Reasoning Procedures II

Non-Termination

- As already mentioned, for $\mathcal{ALC}$ with **general axioms** basic algorithm is **non-terminating**
- **E.g.** if human $\sqsubseteq$ $\exists$has-mother.human $\in \mathcal{T}$, then $\neg$human $\sqcup$ $\exists$has-mother.human added to every node

w $\mathfrak{L}(w) = \{$human, ($\neg$human $\sqcup$ $\exists$has-mother.human), $\exists$has-mother.human$\}$

has-mother

x $\mathfrak{L}(x) = \{$human, ($\neg$human $\sqcup$ $\exists$has-mother.human), $\exists$has-mother.human$\}$

has-mother

y $\mathfrak{L}(y) = \{$human, ($\neg$human $\sqcup$ $\exists$has-mother.human), $\exists$has-mother.human$\}$

Blocking

- When creating new node, check ancestors for equal (superset) label
- If such a node is found, new node is **blocked**

$$\mathcal{L}(w) = \{\text{human}, (\neg\text{human} \sqcup \exists\text{has-mother.human}), \exists\text{has-mother.human}\}$$

$$w \xrightarrow{\text{has-mother}} x$$

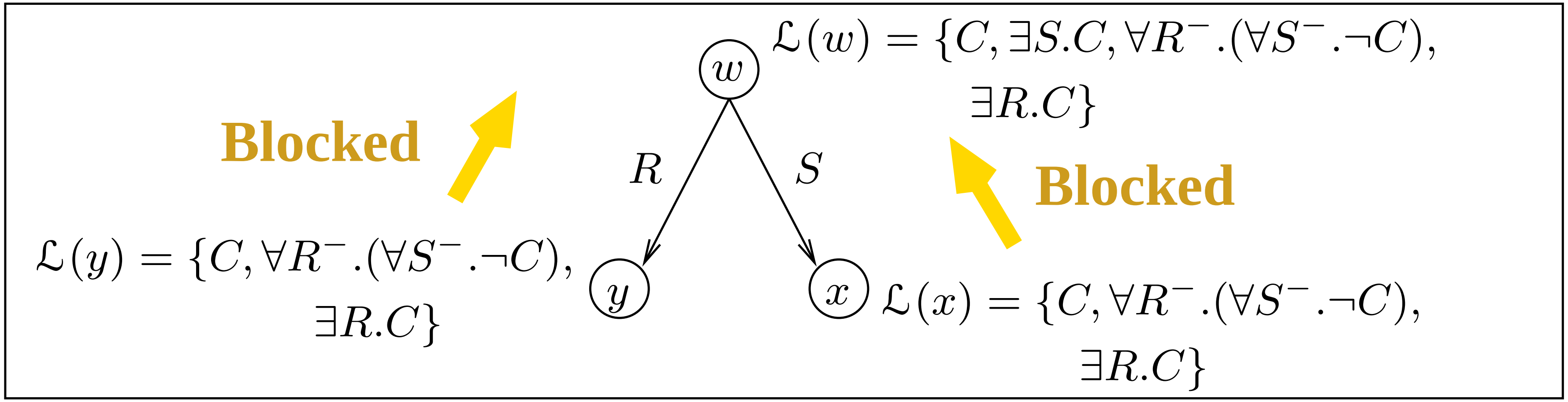
Blocked

$$\mathcal{L}(x) = \{\text{human}, (\neg\text{human} \sqcup \exists\text{has-mother.human})\}$$

