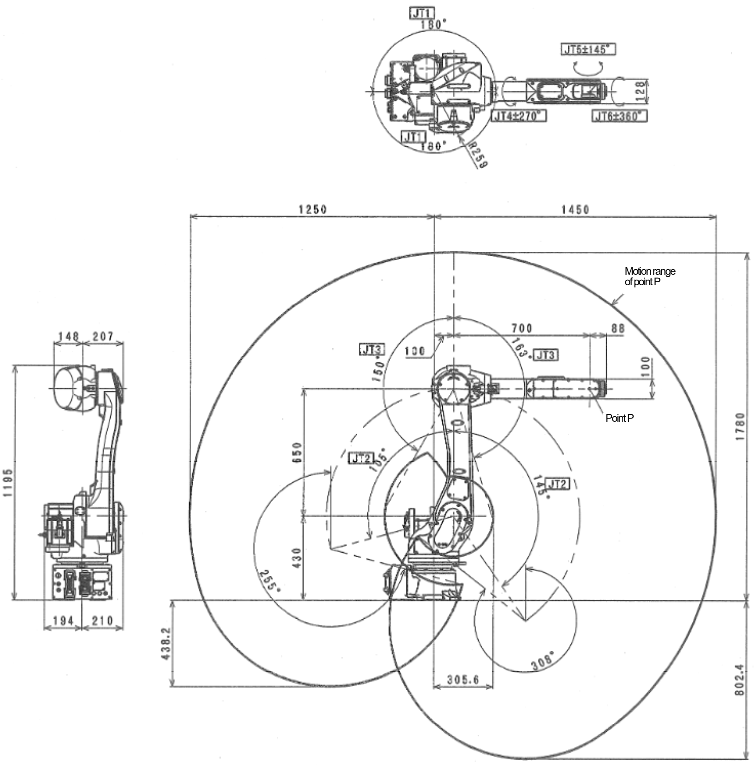
Robotics 1

September 19, 2024

Exercise 1

Figure 1 is taken from the data sheet of the Kawasaki RS010N, a robot with 6 revolute joints and a spherical wrist. Assign the link frames to this robot and fill in the associated table of parameters according to the Denavit–Hartenberg (DH) convention (use the attached sheet). Frame 0 is placed on the ground and frame 6 at the end of the final flange. Provide the numerical values of all parameters, including those of the joint variables θ_i, $i = 1, \ldots, 6$, in the configuration shown.[1]



Model	Vertically articulated robot		
Degree of Freedom of Motion	6		
Motion Range and Maximum Speed	JT	Motion Range	Max. Speed
	1	±180°	250°/s
	2	+145° to -105°	250°/s
	3	+150° to -163°	215°/s
	4	±270°	365°/s
	5	±145°	380°/s
	6	±360°	700°/s

Figure 1: From the data sheet of the Kawasaki RS010N robot. Lengths are in [mm].

[1] Motion ranges in the data sheet do not necessarily correspond to the values assumed by the DH joint variables.

Exercise 2

With reference to the configuration shown in Fig. 1 for the Kawasaki robot, taking into account the (symmetric) limits of joint velocities given in the data sheet, compute numerically the following quantities (expressed in the base frame of the robot).

[a] The velocity $\boldsymbol{v}_P$ of Point P (the center of the spherical wrist) when $\dot{q}_1$, $\dot{q}_2$, and $\dot{q}_3$ assume their maximum absolute values and their signs are chosen so to have the largest possible norm $\|\boldsymbol{v}_P\|$.

[b] The angular velocity $\boldsymbol{\omega}_6^{[b]}$ of the last DH frame when choosing the joint velocities as in **[a]** and with the wrist joints frozen.

[c] The angular velocity $\boldsymbol{\omega}_6^{[c]}$ of the last DH frame when the first three joints are frozen and $\dot{q}_4$, $\dot{q}_5$, and $\dot{q}_6$ assume their maximum *positive* values, according to the direction of the $\boldsymbol{z}_i$ axes $(i = 3, 4, 5)$ in the chosen DH assignment.

Exercise 3

Joints 2 and 3 of the robot in Fig. 1 should move from rest to rest, in minimum time, and in a coordinated way, starting from the lower limit and reaching the upper limit of their respective motion ranges. Assuming that the maximum absolute accelerations of the two joints are

$$A_{max,2} = 5.5 \qquad A_{max,3} = 7 \qquad [\text{rad/s}^2],$$

determine the minimum feasible time T_{min} for the coordinated joint motion. Draw the profiles of the planned velocity and acceleration of the two joints (using radians and not degrees!).

