Robotics 2

January 13, 2025

Exercise 1

a) For the 2R polar robot in Fig. 1 moving in the presence of gravity, derive the dynamic model in the Lagrangian form
$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{1}$$
using the shown generalized coordinates $\boldsymbol{q} = (q_1, q_2)$ and with $\dot{\boldsymbol{M}} - 2\boldsymbol{S}$ being skew-symmetric. Assume that the links have a cylindric form and that their masses are uniformly distributed. Introduce all necessary kinematic or dynamic parameters. Friction phenomena are neglected.

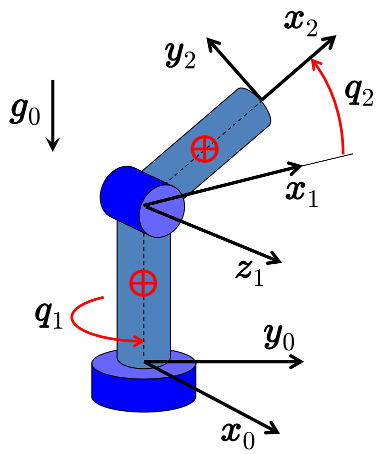

Figure 1: A 2R polar robot

b) For the robot dynamics (1), find a linear parametrization in the form
$$\boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\,\boldsymbol{a} = \boldsymbol{\tau} \tag{2}$$
in terms of a vector $\boldsymbol{a} \in \mathbb{R}^r$ of dynamic coefficients and a $2 \times r$ regressor matrix $\boldsymbol{Y}$. Discuss the minimality of r.

c) Using the explicit elements of the dynamic model (1), write down the expressions of all the control laws that you are aware of that guarantee global regulation to a desired (generic) constant configuration $\boldsymbol{q}_d$. Specify for each law the design conditions for success and the type of convergence/stability achieved.

d) Determine the expression of the inverse dynamics torque needed to reproduce the following desired joint trajectory:
$$q_{d1}(t) = \cos 3t, \qquad q_{d2}(t) = 0.$$

e) Provide then the explicit expression of an adaptive control law that guarantees asymptotic tracking of a desired smooth joint trajectory $\boldsymbol{q}_d(t)$, without any a priori knowledge about the robot dynamic parameters.

Exercise 2

Consider the 3P planar robot in Fig. 2. The robot is commanded by the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^3$. Let a smooth trajectory $\boldsymbol{p}(t) \in \mathbb{R}^2$ be assigned for the end-effector position of this robot.

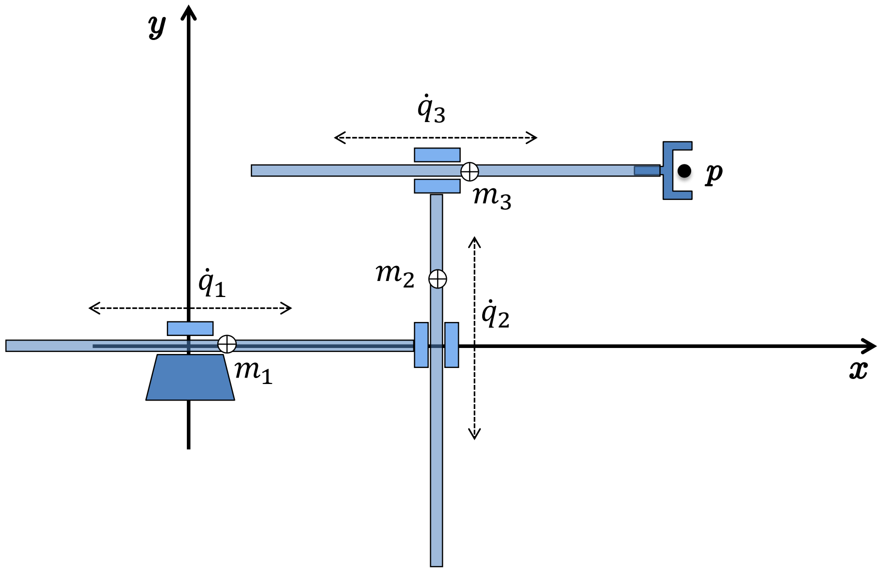

Figure 2: A 3P planar robot

a) Determine the command $\dot{\boldsymbol{q}}_a$ that realizes the assigned end-effector velocity $\dot{\boldsymbol{p}}$ while minimizing the norm $\|\dot{\boldsymbol{q}}\|$.

b) Determine the command $\dot{\boldsymbol{q}}_b$ that realizes the assigned end-effector velocity $\dot{\boldsymbol{p}}$ while minimizing the kinetic energy of the robot.

c) Find, if possible, a distribution of the three link masses m_1, m_2 and m_3 such that the 2×2 Cartesian inertia $\boldsymbol{M_p}$ of this robot is a scaled identity matrix $\mu \boldsymbol{I}_2$, for some scalar $\mu > 0$.

Very important: Provide in all three cases a physical interpretation of your obtained results.

[180 minutes (3 hours); open books]