Blocking with More Expressive DLs

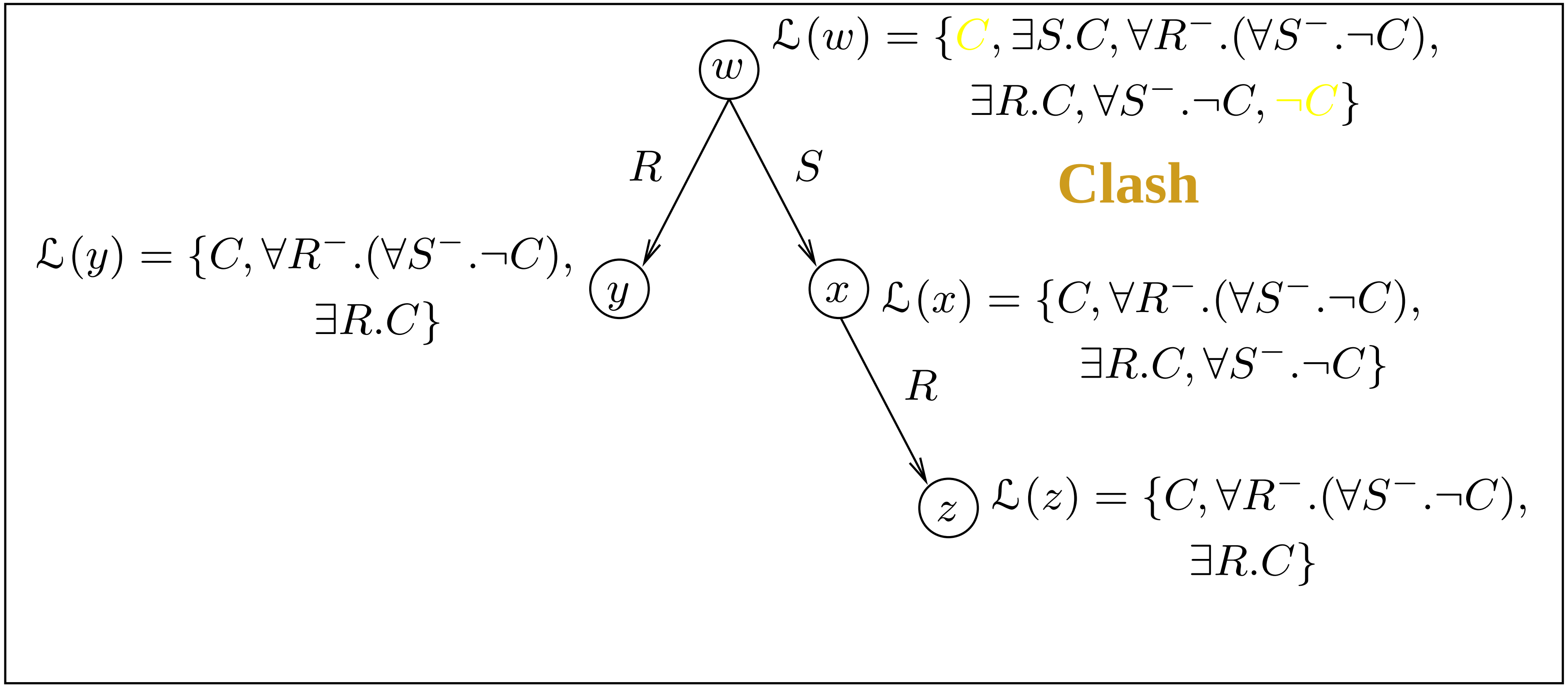
- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

$$\mathcal{T} = \{\top \sqsubseteq \forall R^-.(\forall S^-.\neg C), \top \sqsubseteq \exists R.C\}$$



