Example

We consider FOL interpretations exactly as used in relational databases. This requires to drop functions except for constants. Moreover we assume that the interpretation of constants is the identity function, that is constants are interpreted as themselves. This allows us to drop also the interpretation of constants from our interpretations, which now have the form:

$$\mathcal{I} = (\Delta^{\mathcal{I}}, P_1^{\mathcal{I}}, P_2^{\mathcal{I}}, \ldots, P_n^{\mathcal{I}}).$$

Interpretation: $\mathcal{I}$ is as follows (also given in relational notation):

- $\Delta^{\mathcal{I}} = \{john, paul, george, mick, ny, london, 0, 1, \ldots, 100\}$
- $Person^{\mathcal{I}} = \{(john, 30), (paul, 60), (george, 35), (mick, 35)\}$
- $Lives^{\mathcal{I}} = \{(john, ny), (paul, ny), (george, london), (mick, london)\}$
- $Manages^{\mathcal{I}} = \{(paul, john), (george, mick), (paul, mick)\}$

$Person^{\mathcal{I}}$

name	age
john	30
paul	60
george	35
mick	35

$Lives^{\mathcal{I}}$

name	city
john	ny
paul	ny
george	london
mick	london

$Manages^{\mathcal{I}}$

boss	emp. name
paul	john
george	mick
paul	mick

Query: find name and age of persons who live in the same city as their boss.

$$\exists z, w.\mathrm{Person}(x, y) \land \mathrm{Manages}(z, x) \land \mathrm{Lives}(x, w) \land \mathrm{Lives}(z, w)$$

Example - Interpretation

Consider the following interpretation $\mathcal{I}$:

- $\Delta^{\mathcal{I}}$ is equal to the *active domain*: all objects occurring in any predicate extension.
- $Sailors^{\mathcal{I}}$ see table below
- $Boats^{\mathcal{I}}$ see table below
- $Reserves^{\mathcal{I}}$ see table below

$Sailors^{\mathcal{I}}$

sid	sname
22	dustin
31	lubber
58	rusty

$Boats^{\mathcal{I}}$

bid	color
101	red
102	green
103	red
104	blue

$Reserves^{\mathcal{I}}$

sid	bid	day
22	101	10/10/96
58	103	11/12/96

Example - Queries

- Find the names of the sailors who have reserved boat 103.
- Find the names of the sailors who have reserved a red boat.
- Find the colors of the boats reserved by Bob.
- Find the names of the sailors who have reserved at least one boat.
- Find the names of the sailors who have reserved a red and a green boat.
- Find the names of the sailors who have reserved a red or a green boat.
- Find the names of the sailors who have reserved at least two boats.
- Find the names of the sailors who have not reserved a red boat.
- Find the names of the sailors who have reserved all boats.
- Find the names of the sailors who have reserved all red boats.

Example - Queries

- Find the names of the sailors who have reserved boat 103.

$$\exists x.Sailors(x, y) \wedge \exists w.Reserves(x, 103, z)$$

$$q(y) \leftarrow Sailors(x, y), Reserves(x, 103, z)$$

- Find the names of the sailors who have reserved a red boat.

$$\exists x.Sailors(x, y) \wedge \exists z, w.Reserves(x, z, w) \wedge Boats(z, red)$$

$$q(y) \leftarrow Sailors(x, y), Reserves(x, z, w), Boats(z, red)$$

- Find the colors of the boats reserved by Bob.

$$\exists x.Boats(x, y) \wedge \exists z, w.Reserves(z, y, w) \wedge Sailor(z, Bob)$$

$$q(y) \leftarrow Boats(x, y), Reserves(z, y, w), Sailor(z, Bob)$$

- Find the names of the sailors who have reserved at least one boat.

$$\exists x.Sailors(x, y) \wedge \exists z, w.Reserves(x, z, w)$$

$$q(y) \leftarrow Sailors(x, y), Reserves(x, z, w)$$

- Find the names of the sailors who have reserved a red and a green boat.

$$\exists x.S(x, y) \wedge \exists z, w.R(x, z, w) \wedge B(z, red) \wedge \exists z', w'.R(x, z', w') \wedge B(z', green)$$

Example - Queries

- Find the names of the sailors who have reserved a red or a green boat.

  $\exists x.Sailors(x, y) \wedge \exists z, w.Reserves(x, z, w) \wedge (Boats(z, red) \vee Boats(z, green))$

  $$q(y) \leftarrow Sailors(x, y), Reserves(x, z, w), Boats(z, red)$$
  $$q(y) \leftarrow Sailors(x, y), Reserves(x, z, w), Boats(z, green)$$

- Find the names of the sailors who have reserved at least two boats.

  $\exists x.Sailors(x, y) \wedge \exists z, w.Reserves(x, z, w) \wedge \exists z', w'.Reserves(x, z', w') \wedge z \neq z'$

- Find the names of the sailors who have not reserved a red boat.

  $$\exists x.Sailors(x, y) \wedge \forall z, w.(Reserves(x, z, w) \rightarrow \neg Boats(z, red))$$

- Find the names of the sailors who have reserved all boats.

  $$\exists x.Sailors(x, y) \wedge \forall z, c.(Boats(s, c) \rightarrow \exists w.Reserves(x, z, w))$$

- Find the names of the sailors who have reserved all red boats.

  $$\exists x.Sailors(x, y) \wedge \forall z.(Boats(z, red) \rightarrow \exists w.Reserves(x, z, w))$$