From Aristotle to Pentium

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Logic and CS

Georg Gottlob, Founder of Lixto Software, 2004:

> "Computer science is the continuation of logic by other means."

What on Earth?

Basic Question: What on earth does an obscure, old intellectual discipline have to do with the youngest intellectual discipline?

Cosma R. Shalizi, Santa Fe Institute:

> "If, in 1901, a talented and sympathetic outsider had been called upon (say, by a granting-giving agency) to survey the sciences and name the branch that would be least fruitful in century ahead, his choice might well have settled upon mathematical logic, an exceedingly recondite field whose practitioners could all have fit into a small auditorium. It had no practical applications, and not even that much mathematics to show for itself: its crown was an exceedingly obscure definition of 'cardinal numbers'."

To start: What is logic?

The Most Famous Logician

Figure 1: Rev. Charles Lutwidge Dodgson, 1832–1898

His Very Famous Young Friend

Figure 2: Alice Liddel

What is logic?

Lewis Carroll, Through the Looking Glass:

> "Contrariwise," continued Tweedledee, "if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic."

Andrea Nye, Words of Power:

> "Logic celebrates the unity of a pathological masculine self-identity that cannot listen and recognizes only negation and not difference"

The Most Influential Intellectual

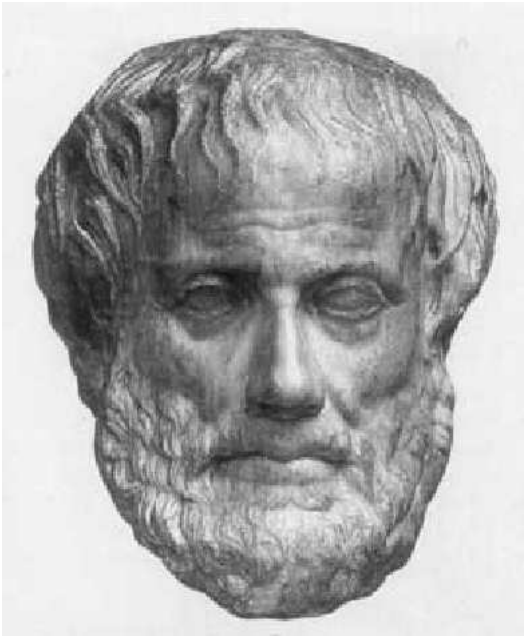


Figure 3: Aristotle, 384–322 BC

Formal Logic

Philosopher: lover of wisdom.

Demagogue: leader of the people

What distinguishes philosophy from demagoguery?

Francis Bacon, 1605:

> "Logic differeth from rhetoric..in this, that logic handleth reason exact and in truth, and rhetoric handleth it as it is planted in popular opinions and manners."

Aristotle: *Syllogism*–Forms of reasoning

- All humans are mortal. All Greeks are humans. Therefore, all Greeks are mortal.
- All bojums are slithy. All toves are bojums. Therefore, all toves are slithy.

Liar's Paradox

Epimenides, Cretan: "All Cretans are liars."

The Epistle of St. Paul to Titus, 1:12: "One of themselves, a prophet of their own, said, Cretans are always liars, evil beasts, lazy gluttons."

Psalms, 116:11: "Everyone is a liar".

Eubulides: "This sentence is false."

Is Eubulides telling the truth or not?

- If the sentence is true, then it is false.
- But if it is false, then it is true.

Key Feature: self-reference

Ramon Lull's Spiritual Logic

Ramon Lull, 1232-1315: use logic to convert the whole world to Christianity

- "If in Thy three properties there were no differences ... the demonstration would give the D to the H of the A with the F and the G as it does with the E, and yet the K would not give significance to the H of any defect in the F or the G; but since diversity is shown in the demonstration that the D makes of the E and the F and the G with the I and the K, therefore the H has certain scientific knowledge of Thy holy and glorious Trinity."
- F. Bacon: "Some persons, more ostentatious than learned, have laboured about a kind of a method not worthy to be called a legitimate method, begging rather a method of imposture, which nevertheless would no doubt be very acceptable to meddling wits. Such was the Art of Lull."
- Lull's contribution: diagrammatic and mechanical aids to inference.

