



Transition Systems and Service Composition

Giuseppe De Giacomo

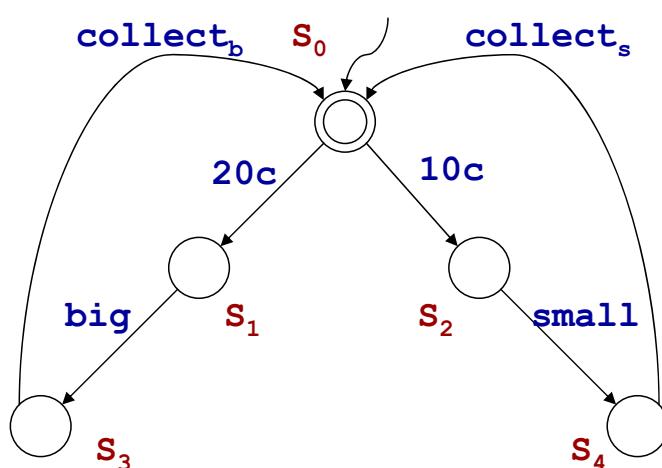
**Seminari di Ingegneria del Software
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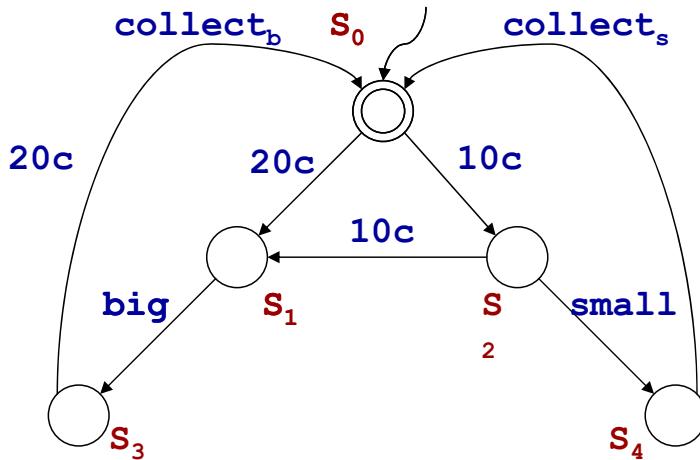
Transition Systems

- A transition system TS is a tuple $T = \langle A, S, S^0, \delta, F \rangle$ where:
 - A is the set of actions
 - S is the set of states
 - $S^0 \subseteq S$ is the set of initial states
 - $\delta \subseteq S \times A \times S$ is the transition relation
 - $F \subseteq S$ is the set of final states
 - Variants:
 - No initial states
 - Single initial state
 - Deterministic actions
 - States labeled by propositions other than Final/ \neg Final
- (c.f. Kripke Structure)

Example (Vending Machine)

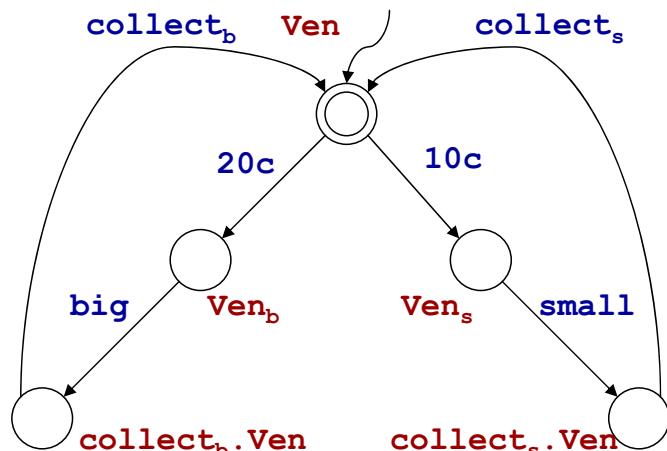


Example (Another Vending Machine)



Process Algebras are Formalisms for Describing TS

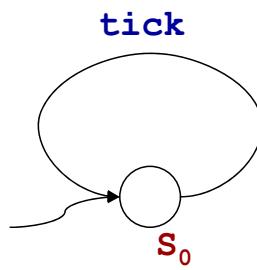
- Trans (a la CCS)
 - $Ven = 20c.Ven_b + 10c.Ven_s$
 - $Ven_b = big.collect_b.Ven$
 - $Ven_s = small.collect_s.Ven$
- Final
 - ✓ Ven



- TS may have infinite states - e.g., this happens when generated by process algebras involving iterated concurrency
- However we have good formal tools to deal only with finite states TS

