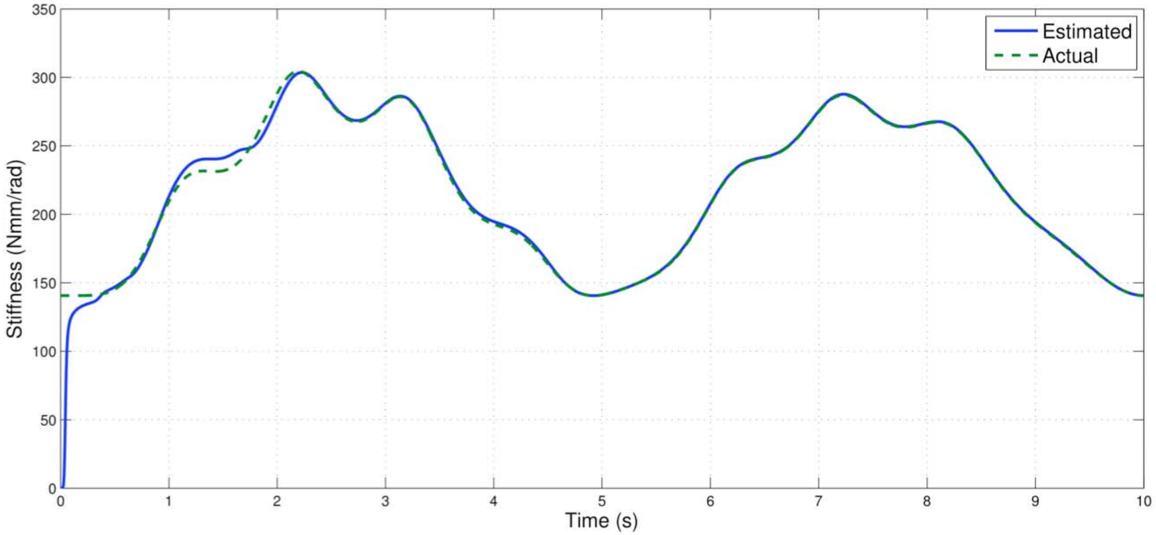
$$P(k) = \left(I - L(k)F^T(k)\right)P(k-1)$$

(since $\sigma(\phi) = \dfrac{\partial f(\phi, \alpha)}{\partial \phi}$)

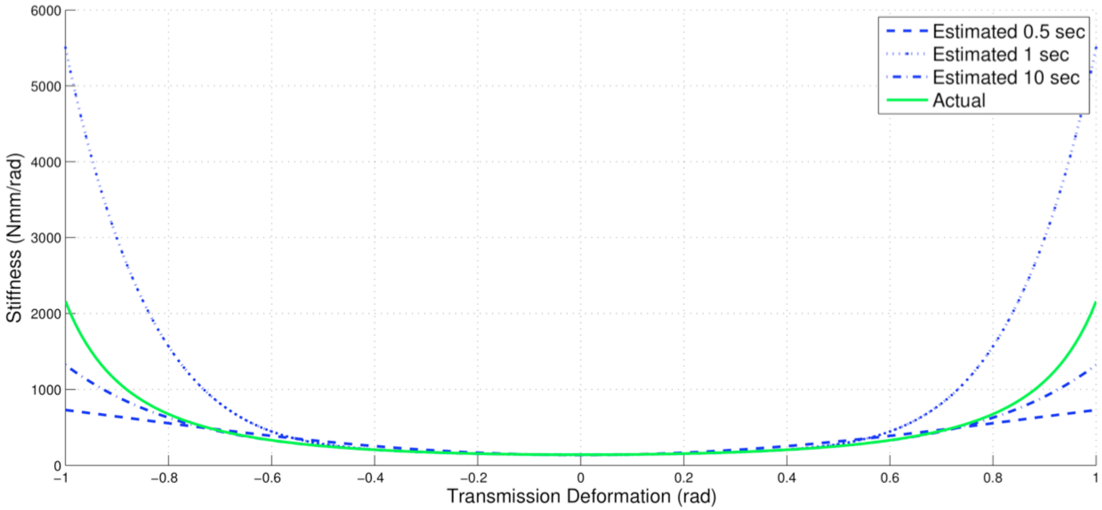
$$\hat{\sigma}(k) = \frac{\partial f(\phi, \hat{\alpha}(k), n)}{\partial \phi} = \sum_{h=1}^{n} \frac{\partial f_h(\phi)}{\partial \phi}\hat{\alpha}_h(k)$$