Lull's Circles

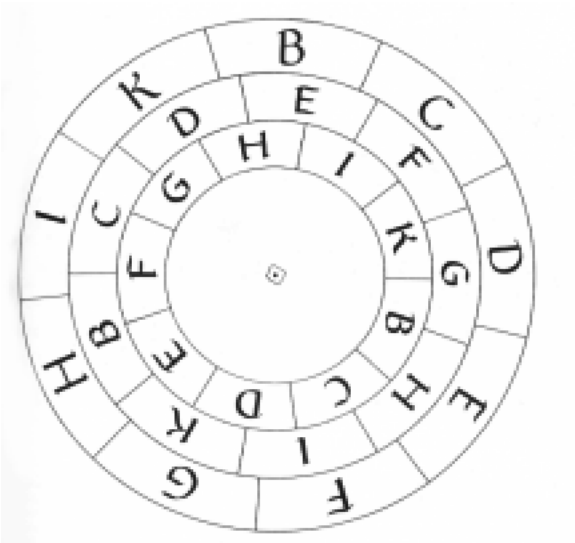


Figure 4: Ars magna, generalis et ultima,c. 1275

The Stoning of Philosopher Ramon Lull

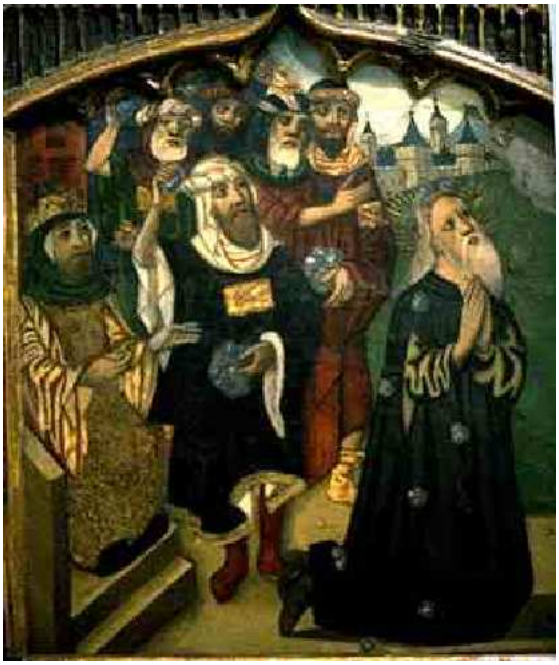


Figure 5: Renaissance, Anonymous, c. 1490

The Man Who Brought Us Calculus and Bits

Figure 6: Gottfried Wilhelm Leibniz, 1646–1716

Leibniz's Dream

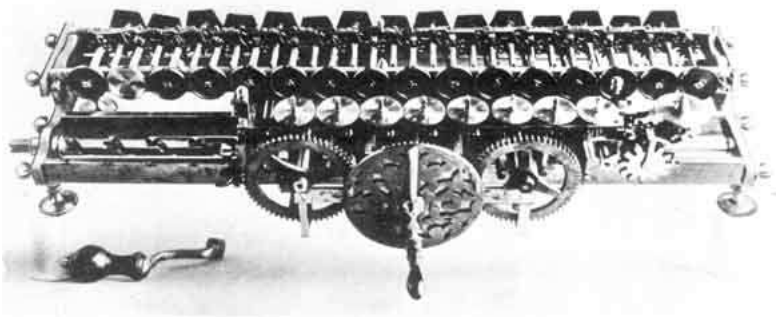
The early years: Taught himself Latin at age 8. Read Aristotle at age 12. Wrote a thesis at age 20 on Aristotelian metaphysics, followed by another Bachelor's thesis on logic in law.

- "When I was young, I found pleasure in the Lullian art".

The Impossible Dream: a universal mathematical language, *lingua characteristica universalis*, in which all human knowledge can be expressed, and calculational rules, *calculus ratiocinator*, carried out by machines, to derive all logical relationships.

> "If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, and say to each other: Calculemus– Let us calculate."

