Robotics 1

November 7, 2024

Exercise 1

Consider the PPR planar robot with a 2-jaw gripper in Fig. 1, shown together with the world frame RF_w. Assign the link frames and fill in the associated table of parameters according to the Denavit–Hartenberg (DH) standard convention. Determine also the homogeneous transformation matrices ${}^w\boldsymbol{T}_0$ and ${}^3\boldsymbol{T}_e$, respectively between the world frame RF_w and the zero-th DH frame RF_0 and between the last DH frame RF_3 and the end-effector frame RF_e placed at the gripper with the usual convention ($\boldsymbol{z}_e$ in the approach direction and $\boldsymbol{y}_e$ in the open/close slide direction of the jaws). Provide the direct kinematics for the end-effector position ${}^w\boldsymbol{p}_e = \boldsymbol{f}(\boldsymbol{q}) \in \mathbb{R}^3$.

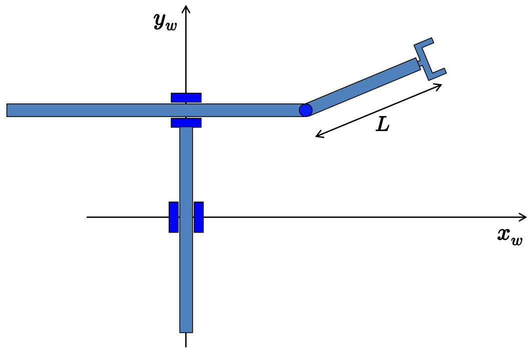

Figure 1: A PPR planar robot with the last link of length L.

Exercise 2

Based on the definition of joint variables in Exercise 1, solve the inverse kinematics problem in closed form for the PPR planar robot in terms of the components of the end-effector task vector $\boldsymbol{r} = ({}^w p_{e,x}, {}^w p_{e,y}, {}^w \alpha_e) \in \mathbb{R}^3$, where ${}^w\alpha_e = \text{atan2}\left\{\boldsymbol{z}_e^T \boldsymbol{y}_w, \boldsymbol{z}_e^T \boldsymbol{x}_w\right\}$ is the planar orientation angle of the gripper. For $\boldsymbol{r} = (1, 0, \pi/2)$ [m,m,rad], give the numerical solution(s) when $L = 0.5$ m.

Exercise 3

With reference to the DH assignment in Exercise 1, determine the 6×3 geometric Jacobian ${}^w\boldsymbol{J}(\boldsymbol{q})$ relating the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^3$ to the end-effector velocities ${}^w\boldsymbol{v}_e$ and ${}^w\boldsymbol{\omega}_e$, all expressed in the world frame. Find then all possible configurations for which this matrix loses rank. Next, with the robot in the configuration $\bar{\boldsymbol{q}} = (1, 1, 0)$ [m,m,rad] and with $L = 0.5$ m:

a) compute the joint velocity $\dot{\boldsymbol{q}}_a$ producing the end-effector velocity ${}^w\boldsymbol{v}_e = (-2, 1, 0)$ [m/s];

b) compute the joint velocity $\dot{\boldsymbol{q}}_b$ producing the angular velocity ${}^w\boldsymbol{\omega}_e = (0, 0, -3)$ [rad/s] of the end-effector frame;

c) determine whether the end-effector (twist) velocity $\left({}^w\boldsymbol{v}_e^T \; {}^w\boldsymbol{\omega}_e^T\right)^T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}^T$ is admissible for this robot and, if so, compute the joint velocity $\dot{\boldsymbol{q}}_c$ that realizes it.

Exercise 4

For the given PPR robot. with a generic length L of the third link, consider the joint variables as in Exercise 1 and the task variables as in Exercise 2. Plan a path $\boldsymbol{r}_d(s)$, parametrized by $s \in [0, 1]$, made by a circle of radius $R = 3L/4$ traced counterclockwise, with the gripper always oriented along the path normal and pointing outside the circle. At the start ($s = 0$), the path should be matched with the initial robot configuration $\boldsymbol{q}_0 = (1, 2, -\pi/4)$ [m,m,rad].

[180 minutes (3 hours); open books]