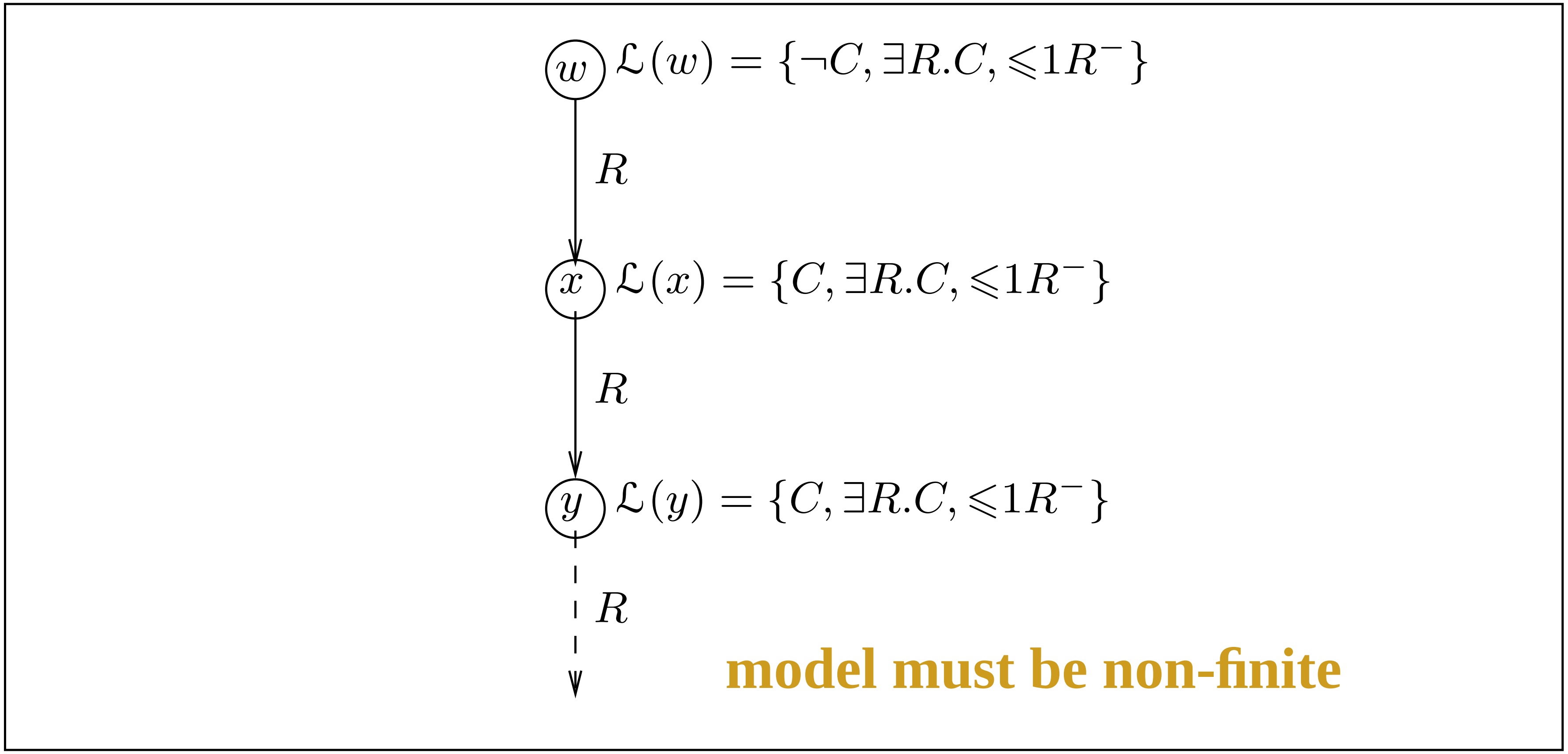
Dynamic Blocking

- Solution (for inverse roles) is **dynamic blocking**
  - Blocks can be established broken and re-established
  - Continue to expand $\forall R.C$ terms in blocked nodes
  - Check that cycles satisfy $\forall R.C$ concepts



Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{\top \sqsubseteq \exists R.C, \top \sqsubseteq \leqslant 1 R^-\}$



Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{\top \sqsubseteq \exists R.(C \sqcap \exists R^{-}.\neg C), \top \sqsubseteq \leqslant 1 R^{-}\}$

w $\mathcal{L}(w) = \{\neg C, \exists R.(C \sqcap \exists R^{-}.\neg C), \leqslant 1 R^{-}\}$

R

x $\mathcal{L}(x) = \{(C \sqcap \exists R^{-}.\neg C), \exists R.(C \sqcap \exists R^{-}.\neg C), \leqslant 1 R^{-}, C, \exists R^{-}.\neg C\}$

R^{-} **Blocked**

y $\mathcal{L}(y) = \{(C \sqcap \exists R^{-}.\neg C), \exists R.(C \sqcap \exists R^{-}.\neg C), \leqslant 1 R^{-}, C, \exists R^{-}.\neg C\}$

But $\exists R^{-}.\neg C \in \mathcal{L}(y)$ not satisfied

Inconsistency due to $\leqslant 1 R^{-} \in \mathcal{L}(y)$ and $C \in \mathcal{L}(x)$

Double Blocking I

☞ Problem due to $\exists R^{-}.\neg C$ term **only** satisfied in **predecessor** of blocking node

$$\begin{array}{l} w \quad \mathfrak{L}(w) = \{\neg C, \exists R.(C \sqcap \exists R^{-}.\neg C), \leqslant 1 R^{-}\} \\ \downarrow R \\ x \quad \mathfrak{L}(x) = \{(C \sqcap \exists R^{-}.\neg C), \exists R.(C \sqcap \exists R^{-}.\neg C), \leqslant 1 R^{-}, C, \exists R^{-}.\neg C\} \end{array}$$

☞ Solution is **Double Blocking** (pairwise blocking)

- Predecessors of blocked and blocking nodes also considered
- In particular, $\exists R.C$ terms satisfied in predecessor of blocking node must also be satisfied in predecessor of blocked node $\neg C \in \mathfrak{L}(w)$

Double Blocking II

- Due to pairwise condition, block no longer holds
- Expansion continues and contradiction discovered

$$\begin{tikzcd} w \arrow[d, "R"] \\ x \\ y \arrow[u, "R"'] \end{tikzcd}$$

$$\mathcal{L}(w) = \{\neg C, \exists R.(C \sqcap \exists R^{-}.\neg C), \leqslant 1 R^{-}\}$$

$$\mathcal{L}(x) = \{(C \sqcap \exists R^{-}.\neg C), \exists R.(C \sqcap \exists R^{-}.\neg C), \leqslant 1 R^{-}, C, \exists R^{-}.\neg C, \neg C\}$$

Clash

$$\mathcal{L}(y) = \{(C \sqcap \exists R^{-}.\neg C), \exists R.(C \sqcap \exists R^{-}.\neg C), \leqslant 1 R^{-}, C, \exists R^{-}.\neg C\}$$