Exercise 4

Assume that the robot in Fig. 1 mounts two optical encoders of the incremental type on the motor axes of joint 2 and 3, respectively with $N_2 = 4000$ and $N_3 = 2600$ pulses per turn (after electronic quadrature), while the reduction ratios of the corresponding transmission gears are $n_{r2} = 40$ and $n_{r3} = 20$. When the robot is in the shown configuration, an instantaneous displacement is commanded to Point P in the upward vertical direction $\boldsymbol{z}_0$. What is the minimum displacement Δz of Point P in this direction that can be measured by the encoders?

[180 minutes (3 hours); open books]

Solution

September 19, 2024

Exercise 1

A possible assignment of DH frames for the Kawasaki RS010N robot is shown in Fig. 2, with Tab. 1 containing the corresponding constant parameters, as well as the values of the joint variables $\boldsymbol{\theta}$ in the configuration shown. In the figure, the x_i axes are shown in blue, the y_i axes in green, and the z_i axes in red. Off-plane axes are not indicated, but they complete as usual a right-handed frame.

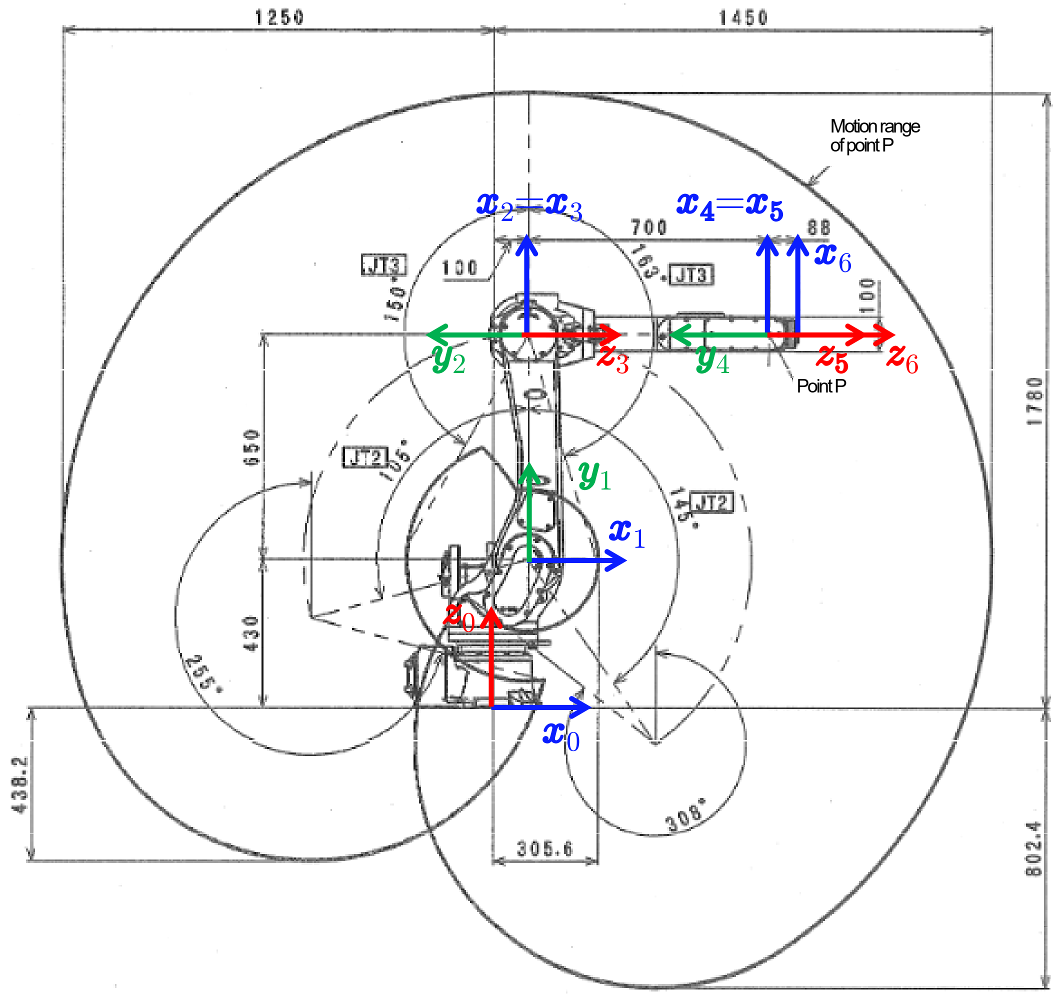


Figure 2: A possible assignment of DH frames for the Kawasaki RS010N robot.