Leibniz's Wheel, 1671



A Man of Acutist Intellect and Manifold Learning

Figure 7: George Boole, 1815–1864

Boole's Symbolic Logic

Boole's insight: Aristotle's syllogisms are about *classes* of objects, which can be treated *algebraically*.

> "If an adjective, as 'good', is employed as a term of description, let us represent by a letter, as y, all things to which the description 'good' is applicable, i.e., 'all good things', or the class of 'good things'. Let it further be agreed that by the combination xy shall be represented that class of things to which the name or description represented by x and y are simultaneously applicable. Thus, if x alone stands for 'white' things and y for 'sheep', let xy stand for 'white sheep'.

The Mathematical Analysis of Logic, 1847

> "The design of the following treatise is to investigate the fundamental laws of the operations of the mind by which reasoning is performed; to give expressions to them in the symbolic language of a calculus, and upon this foundation to establish the science of logic and construct its methods."

Boolean Algebra

New axiom: $x = xx$

Consequently:

- $x = xx$
- $x - xx = 0$
- $x(1 - x) = 0$

Principle of Contradiction: *Nothing can both belong and fail to belong to a given class*.

Boole: $x(1 - x) = 0$ expresses

> "the Principle of Contradiction, which Aristotle has described as the most fundamental axiom of all philosophy."

Economist and Logician

Figure 8: William Stanley Jevons, 1835–1882

Jevons: Logic Machines

William Stanley Jevons:

- "I have given much attention, therefore, to lessening both the manual and mental labour of the process, and I shall describe several devices which may adopted for saving trouble and risk of mistake."
- "As I awoke in the morning, the sun was shining brightly into my room. There was a consciousness on my mind that I was the discoverer of the true logic of the future. For a few minutes I felt such a delight such as one can seldom hope to feel."
- "The machine represents a mind endowed with powers of thought, but wholly devoid of knowledge. ... It cannot be asserted indeed that the machine entirely supersedes the agency of concious thought."
- "I must remark that these mechanical devices are not likely to posses much practical utility. We do not require in common life to be constantly solving complex logical questions..

Logical Piano

Figure 9: The First Logic Machine, 1870

From Boole to the Pentium

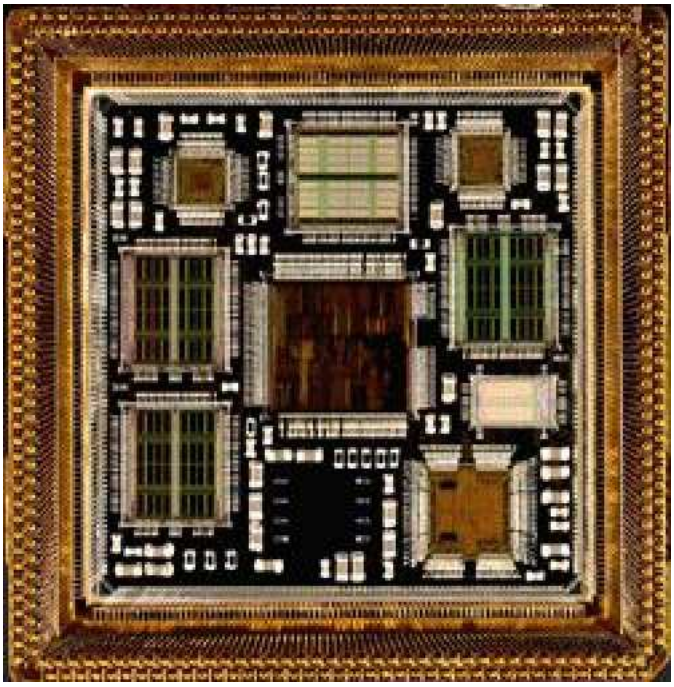


Figure 10: Pentium Chip, 1993

A Bright Young Student

Figure 11: Claude Elwood Shannon, 1916–2001

A Mere Master's Thesis

A Symbolic Analysis of Relay and Switching Circuits, MIT, 1937:

> "Shannon noted that the switches were always either open or closed, or on and off. This led him to think about a mathematical way to describe the open and closed states, and he recalled the logical theories of mathematician George Boole. Shannon theorized that a switch in the on position would equate to a Boolean one. In the off position, it was a zero."

