Robotics 1

January 13, 2025

[students with midterm]

Exercise 1

For the PRR planar robot in Fig. 1, consider first the task vector $\boldsymbol{r}$ made by the position $\boldsymbol{p} \in \mathbb{R}^2$ of the end-effector and by its angle $\phi \in \mathbb{R}$ with respect to the $\boldsymbol{x}_w$ axis. Compute the corresponding Jacobian $\boldsymbol{J_r}(\boldsymbol{q})$ and find all its singularities. With the robot in a generic singular configuration $\boldsymbol{q}_s$:

a) provide the expression of all joint velocities $\dot{\boldsymbol{q}}$ that produce no task velocity $\dot{\boldsymbol{r}}$;

b) determine all task velocities $\dot{\boldsymbol{r}}$ that cannot be instantaneously realized.

Next, consider only the two-dimensional task vector $\boldsymbol{r} = \boldsymbol{p}$ for the same robot and find all singularities of the corresponding Jacobian $\boldsymbol{J_p}(\boldsymbol{q})$. When the robot is in a configuration $\boldsymbol{q}_s$ with all strictly positive joint values and such that the matrix $\boldsymbol{J_p}(\boldsymbol{q}_s)$ loses rank:

c) provide the expression of all forces $\boldsymbol{f} \in \mathbb{R}^2$ applied to the end-effector that need no joint force/torque $\boldsymbol{\tau} \in \mathbb{R}^3$ to be balanced;

d) determine the $\boldsymbol{\tau}$ that statically balances a force $\boldsymbol{f} = \begin{pmatrix} 3 & 1 \end{pmatrix}^T$ [N] applied to the end-effector.

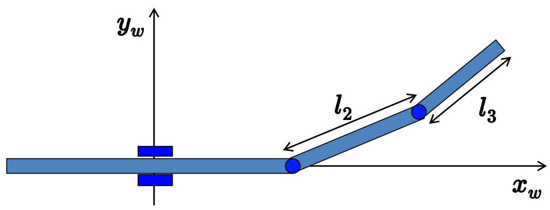

Figure 1: A PRR planar robot with the last two links of length l_2 and $l_3 \neq l_2$.

Exercise 2

A cylindrical robot has the direct kinematics of its end-effector position expressed by

$$\boldsymbol{p}(\boldsymbol{q}) = \begin{pmatrix} q_3 \cos q_2 \\ q_3 \sin q_2 \\ q_1 \end{pmatrix}.$$

When the desired position is $\boldsymbol{p}_d = \begin{pmatrix} 1 & -1 & 3 \end{pmatrix}^T$ [m], provide the first few iterations of a Newton algorithm for the numerical solution of the inverse kinematics problem in the following two cases:

a) starting from the initial guess $\boldsymbol{q}_a^{[0]} = \begin{pmatrix} -2 & 0.7\pi & \sqrt{2} \end{pmatrix}^T$ [m,rad,m];

b) starting from the initial guess $\boldsymbol{q}_b^{[0]} = \begin{pmatrix} 2 & \pi/4 & \sqrt{2} \end{pmatrix}^T$ [m,rad,m].

In case of convergence, the algorithm should stop as soon as $\|\boldsymbol{e}^{[k]}\| = \|\boldsymbol{p}_d - \boldsymbol{p}(\boldsymbol{q}^{[k]})\| \leq \epsilon = 0.1$ mm.

Exercise 3

A 2R planar robot with link lengths $l_1 = 1.2$, $l_2 = 0.8$ [m] is at rest at $t = 0$ in the configuration $\boldsymbol{q}_0 = \boldsymbol{0}$ (stretched along the $\boldsymbol{x}_0$ axis). A pointwise target moves at constant speed $v = 1.5$ m/s on a straight line with an angle $\delta = 15°$ from the $\boldsymbol{x}_0$ axis, being in $\boldsymbol{p}_0 = \begin{pmatrix} -2 & 1 \end{pmatrix}^T$ [m] at $t = 0$ and entering after in the robot workspace. Solve the following rendez-vous problem:

a) define a trajectory that will bring the robot end-effector on the target when the latter crosses the $\boldsymbol{y}_0$ axis; the end-effector should have then the same velocity $\boldsymbol{v}_t \in \mathbb{R}^2$ of the target;

b) provide the rendez-vous time $t_{rv} > 0$ and the expression of the command $\dot{\boldsymbol{q}}(t) \in \mathbb{R}^2$, $t \in [0, t_{rv}]$.

How would you modify the velocity command $\dot{\boldsymbol{q}}(t)$ as a function of $\boldsymbol{q}(t)$ so as to reach the target at the rendez-vous position if the robot starts from a configuration close but different from $\boldsymbol{q}_0$?

[180 minutes (3 hours); open books]