CONTROL SYSTEMS - 3/6/2024

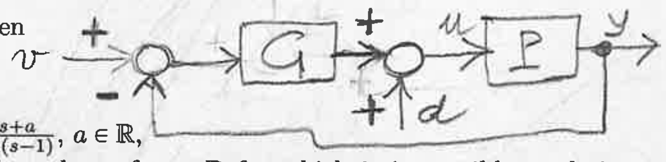
[time 3 hours; no textbooks or notes; no programmable calculators; all the mathematical passages must be motivated and clearly explained]

Ex. # 1) Given $\mathbf{P}(s) = \frac{0.1}{s+1}$ design a controller $\mathbf{G}(s)$ with dimension $\leqslant 2$ such that

(i) the closed-loop system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable (use the Nyquist criterion) with steady state error $|\mathbf{e}_{ss}(t)| \leqslant 1$ to inputs $\mathbf{v}(t) = t$

(ii) the open-loop system $\mathbf{PG}(s)$ has crossover frequency $\omega_t^* \leqslant 1$ rad/sec and phase margin $m_\phi^* \geqslant 60°$.

Ex. # 2) Given



with $\mathbf{P}(s) = \frac{s+a}{s^2(s-1)}$, $a \in \mathbb{R}$,
i) determine the values of $a \in \mathbb{R}$ for which it is possible to design a controller $\mathbf{G}(s)$ with dimension 2 such that the feedback system $\mathbf{W}(s) = \frac{\mathbf{PG}(s)}{1+\mathbf{PG}(s)}$ is asymptotically stable with poles in $\{s \in \mathbb{C} : \text{Re}(p) < -3\}$, with steady state output response $\mathbf{y}_{ss}(t) = 0$ to disturbances $\mathbf{d}(t) = 1$ and $|\mathbf{y}_{ss}(t)| \leqslant 0.01$ to disturbances $\mathbf{d}(t) = t$.
Choose any admissible of $a \in \mathbb{R}$ and draw the root locus of $\mathbf{PG}(s)$ (using the Routh criterion for determining the curves in the left- and right-half complex plane).

Ex. # 3) Given the system

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} := \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} \mathbf{u} \qquad (1)$$

i) Decompose the system into controllable and uncontrollable subsystems, determining the invariant spectrum of A under feedback transformations, and determine a feedback law $\mathbf{u} = F\mathbf{x}$ such that the eigenvalues of $A + BF$ are all in -2.
ii) Determine if there exists a control law $\alpha(t)$ such that the state $\mathbf{x}(t)$ of (1) with $\mathbf{u}(t) = \alpha(t)$ satisfies $\mathbf{x}(0) = x_0 := (1,0)^T$ and $\mathbf{x}(t_f) = x_f := (3,0)^T$ with $t_f = 3$ sec. If yes, determine $\mathbf{u}(t)$.
iii) If $x_0 := (1,-1)^T$ for which $x_f \in \mathbb{R}^2$ there would exist a control law $\alpha(t)$ such that the state $\mathbf{x}(t)$ of (1) with $\mathbf{u}(t) = \alpha(t)$ satisfies $\mathbf{x}(0) = x_0$ and $\mathbf{x}(t_f) = x_f$ for some $t_f \geqslant 0$?