Herman Goldstine, ENIAC Project Manager: "This thesis helped to change digital circuit design from art to science."

Howard Gardner, MacArthur Fellow, 1981: "possibly the most important, and also the most famous, master's thesis of the century."

A Figure of Mysterious Greatness

Charles Sanders Peirce.

Mathematician, astronomer, chemist, geodesist, surveyor, cartographer, metrologist, spectroscopist, engineer, inventor, psychologist, philologist, lexicographer, historian of science, mathematical economist, lifelong student of medicine, book reviewer, dramatist, actor, short story writer, phenomenologist, semiotician, logician, rhetorician and metaphysician

Logical Machines

In 1886, Alan Marquand, Peirce's student, published in the Proceedings of the American Academy of Arts and Science an article on "*A New Logical Machine*".

Charles Kendall Adams, Cornell's President: "Dear Sir: Please accept my thanks for your "New Logical Machine." Could you invent a machine that will do the work of a College President? I give you my order in advance."

On Dec. 30, 1886, Peirce wrote to Marquand, suggesting that he build a machine "for really very difficult mathematical problems, I think electricity would be the best thing to rely on." Peirce drew up a wiring diagram, for "multiplication and addition in logic", but Marquand never built that machine.

Peirce on Computing

1889: "A logical machine is a machine which, being fed with premises produces the necessary conclusions from them. The value of logical machines seems to lie in their showing how far reasoning is a mechanical process. Calculating machines are specialized logical machines."

1887: "Precisely how much the business of thinking a machine could possible be made to perform and what part of it must be left to the living mind is a question not without conceivable practical importance; the study of it at any rate not fail to throw needed light on the natue of the reasoning process."

Mathematics: Queen and Maidservant of Science

E.P. Wigner (1960): *On the Unreasonable Effectiveness of Mathematics in the Natural Sciences*

- Euclid: "The laws of nature are but the mathematical thoughts of God."
- Galileo: "The universe cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in a mathematical language, and the letters are triangles, circles and other geometrical figures, without which it is humanly impossible to comprehend a single word."
- *The empirical law of epistemology:* The mathematical formulation of the laws of nature is both appropriate and accurate; mathematics is the *correct* language for formulating the laws of nature.

Question: What makes mathematics so reliable?
Answer: *Proofs – the high road to truth!*

The Most Influential Mathematician

Figure 12: Euclid, 325–265 BC

High-School Mathematics

Dichotomy:

- *Geometry*: logically perfect science
  - On the gates of his academy, Plato has inscribed: "let no ungeometrical person enter here."
- *Algebra and Calculus*: shaky foundations

19th Century Reversal:

- Bolyai and Lobachevsky: non-Euclidean geometry
- Bolzano, Cauchy, Cantor, Dedekind and Weierstrass: formalization of calculus in terms of infinity.

Central Issue: What is a rigorous proof?

The Infinite

Old Controversy:

- Aristotle: "Infinitum Actu Non Datur" - there is no actual infinity.
- St. Augustine, 4th Century: "Individual numbers are finite but as a class are infinite. Does that mean that God does not know all the numbers, because of their infinity? No one could be insane enough to say that."

Amir Aczel, "The Mystery of the Aleph":

> "First discovered by the Greeks between the 5th and 6th centuries B.C., the concept of infinity was so overwhelming, so bizarre, so contrary to every human intuition, that it confounded the ancient philosophers and mathematicians who discovered it, causing pain, insanity and at least one murder."

Infinitely Controversial

Cantor's Theorem, 1874: There are infinitely many infinities! (Proof uses diagonalization.)

Mathematical Controversy:

- Leibniz: "I am so in favor of the actual infinite that instead of admitting that Nature abhors it, I hold that Nature makes frequent use of it everywhere."
- Gauss: "I protest above all the use of an infinite quantity as a completed one, which in mathematics is never allowed."
- Kronecker (on the Bolzano-Weierstrass Theorem): "obvious sophism."
- Hilbert: "No one shall be able to expel us from the paradise that Cantor created for us."
- Frege: "For the infinite will eventually refuse to be excluded from arithmetics . . . Thus we can foresee that this issue will provide for a momentous and decisive battle."