Stiffness estimation results
for VSA-II

evolution
of estimation
in time

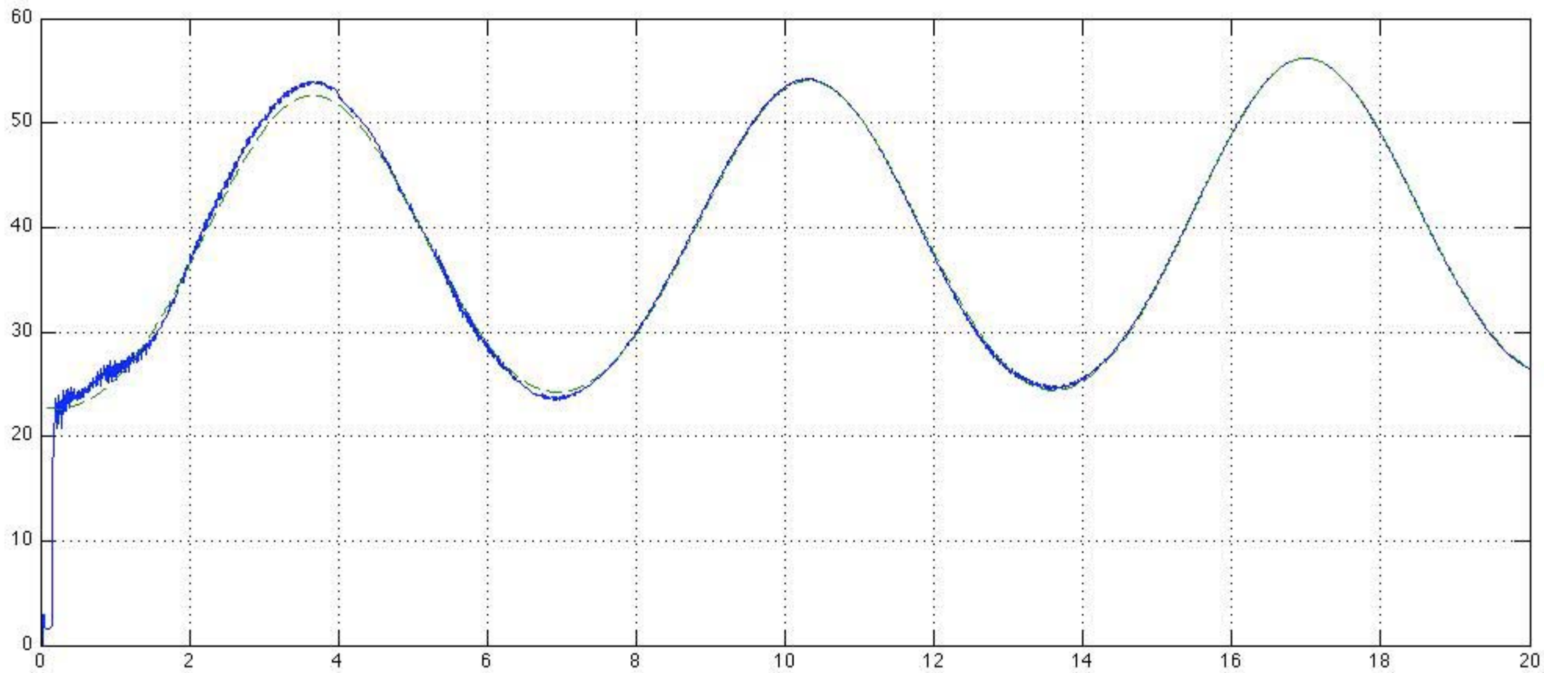


stiffness
profile
estimation



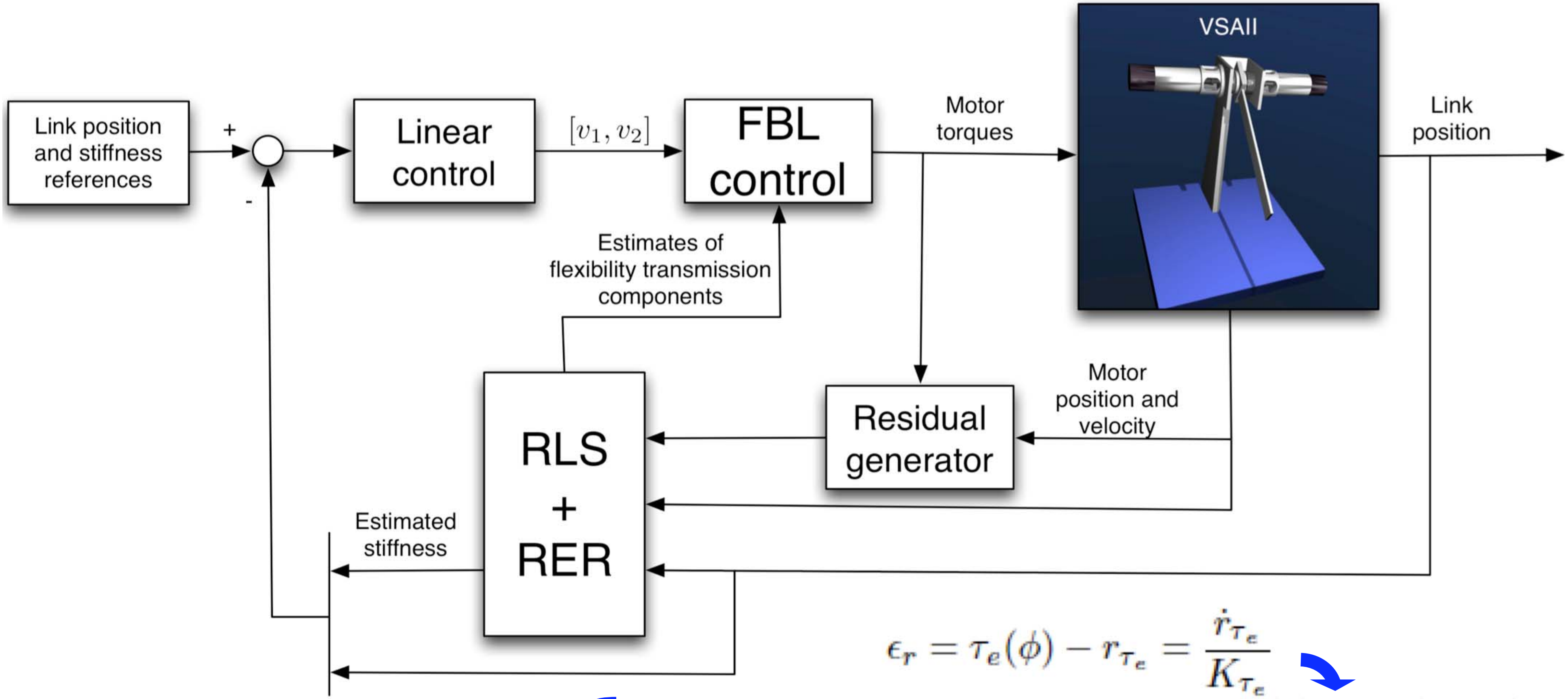
Stiffness estimation results
weighted method suitably adapted for IIT AwAS

- including discretization (T=5 msec), encoder quantization (4096 ppr)



using experimental data from the AwAS-I

Feedback linearization using stiffness estimation



$$\epsilon_r = \tau_e(\phi) - r_{\tau_e} = \frac{\dot{r}_{\tau_e}}{K_{\tau_e}}$$

RER = Residual Error Recovery to improve update in RLS

using backward differences for the residual derivative

$$\epsilon_r(k) = \frac{r_{\tau_e}(k) - r_{\tau_e}(k-1)}{T K_{\tau_e}}$$

$$\Delta\hat{\alpha}(k) = L(k)\left(r_{\tau_e}(k) + \epsilon_r(k) - F^T(k)\hat{\alpha}(k-1)\right)$$

