Robotics 1
January 13, 2025

Exercise 1

The end-effector of a robot manipulator has an orientation specified by the YXY sequence of RPY-type angles $(\alpha, \beta, \gamma) = (\pi/4, \pi, -\pi/4)$ [rad]. Find the values $(\boldsymbol{r}, \theta)$ of this orientation as defined by the axis-angle method. Check the correctness of the obtained result!

Exercise 2

A DC motor moves a rotating robot link of length $L = 0.75$ m through an harmonic drive (HD) transmission element. The motor is equipped with an incremental encoder mounted on its axis having $N_e = 800$ optical pulses/turn and quadrature electronics. If we wish to have a resolution of at least $\epsilon = 10^{-4}$ m at the tip of the link, what is the minimum number of teeth that the circular spline of the HD should have? If the motor rotates clockwise, will the link rotate clockwise as well?

Exercise 3

Consider the 2P3R spatial robot in Fig. 1, shown together with a world frame RF_w. This 5-dof robot is hanging from the ceiling at an height $H > 0$ from the plane $(\boldsymbol{x}_w, \boldsymbol{y}_w)$.

- Assign the link frames and fill in the associated table of parameters according to the Denavit–Hartenberg (DH) convention (use the extra sheet). The origin O_5 of the last DH frame should be placed at the center of the gripper.
- Provide the symbolic expression of the end-effector position, both as ${}^0\boldsymbol{p}_5$ (i.e., in terms of the robot base frame) and as ${}^w\boldsymbol{p}_5$ (expressed in the world frame).
- Compute the 6×5 geometric Jacobian $\boldsymbol{J}(\boldsymbol{q})$, either in the base or in the world frame.

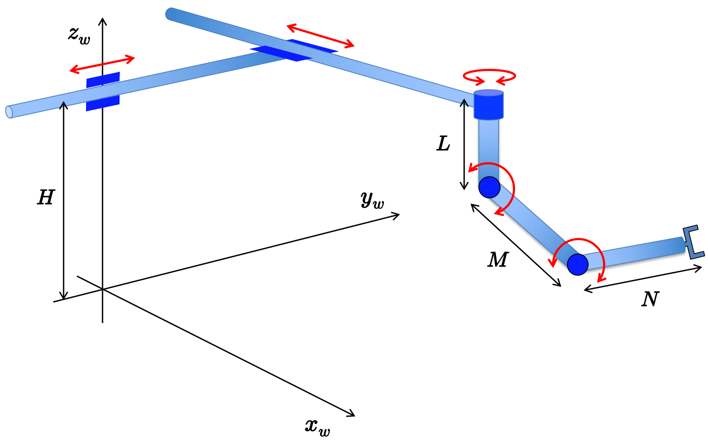

Figure 1: A 2P3R spatial robot hanging from the ceiling.

Exercise 4

A cylindrical robot has the direct kinematics of its end-effector position expressed by

$$\boldsymbol{p}(\boldsymbol{q}) = \begin{pmatrix} q_3 \cos q_2 \\ q_3 \sin q_2 \\ q_1 \end{pmatrix}.$$

When the desired position is $\boldsymbol{p}_d = \begin{pmatrix} 1 & -1 & 3 \end{pmatrix}^T$ [m], provide the first few iterations of the Newton algorithm for the numerical solution of the inverse kinematics problem starting from the initial guess $\boldsymbol{q}^{[0]} = \begin{pmatrix} -2 & 0.7\pi & \sqrt{2} \end{pmatrix}^T$ [m,rad,m]. The algorithm should stop as soon as

$$\left\| \boldsymbol{e}^{[k]} \right\| = \left\| \boldsymbol{p}_d - \boldsymbol{p}(\boldsymbol{q}^{[k]}) \right\| \leq \epsilon = 0.1 \text{ mm}.$$

Exercise 5

A 2R planar robot with link lengths $l_1 = 1.2$, $l_2 = 0.8$ [m] is at rest at $t = 0$ in the configuration $\boldsymbol{q}_0 = \mathbf{0}$ (stretched along the $\boldsymbol{x}_0$ axis). A pointwise target moves at constant speed $v = 1.5$ m/s on a straight line with an angle $\delta = 15°$ from the $\boldsymbol{x}_0$ axis, being in $\boldsymbol{p}_0 = \begin{pmatrix} -2 & 1 \end{pmatrix}^T$ [m] at $t = 0$ and entering after in the robot workspace. Solve the following rendez-vous problem:

a) define a trajectory that will bring the robot end-effector on the target when the latter crosses the $\boldsymbol{y}_0$ axis; the end-effector should have then the same velocity $\boldsymbol{v}_t \in \mathbb{R}^2$ of the target;
b) provide the rendez-vous time $t_{rv} > 0$ and the expression of the command $\dot{\boldsymbol{q}}(t) \in \mathbb{R}^2$, $t \in [0, t_{rv}]$.

How would you modify the velocity command $\dot{\boldsymbol{q}}(t)$ as a function of $\boldsymbol{q}(t)$ so as to reach the target at the rendez-vous position if the robot starts from a configuration close but different from $\boldsymbol{q}_0$?

[270 minutes (4.5 hours); open books]