CONTROL SYSTEMS - 25/10/2024

[time 3 hours; no textbooks or notes; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) Given $\mathbf{P}(s) = \frac{1}{s-1}$ design a controller $\mathbf{G}(s)$ with minimal dimension such that the feedback system $W(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$

(i) is asymptotically stable (use the Nyquist criterion)

(ii) has zero steady-state error $\mathbf{e}_{ss}(t)$ with constant inputs

and the open loop system $\mathbf{PG}(s)$ has crossover frequency $\omega_t^* = 10$ rad/sec and phase margin $m_\phi^* \geqslant 30°$.

Ex. # 2) Given $\mathbf{P}(s) = \frac{s+2}{s^2+1}$, determine a one-dimensional controller $G(s)$ such that the feedback system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ has the following properties:
i) it is asymptotically stable
ii) its steady state error $\mathbf{e}_{ss}(t)$ to constant and sinusoidal (with frequency $\omega = 1$ rad/sec) inputs is 0.
Draw the root locus of $\mathbf{PG}(s)$. Determine if a one-dimensional controller $\mathbf{G}(s)$ can be designed in such a way that $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$, besides satisfying (i) and (ii), has all the poles $\mathbf{p}$ with $\mathrm{Re}(\mathbf{p}) < -\alpha$ for some $\alpha > 0$. Determine all the values of $\alpha > 0$ for which this is possible.

Ex. # 3) Given

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} = \begin{pmatrix} -1 & 1 \\ 0 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathbf{u},\ \mathbf{y} = C\mathbf{x} = \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x},$$

(i) determine the indistinguishable pairs of states $(\mathbf{x}_a, \mathbf{x}_b)$ from the output $\mathbf{y}$ and the reachable states $\mathbf{x}_f$ from the origin $\mathbf{x}_0 = 0$ and from $\mathbf{x}_0 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$,
(ii) decompose the system into observable and unobservable subsystems, and find, if possible, K such that $A - KC$ has the spectrum $\{-2, -3\}$,
(ii) decompose the system into controllable and uncontrollable subsystems, and find, if possible, F such that $A + BF$ has the spectrum $\{-2, -3\}$.