Estimated quantities needed in FBL control law of VSA-II

deformation torque $\hat{\tau}_e(\phi) = f(\phi, \hat{\alpha}, n) = \sum_{h=1}^{n} \phi^{2h-1}\,\hat{\alpha}_h$

stiffness $\hat{\sigma}(\phi) = \sum_{h=1}^{n} (2h-1)\phi^{2h-2}\,\hat{\alpha}_h$

$\hat{\alpha}$ ⇒

stiffness derivative $\dfrac{\partial\hat{\sigma}(\phi)}{\partial\phi} = \sum_{h=2}^{n} (4h^4 - 6h + 2)\phi^{2h-3}\,\hat{\alpha}_h$

stiffness second derivative $\dfrac{\partial^2\hat{\sigma}(\phi)}{\partial\phi^2} = \sum_{h=2}^{n} (8h^5 - 24h^2 + 22h - 6)\phi^{2h-4}\,\hat{\alpha}_h$

dropping index $i = 1, 2$ for the two transmissions

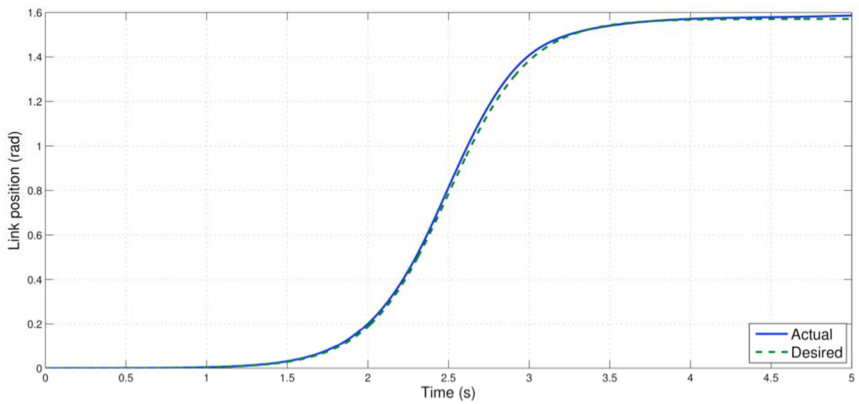
to be ⇓ used in

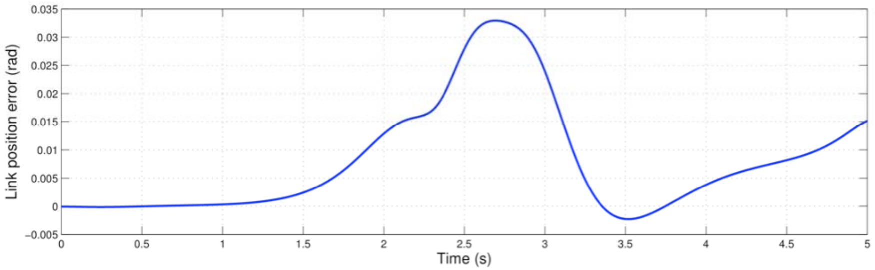
$$\begin{pmatrix}\tau_1\\ \tau_2\end{pmatrix} = A^{-1}(x)\left(\begin{pmatrix}v_1\\ v_2\end{pmatrix} - b(x)\right)$$