i	α_i	a_i	d_i	θ_i
1	$\pi/2$	100	430	0
2	0	650	0	$\pi/2$
3	$\pi/2$	0	0	0
4	$-\pi/2$	0	700	0
5	$\pi/2$	0	0	0
6	0	0	88	0

Table 1: DH parameters tor the frame assignment in Fig. 2 (units in [rad] or [mm]).

Exercise 2

For all three cases, we need to compute the 6×6 geometric Jacobian of the robot which, in view of the presence of a spherical wrist, takes the form

$$\boldsymbol{J}(\boldsymbol{q}) = \begin{pmatrix} \boldsymbol{J}_L(\boldsymbol{q}) \\ \boldsymbol{J}_A(\boldsymbol{q}) \end{pmatrix} = \begin{pmatrix} \boldsymbol{J}_{L,P}(\boldsymbol{q}_b) & \boldsymbol{O} \\ \boldsymbol{J}_{A,b}(\boldsymbol{q}_b) & \boldsymbol{J}_{A,w}(\boldsymbol{q}) \end{pmatrix},$$

where we $\boldsymbol{q}_b = (q_1, q_2, q_3)$ is the vector of the first three (base) joints. Accordingly, $\boldsymbol{q}_w = (q_4, q_5, q_6)$ will be the vector of the last three (wrist) joints.

[**a**] For the velocity of Point P, only the first three joints matter. Moreover, to compute $\boldsymbol{v}_P$ one can equivalently use the 3×3 analytic Jacobian obtained by differentiation of the direct kinematics of Point P, which coincides with the origin of frame 4 in the DH convention. Using the parameters in Tab. 1, one has

$$\begin{aligned} \boldsymbol{p}_{P,hom}(\boldsymbol{q}) &= \begin{pmatrix} \boldsymbol{p}_P(\boldsymbol{q}) \\ 1 \end{pmatrix} = {}^0\boldsymbol{A}_1(q_1)\,{}^1\boldsymbol{A}_2(q_2)\,{}^2\boldsymbol{A}_3(q_3)\,{}^3\boldsymbol{A}_4(q_4) \begin{pmatrix} \boldsymbol{0} \\ 1 \end{pmatrix} \\ &= {}^0\boldsymbol{A}_1(q_1)\,{}^1\boldsymbol{A}_2(q_2)\,{}^2\boldsymbol{A}_3(q_3) \begin{pmatrix} 0 \\ 0 \\ d_4 \\ 1 \end{pmatrix} = \begin{pmatrix} c_1(a_1 + a_2 c_2 + d_4 s_{23}) \\ s_1(a_1 + a_2 c_2 + d_4 s_{23}) \\ d_1 + a_2 s_2 - d_4 c_{23} \\ 1 \end{pmatrix}. \end{aligned}$$

Thus, we obtain

$$\boldsymbol{J}_{L,P}(\boldsymbol{q}_b) = \frac{\partial \boldsymbol{p}_P}{\partial \boldsymbol{q}_b} = \begin{pmatrix} -s_1(a_1 + a_2 c_2 + d_4 s_{23}) & c_1(d_4 c_{23} - a_2 s_2) & d_4 c_1 c_{23} \\ c_1(a_1 + a_2 c_2 + d_4 s_{23}) & s_1(d_4 c_{23} - a_2 s_2) & d_4 s_1 c_{23} \\ 0 & a_2 c_2 + d_4 s_{23} & d_4 s_{23} \end{pmatrix}. \quad (1)$$

When evaluated using the numerical DH parameters (with length expressed in [m]) and in the configuration shown in Fig. 2, we have

$$\boldsymbol{J}_{L,P} = \begin{pmatrix} 0 & -0.65 & 0 \\ 0.8 & 0 & 0 \\ 0 & 0.7 & 0.7 \end{pmatrix}.$$