"The Second Most Important Logician"

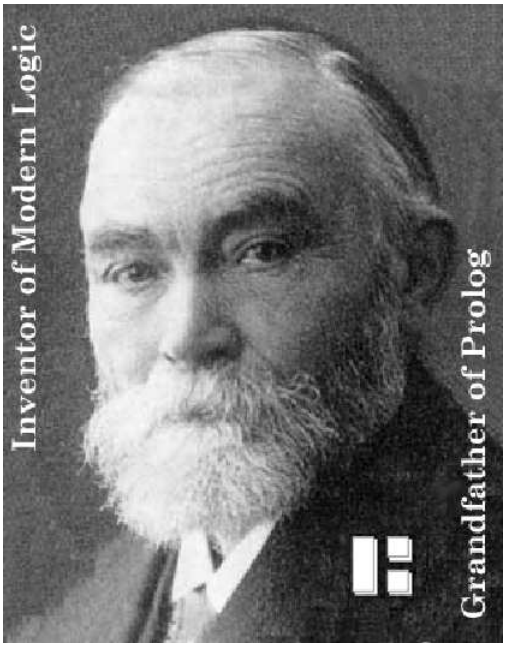


Figure 13: Friedrich Ludwig Gottlob Frege, 1848–1925

Getting Closer to Leibniz's Dream

Frege's Contribution, Begriffsschrift, 1879:

- Objects, e.g., 2
- Predicates (relationships), e.g., $2 < 3$
- Operations (functions), e.g., $2 + 3$
- Logical operations (a lá Boole), e.g., "and"
- Quantifiers, e.g., "for all"

Back to Aristotle:

- "All men are mortal"
- "For all x, if x is a man, then x is mortal"
- $(\forall x)(Man(x) \rightarrow Mortal(x))$: *First-Order Logic*

van Heijenoort, From Frege to Gödel, 1967: "perhaps the most important single work ever written in logic".

Grandson of Prime Minister

Figure 14: Bertrand Arthur William Russell, 1872–1970

Russell's Letter

A letter from Russell to Frege, June 16, 1902:

> "I find myself in agreement with you in all essentials . . . I find in your work discussions, distinctions, and definitions that one seeks in vain in the work of other logicians . . . There is just one point where I have encountered a difficulty."

Appendix to Frege's 1903 volume:

> "There is nothing worse that can happen to a scientist than to have the foundation collapse just as the work is finished. I have been placed in this position by a letter from Mr. Bertrand Russell."

Despair: Russell's criticism dealt a shattering blow to Frege's life work. At the end of his life, Frege was a man of extreme right-wing opinions, bitterly opposed to the parliamentary system, democrats, liberals, Catholics, the French, and, above all, the Jews.

Russell's Paradox

Central to Frege's Work: *sets*, i.e., collections of objects.

Russell: Is the collection of all sets that do not include themselves as a member a set? Frege's system is inconsistent!

Analogy:

Consider all men in a small town as members of a set. Imagine a barber putting up a sign ,"I shave all those men, and only those men, who do not shave themselves."

We can divide the set of men in this town into two sets, those who shave themselves, and those who are shaved by the barber. To which set does the barber himself belong?

Epimenides has spoken from his grave, after a slumber of 2,500 years. Russell has launched a foundational crisis in mathematics!

Principia Mathematica

Russell: "Every morning I would sit down before a blank sheet of paper. Throughout the day, with a brief interval for lunch, I would stare at the blank sheet. Often when evening came it was still empty. It seemed quite likely that the whole of the rest of my life might be consumed in looking at that blank sheet of paper."

Outcome: *Principia Mathematica*, by Russell and Whitehead, 1910-1913

- ten-year, monumental work
- three volumes, 2,000 pages
- systematic derivation of mathematics from logic
- avoided obvious paradoxes

Russell: "my own intellect never quite recovered from the strain of writing it,... I turned aside from mathematical logic with a kind of nausea."

