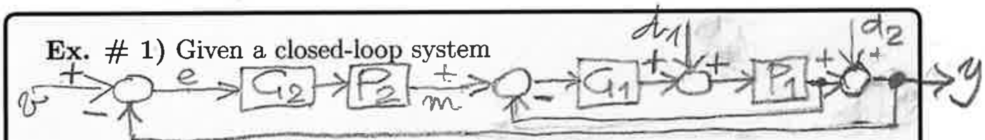
CONTROL SYSTEMS - 2/9/2024

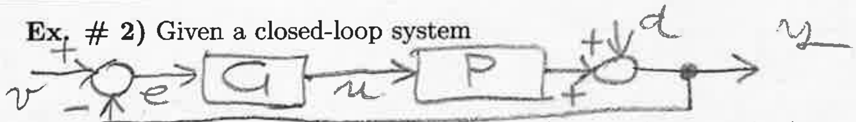
[time 3 hours; no textbooks or notes; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) Given a closed-loop system



where $\mathbf{P}_1(s) = \frac{2.1s+0.1}{s-1}$, $\mathbf{P}_2(s) = \frac{1}{2.1s+0.1}$ design a one-dimensional controller $\mathbf{G}_1(s)$ and a two-dimensional controller $\mathbf{G}_2(s)$ such that
(i) the closed-loop system (from v, d_1 and d_2 to y) is asymptotically stable (use Nyquist criterion for assessing stability) with steady-state error $\mathbf{e}_{ss}(t) = 0$ to inputs $\mathbf{v}(t) = 1$, steady-state output response $\mathbf{y}_{ss}(t) = 0$ to disturbances $\mathbf{d}_1(t) = 1$, $\mathbf{d}_2(t) = 1$ and $\mathbf{d}_1(t) = t$,
(ii) the open-loop system (from e to y) has the largest as possible phase.

Ex. # 2) Given a closed-loop system



where $\mathbf{P} : \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$, $\mathbf{y} = C\mathbf{x}$, with $A = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $C = \begin{pmatrix} -1 & -2 \end{pmatrix}$, design a minimal dimensional controller $\mathbf{G}(s)$ such that the closed-loop system is asymptotically stable with poles in $\mathcal{S} = \{s \in \mathbb{C} : \mathrm{Re}(s) \leqslant -0.3\}$ and steady state output response $\mathbf{y}_{ss}(t) = 0$ to constant disturbances $\mathbf{d}(t) = 1$ and sinusoidal disturbances $\mathbf{d}(t) = \sin(t)$.
Draw accurately the root locus of $\mathbf{P}(s)\mathbf{G}(s)$, using the Routh table for determining the crossing points of the imaginary axis and all the real values of the gain K for which the closed-loop system $\frac{K\mathbf{P}(s)\mathbf{G}(s)}{1+K\mathbf{P}(s)\mathbf{G}(s)}$ maintains its asymptotic stability.

Ex. # 3) Given $\mathbf{P} : \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$, $\mathbf{y} = C\mathbf{x}$, with $A = \begin{pmatrix} -3 & 1 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$,
(i) find $u(t)$ and $t_f > 0$ such that $x(t_f) = (10, 10)^\top$ with $x(0) = (1, 1)^\top$
(ii) for which choices of the matrix C the system $\mathbf{P}$ is observable? for which choices of the matrix C the eigenvalues of the unobservable subsystem of $\mathbf{P}$ are in $\mathbb{C}^-$?
(iii) set $C = \begin{pmatrix} 1 & 0 \end{pmatrix}$ and calculate the forced output response of $\mathbf{P}$ to an input $u(t) = 1$ and, if there exists, its steady-state value.