The maximum velocity in norm of Point P is obtained when taking the velocities of joints 2 and 3 at their limit value and with the *same* (positive or negative) sign, independently of the velocity of joint

1 (which can be as well positive or negative in the solution). Taking into account the conversion from [°/s] to [rad/s], one has for example with all positive (and maximum) joint velocities

$$\boldsymbol{v}_P = \boldsymbol{J}_{L,P} \begin{pmatrix} \dot{q}_{max,1} \\ \dot{q}_{max,2} \\ \dot{q}_{max,3} \end{pmatrix} = \boldsymbol{J}_{L,P} \begin{pmatrix} 250\,°/\text{s} \\ 250\,°/\text{s} \\ 215\,°/\text{s} \end{pmatrix} \cdot \frac{\pi}{180°} = \begin{pmatrix} -2.8362 \\ 3.4907 \\ 5.6810 \end{pmatrix} \text{[m/s]} \quad \Rightarrow \quad \|\boldsymbol{v}_P\| = 7.2459,$$

while flipping for instance the sign of the second joint velocity one obtains

$$\boldsymbol{v}_P = \boldsymbol{J}_{L,P} \begin{pmatrix} \dot{q}_{max,1} \\ -\dot{q}_{max,2} \\ \dot{q}_{max,3} \end{pmatrix} = \begin{pmatrix} 2.8362 \\ 3.4907 \\ -0.4276 \end{pmatrix} \text{[m/s]} \quad \Rightarrow \quad \|\boldsymbol{v}_P\| = 4.5179,$$

namely, a lower norm for the velocity of point P.

[b] The 3×6 angular part $\boldsymbol{J}_A$ of the geometric Jacobian is computed using the $\boldsymbol{z}_i$ axes of the DH frames, for $i = 0, 1, \ldots, 5$:

$$\boldsymbol{z}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \boldsymbol{z}_1 = {}^0\boldsymbol{R}_1(q_1)\boldsymbol{z}_0 \qquad \ldots \qquad \boldsymbol{z}_5 = {}^0\boldsymbol{R}_1(q_1)\,{}^1\boldsymbol{R}_2(q_2)\ldots{}^4\boldsymbol{R}_5(q_5)\boldsymbol{z}_0,$$

where the rotation matrices are extracted from the DH homogeneous transformation matrices ${}^{i-1}\boldsymbol{A}_i$, for $i = 1, \ldots, 5$. We obtain

$$\boldsymbol{J}_{A,b}(\boldsymbol{q}_b) = \begin{pmatrix} \boldsymbol{z}_0 & \boldsymbol{z}_1 & \boldsymbol{z}_2 \end{pmatrix} = \begin{pmatrix} 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{pmatrix}$$

and

$$\boldsymbol{J}_{A,w}(\boldsymbol{q}) = \begin{pmatrix} \boldsymbol{z}_3 & \boldsymbol{z}_4 & \boldsymbol{z}_5 \end{pmatrix} = \begin{pmatrix} c_1 s_{23} & s_1 c_4 - c_1 s_4 c_{23} & s_5(s_1 s_4 + c_1 c_4 c_{23}) + c_5 c_1 s_{23} \\ s_1 s_{23} & -c_1 c_4 - s_1 s_4 c_{23} & c_5 s_1 s_{23} - s_5(c_1 s_4 - c_4 s_1 c_{23}) \\ -c_{23} & -s_{23} s_4 & s_{23} c_4 s_5 - c_{23} c_5 \end{pmatrix}.$$

Evaluating these matrices as before in the configuration shown in Fig. 2, we have

$$\boldsymbol{J}_{A,b} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \end{pmatrix} \qquad \boldsymbol{J}_{A,w} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

As a result, when $\dot{\boldsymbol{q}}_b$ is chosen as in the solution used in **[a]** and $\dot{\boldsymbol{q}}_w = \boldsymbol{0}$, one has

$$\boldsymbol{\omega}_6^{[b]} = \boldsymbol{J}_{A,b} \begin{pmatrix} \dot{q}_{max,1} \\ \dot{q}_{max,2} \\ \dot{q}_{max,3} \end{pmatrix} = \begin{pmatrix} 0 \\ -8.1158 \\ 4.3633 \end{pmatrix} \text{[rad/s]}.$$