see slide #33

Tracking results
with on-line stiffness estimation

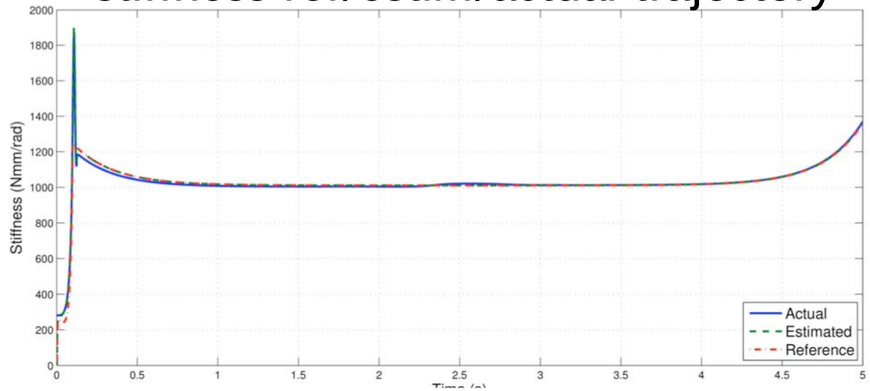
link position desired/actual trajectory

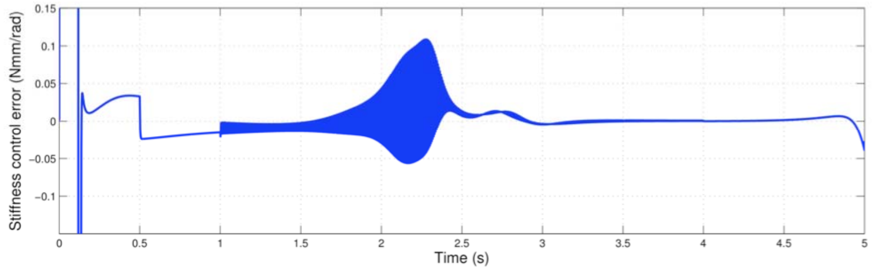




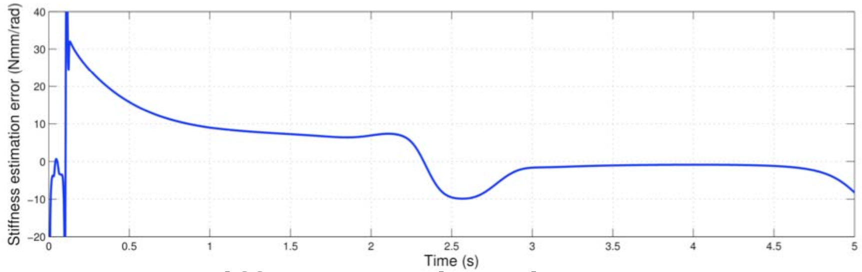
link position error

stiffness ref/estim/actual trajectory





reference-estimated stiffness (control) error



stiffness estimation error

References

- A. De Luca, R. Mattone, "Sensorless robot collision detection and hybrid force/motion control," *IEEE ICRA*, Barcelona, pp. 999-1004, 2005
- A. De Luca, A. Albu-Schäffer, S. Haddadin, G. Hirzinger, "Collision detection and safe reaction with the DLR-III lightweight manipulator arm," *IEEE/RSJ IROS*, Beijing, pp. 1623-1630, 2006
- G. Palli, C. Melchiorri, A. De Luca, "On the feedback linearization of robots with variable joint stiffness," *IEEE ICRA*, Pasadena, pp. 1753-1759, 2008
- S. Haddadin, A. Albu-Schäffer, A. De Luca, G. Hirzinger, "Collision detection and reaction: A contribution to safe physical human-robot interaction," *IEEE/RSJ IROS*, Nice, pp. 3356-3363, 2008
- A. De Luca, F. Flacco, A. Bicchi, R. Schiavi, "Nonlinear decoupled motion-stiffness control and collision detection/reaction for the VSA-II variable stiffness device," *IEEE/RSJ IROS*, St. Louis, pp. 5487-5494, 2009
- A. De Luca, F. Flacco, "Dynamic gravity cancellation in robots with flexible transmissions," *49th IEEE CDC*, Atlanta, pp. 288-295, 2010
- A. De Luca, F. Flacco, "A PD-type regulator with exact gravity cancellation for robots with flexible joints," *IEEE ICRA*, Shanghai, 2011
- F. Flacco, A. De Luca, "Residual-based stiffness estimation in robots with flexible transmissions," *IEEE ICRA*, Shanghai, 2011
- F. Flacco, A. De Luca, "Stiffness estimation and nonlinear control of robots with variable stiffness actuation," *18th IFAC World Congress*, Milano, 2011