CONTROL SYSTEMS - 1/7/2024

[time 3 hours; no textbooks or notes; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) Given $\mathbf{P}(s) = \frac{1}{s-1}$ design a minimal dimensional controller $\mathbf{G}(s)$ such that

(i) the closed-loop system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable (use the Nyquist criterion) with steady state error $|\mathbf{e}_{ss}(t)| \leqslant 0.1$ to inputs $\mathbf{v}(t) = t$

(ii) the open-loop system $\mathbf{PG}(s)$ has crossover frequency $\omega_t^* = 0.1$ rad/sec.

Ex. # 2) Given a process $\mathbf{P}(s) = -\frac{(s-1)(s+a)}{(s+1)(1+s+s^2)}$ with $a \in (-\infty, -1) \cup (1, +\infty)$
i) draw the root locus of $\mathbf{P}$ (using the Routh criterion for determining the curves in the left- and right-half complex plane)
ii) with $a \in (1, +\infty)$ design a controller $\mathbf{G}(s)$ with minimal dimension such that the feedback system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable with steady state error response $\mathbf{e}_{ss}(t) = 0$ to constant inputs $\mathbf{v}(t)$
ii) with $a \in (-\infty, -1)$ design a controller $\mathbf{G}(s)$ with minimal dimension such that the feedback system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable with steady state error response $\mathbf{e}_{ss}(t) = 0$ to constant inputs $\mathbf{v}(t)$

Ex. # 3) Given the process

$$\mathbf{P}(s) = \frac{1}{(s+k)^2 + \frac{3}{4}} \tag{1}$$

where $k \in \mathbb{R}$,
i) set $k = \frac{1}{2}$ and compute the maximal overshooting of the forced output response $\mathbf{y}(t)$ to inputs $\mathbf{v}(t) = 1$
ii) either increasing or decreasing k from the reference value $\frac{1}{2}$ is it possible to decrease the maximal overshooting? Motivate the answer with either accurate calculations or rigorous proof.