Exercise A farm grows maize (corn) and calves, and can work using three different processes. With the first procedure 1000 calves are raised using 30 man-months of work, 20 hectares of land, 200 tons of corn. With the second process 100 tons of corn are produced using 20 man-months of work and 40 hectares of land. The third process produces 200 tons of corn and 500 calves using 40 man-months of work and 50 hectares of land.
Corn is sold for 500 euros per ton, calves for 200 euros each.
The company owns 70 hectares of land, has 50 man-months of work and must be self-sufficient with regards to corn. The purpose of the company is to maximize profit.

Question a) Write the linear model to find how much to produce according to each procedure, taking into account that each procedure can be applied several times and even partially (e.g. the first procedure applied 0.5 times means producing 500 calves using 15 man-months of work, 10 hectares and 100 tons of corn.) Since the number of calves is high, and we are talking about average values, there is no need to require that the number of calves must be integer.
Question b) Suppose that the second and third procedures cannot be used at the same time. Modify the formulation obtained in point (a) in order to take this further constraint into account.

Solution a) We can describe the 3 possibilities as follows:

procedure	needs	produces
1	30 man-months, 20 hectares 200 t. corn	1000 calves
2	20 man-months, 40 hectares	100 t. corn
3	40 man-months, 50 hectares	200 t. corn, 500 calves

We need to decide how much of each procedure we must utilize, so we use the following variables:

$$x_i = \text{number of times we use procedure } i$$

Since each procedure can be applied several times and even partially, those variables are real-valued but non-negative.
Profit is given by calves and corn sold, that is, the amount produced minus the amount consumed. Hence, it is

$$200\,(1000\,x_1 + 500\,x_3) + 500\,(100\,x_2 + 200\,x_3 - 200\,x_1)$$

Two constraints are those on the quantity of hectares of land and the number of man-months of work available. It should be noted that saving on these quantities does not provide any gains. A third constraint is given by the fact that the quantity of corn used cannot exceed that produced (self-sufficiency). The linear model is therefore:

$$\max 200\,(1000\,x_1 + 500\,x_3) + 500\,(100\,x_2 + 200\,x_3 - 200\,x_1)$$

$$\begin{aligned}
&20\,x_1 + 40\,x_2 + 50\,x_3 \leq 70\\
&30\,x_1 + 20\,x_2 + 40\,x_3 \leq 50\\
&-200\,x_1 + 100\,x_2 + 200\,x_3 \geq 0\\
&x_1, x_2, x_3 \geq 0\\
&x_1, x_2, x_3 \in R
\end{aligned}$$

Solution b) Since the x variables are real-valued, to express mutual exclusivity of second and third procedures we need to introduce a binary variable

$$y = \begin{cases} 1 & \text{if we use (even partially) procedure 2 (and so we cannot use procedure 3)} \\ 0 & \text{otherwise (not using procedure 2 so we can use procedure 3)} \end{cases}$$

Now we add to the previous model the following constraints, with M larger than any possible value of x_2 and x_3

$$x_2 \leq My \qquad\qquad x_3 \leq M(1\text{-}y)$$