[c] Conversely, when $\dot{\boldsymbol{q}}_b = \boldsymbol{0}$ and $\dot{\boldsymbol{q}}_w$ has its components at their maximum positive values (i.e., rotating counterclockwise around the joint axes $\boldsymbol{z}_i$ of the wrist), one obtains

$$\boldsymbol{\omega}_6^{[c]} = \boldsymbol{J}_{A,w} \begin{pmatrix} \dot{q}_{max,4} \\ \dot{q}_{max,5} \\ \dot{q}_{max,6} \end{pmatrix} = \begin{pmatrix} 18.5878 \\ -6.6323 \\ 0 \end{pmatrix} \text{[rad/s]}.$$

Exercise 3

This trajectory planning problem is tackled first separately for each of the two joints. The rest-to-rest minimum time problem under bounds on velocity and acceleration is solved using a trapezoidal velocity profile (bang-coast-bang in acceleration), which may also collapse into a triangular velocity profile (without the coast phase) if the required displacement L is too short with respect to the given bounds (i.e., if $L \leq V_{max}^2/A_{max}$). However, this is not the case here. Once expressed in radians, the required displacements (from the lower to the upper limit of the motion range) are

$$L_2 = Q_{max,2} - Q_{min,2} = (145° - (-105°)) \cdot \frac{\pi}{180°} = 4.36 \text{ rad}$$
$$L_3 = Q_{max,3} - Q_{min,3} = (150° - (-163°)) \cdot \frac{\pi}{180°} = 5.46 \text{ rad},$$

and the check for the existence of a coast phase with constant (maximum) cruising velocity, with $V_{max,2} = 4.36$ and $V_{max,3} = 3.75$ [rad/s] (converted from the values in [°/s] of the data sheet)

$$L_2 = 4.36 > 3.46 = \frac{V_{max,2}^2}{A_{max,2}} \qquad L_3 = 5.46 > 2.01 = \frac{V_{max,3}^2}{A_{max,3}}$$

are satisfied in both cases. Therefore, the minimum time for the desired motion of the two joints when considered separately are

$$T_{min,2} = \frac{L_2}{V_{max,2}} + \frac{V_{max,2}}{A_{max,2}} = 1.79 \text{ s} \qquad T_{min,3} = \frac{L_3}{V_{max,3}} + \frac{V_{max,3}}{A_{max,3}} = 1.99 \text{ s},$$

with associated rising times $T_{s,2} = V_{max,2}/A_{max,2} = 0.79$, $T_{s,3} = V_{max,3}/A_{max,3} = 0.54$ [s].

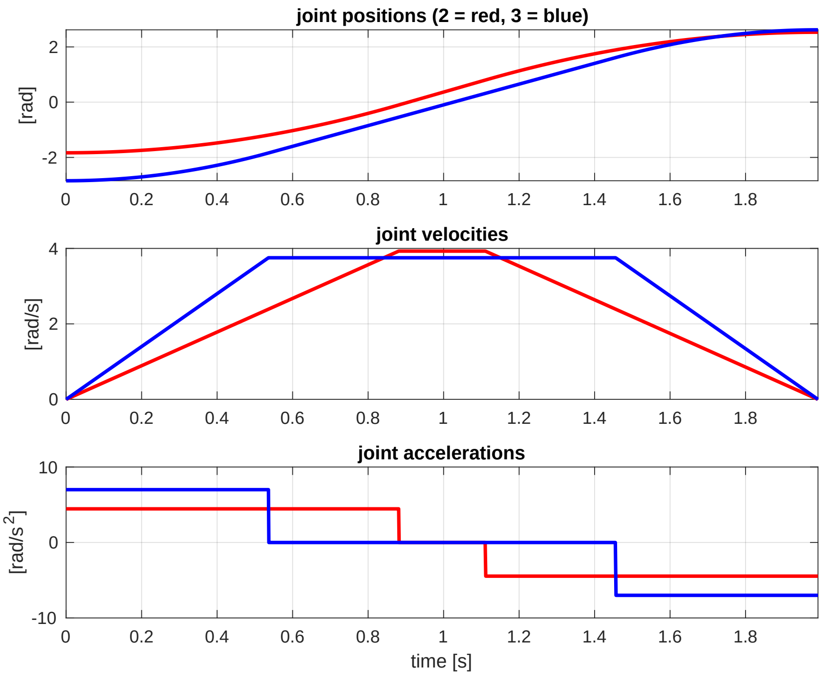


Figure 3: The coordinated minimum-time trajectory for the two joints 2 and 3.