But: Who could be sure there were not contradictions lurking undetected in Principia Mathematica?

The King of Mathematics

Figure 15: David Hilbert, 1862–1943

Hilbert's Program

Hilbert's Program (1922-1930):

Formalize mathematics and establish that:

- Mathematics is *consistent*: a mathematical statement and its negation cannot ever both be proved.
- Mathematics is *complete*: all true mathematical statements can be proved.
- Mathematics is *decidable*: there is a mechanical way to determine whether a given mathematical statement is true or false.

Hilbert

Hilbert wanted to "dispose of the foundational questions once and for all".

> "Every mathematical problem must necessarily be susceptible to an exact statement, either in the form of an actual answer to the question asked, or by the proof of the impossibility of its solution."
>
> "Once a logical formalism is established one can expect that a systematic, so-to-say computational, treatment of logic formulas is possible, which would somewhat correspond to the theory of equations in algebra."
>
> "Every mathematician certainly shares the conviction that every mathematical problem is necessarily capable of strict resolution. We hear within us the perpetual call. There is the problem, seek its solution. You can find it by pure reason."

We Must Know!

In 1930 Hilbert retired and the city of Königsberg made him an honorary citizen of the city. On September 8, he gave a radio address on "Natural Philosophy and Logic", which ended with six famous words showing his enthusiasm for mathematics and his life's devotion to solving mathematical problems:

> *"Wir müssen wissen, wir werden wissen–*
> *We must know, we shall know."*

The Demise of Hilbert's Program

- K. Gödel (1930-3):
  - *Incompleteness* of ordinary arithmetic
    - There is no systematic way of resolving all mathematical questions.
  - *Impossibility* of proving consistency of mathematics
- A. Church and A. Turing (1936-1937):

  Undecidability of first-order logic:
  - The set of all true first-order logic formulas is *not computable* – there is no systematic computational way to decide the truth of first-order logic formulas.

"der Herr Warum"

Figure 16: Kurt Friedrich Gödel, 1906–1978

Figure 17: Gödel and Einstein, IAS, Princeton

Death of a Program

Königsberg, September 7, 1930: Conference on The Epistemology of The Exact Sciences

- Gödel: ”One may, in fact, exhibit sentences, which, although intensionally correct (true), are not provable in the formal system of Mathematics.”

Post-Gödel von Neumann:

- von Neumann has been working on the consistency of arithmetics (Hilbert’s 2nd Problem).
- After Königsberg: “I will have nothing more to do with logic. I will never read another paper on logic.”

Socially Inept, with Erratic Grooming and a Grating Voice

Figure 18: Alan Mathison Turing, 1912–1954

Birth of Computer Science

Church, Gödel, Kleene, Post, Turing: Mathematical proofs have to be “machine checkable” - *computation* lies at the heart of mathematics!

Fundamental Question: What is “machine checkable”?

Computer science was born out of the ruins of Hilbert’s Program:

- *Algorithm*: a procedure for solving a problem by carrying out a precisely determined sequence of simpler, unambiguous steps.
- Turing:
  - distinction between *hardware* and *software*
  - a *universal* machine: a machine that can execute arbitrary programs
    - a machine “which can be made to do the work of any special-purpose machine, that is to carry out any piece of computing, if a tape bearing suitable ‘instructions’ is inserted into it.”
- Church: a *programming language*– λ-calculus, a universal language for expressing algorithms