At this stage, since a coordinated joint motion is required, the minimum coordinated motion time will be

$$T_{min} = \max\{T_{min,2}, T_{min,3}\} = 1.99 \text{ s},$$

and the fastest joint, namely joint 2, must be slowed down. While joint 3 will keep the same trapezoidal profile computed independently, with cruise speed $V_{max,3} = 3.75$ rad/s, acceleration $A_{max,3} = 7$ rad/s^2 and rising time $T_{s,3} = 0.53$ s, the trajectory of joint 2 will be uniformly scaled in time by a factor

$$k = \frac{T_{min}}{T_{min,2}} = \frac{1.99}{1.79} = 1.11 > 1,$$

thus specifying scaled rising time $T_2 > T_{s,2}$, cruise speed $V_2 < V_{max,2}$ and acceleration $A_2 < A_{max,2}$ as

$$T_2 = k\,T_{s,2} = 0.88 \text{ s} \qquad V_2 = \frac{V_{max,2}}{k} = 3.92 \text{ rad/s} \qquad A_2 = \frac{A_{max.2}}{k^2} = 4,46 \text{ rad/s}^2.$$

The resulting coordinated motion trajectories of the two joints are shown in Fig. 3, together with their trapezoidal velocity and bang-coast-bang acceleration profiles.

Exercise 4

The resolutions of the two encoders on the motor side are

$$r_{m2} = \frac{2\pi}{N_2} = 157.08 \cdot 10^{-5} \text{ rad} \qquad r_{m3} = \frac{2\pi}{N_3} = 241.66 \cdot 10^{-5} \text{ rad},$$

while on the link side of the gears we have

$$r_2 = \frac{r_{m2}}{n_{r2}} = 3.93 \cdot 10^{-5} \text{ rad} \qquad r_3 = \frac{r_{m3}}{n_{r3}} = 12.08 \cdot 10^{-5} \text{ rad}.$$

The part of the Jacobian $\boldsymbol{J}_{L,P}(\boldsymbol{q}_b)$ in eq. (1) that is involved in the assignment of a vertical velocity to Point P is given by the 2×2 matrix made by the first and third rows (respectively, along the $\boldsymbol{x}_0$ and $\boldsymbol{z}_0$ directions) and the second and third columns (corresponding to $\dot{q}_2$ and $\dot{q}_3$), namely

$$\bar{\boldsymbol{J}}_P(\boldsymbol{q}_b) = \begin{pmatrix} c_1(d_4 c_{23} - a_2 s_2) & d_4 c_1 c_{23} \\ a_2 c_2 + d_4 s_{23} & d_4 s_{23} \end{pmatrix}.$$

Evaluating this matrix in the configuration shown in Fig. 1 gives

$$\bar{\boldsymbol{J}}_P = \begin{pmatrix} -0.64 & 0 \\ 0.7 & 0.7 \end{pmatrix}.$$

Thus, a desired instantaneous displacement $\boldsymbol{\Delta p}_P = \begin{pmatrix} 0 & \Delta z \end{pmatrix}^T$ m of Point P along the upward vertical direction will be realized by the joint displacement

$$\boldsymbol{\Delta\bar{q}} = \begin{pmatrix} \Delta q_2 \\ \Delta q_3 \end{pmatrix} = \bar{\boldsymbol{J}}_P^{-1} \begin{pmatrix} 0 \\ \Delta z \end{pmatrix} = \begin{pmatrix} 0 \\ 10\Delta z/7 \end{pmatrix} \text{ [rad]}.$$

From this relation, it follows that the desired displacement $\boldsymbol{\Delta p}_P$ will not require any motion of joint 2 (so, no motion will be measured instantaneously by the encoder at this joint). On the other hand, the encoder at joint 3 will detect a Cartesian displacement of Point P along the $\boldsymbol{z}_0$ direction as long as this is larger or equal than its resolution beyond the transmission gear, or

$$\frac{10\Delta z}{7} \geq r_3 \qquad \Rightarrow \qquad \Delta z \geq 0.7 \cdot 12.08 \cdot 10^{-5} \text{ m} = 8.45 \cdot 10^{-5} \text{ m} = 0.0845 \, \text{mm}.$$

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