Machina Ratiocinatrix

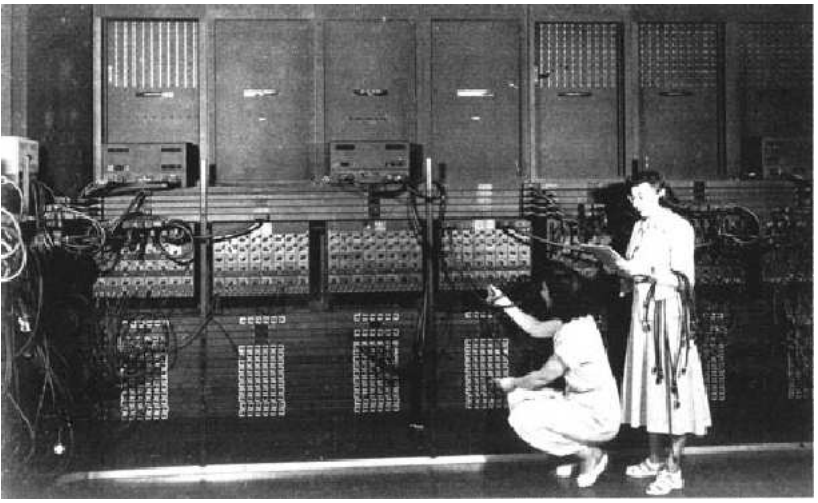


Figure 19: Betty Jennings and Frances Bilas programming the Eniac, 1946

The Last of The Great Mathematicians

Figure 20: John Louis von Neumann, 1903–1957

von Neumann and the EDVAC

ENIAC: Feb. 14, 1946

- 18,000 vacuum tubes, $100 \times 10 \times 3$ foot^3, 30 tons, 150 KW
- added 5,000 numbers in one second,
- a marvel of engineering, little theoretical basis

von Neumann

- Joined the project in August 1944
- June 1945: "A First Draft of a Report on the EDVAC" June 1946: "Preliminary discussion of the logical design of an electronic computing instrument"
- Design based on Boole's and Turing's ideas
- Modern computers: *"von Neumann architecture"*
- 1946: "I am thinking about something much more important than bombs. I am thinking about computers."

Closing the Circle

By the early 1950s, dozens of "johniacs", *reasoning machines*, have been built around the world. Leibniz's dream came true!

From	*reasoning*
to	*patterns of reasoning*
to	*logic*
to	computers
to	*computers that reason*

Logic in Computer Science

- During the past 50 years, there has been an extensive and growing interaction between logic and computer science.
- Concepts and methods of logic occupy a central place in computer science, insomuch that logic has been called *the calculus of computer science*.

M. Davis (1988): *Influences of Mathematical Logic on Computer Science*:

> "When I was a student, even the topologists regarded mathematical logicians as living in outer space. Today the connections between logic and computers are a matter of engineering practice at every level of computer organization."

Logicians' Fate - Philosophy and Mathematics

Christos H. Papadimitriou, 2001:

"And then there is the human story, the unbelievable and total sadness of it. Leibniz never had a chance to advance his dream, and he was consumed by his famous row with Isaac Newton over their simultaneous invention of the calculus. Boole lived in poverty and died relatively young of pneumonia, having walked to his lectures through rain. Cantor died in deep depression, his ingenious work ignored. Although Frege's reputation survived well the defect that Russell found in his system, it was marred forever by his vehement racism and anti-Semitism. Hilbert lived until 1943, in a strange denial of the evil that was destroying his country, whereas Gödel fled to Princeton, where, possessed by an advancing paranoia, he starved himself to death in 1978. Still, the most tragic end is Turing's. His apparent suicide in 1954 was presumably the result of his continued persecution by the country that he had served so brilliantly and crucially."

Logicians' Fate - Computer Science

Turing Award: The Association for Computing Machinery's most prestigious technical award

- Hoare, 1980: for fundamental contributions to the definition and design of programming languages.
- Codd, 1981: for fundamental contributions to the theory and practice of database management systems.
- Cook, 1982: for the advancement of our understanding of the complexity of computation.
- Milner, 1991: for the developments of logical formalisms in computer science.
- Pnueli, 1996: for seminal work introducing temporal logic into computing science.
- Clarke, Emerson, Sifakis, 2007: for the development of model checking into a highly effective verification technology

Epimenides Is Alive and Well

Postmodernism: "There is no universal truth"

Question: Is the above universally true?

The Patron Saint of Cybernetics

Leibniz:

> “Once the characteristic numbers are established for most concepts, mankind will then posses a new instrument that will enhance the capabilities of the mind to a far greater extent than optical instruments strengthen the eyes.”

Self Reference

“You are not reading this.”

More Reading

Martin Davis, Engines of Logic, 2001.