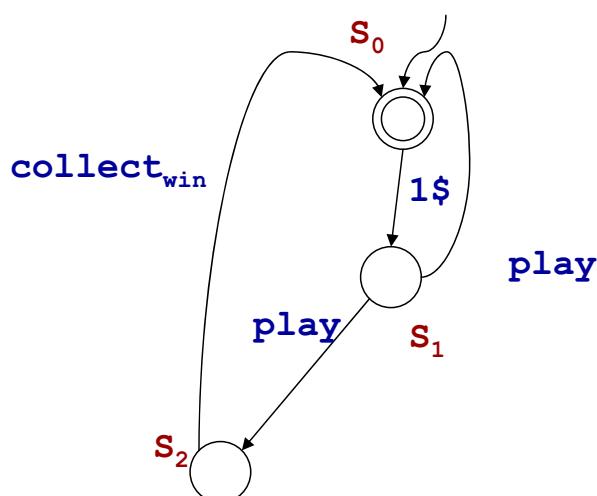
Example (Clock)

TS may describe (legal) nonterminating processes

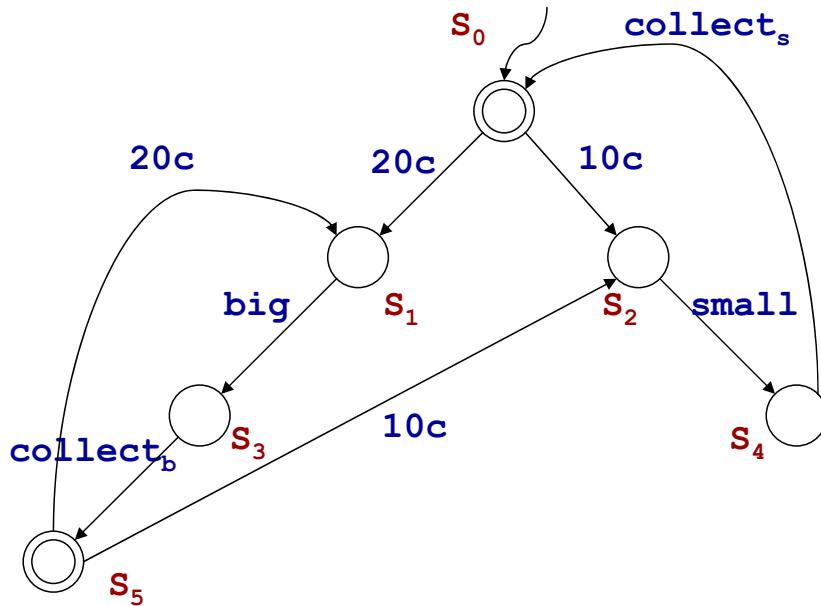


Example (Slot Machine)

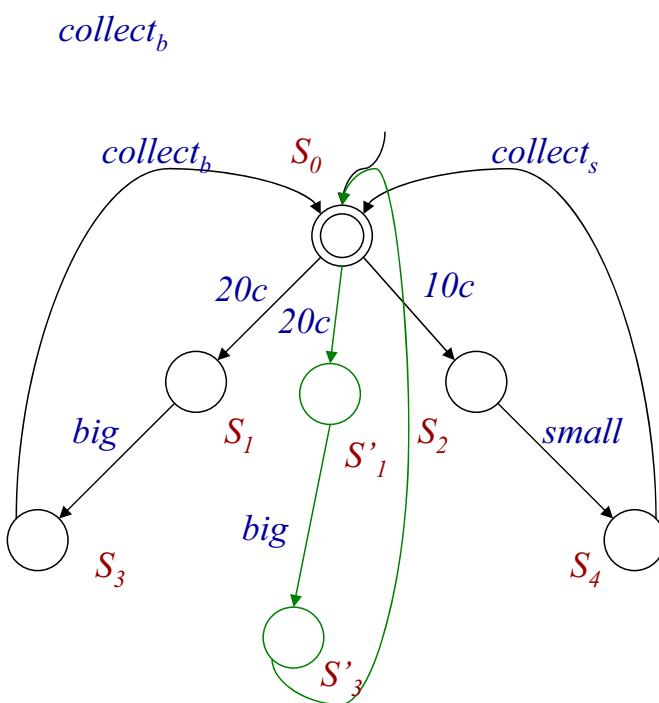
Non-deterministic transitions express
choice that is not under the control of clients



Example (Vending Machine - Variant 1)



Example (Vending Machine - Variant 2)



Bisimulation



- A binary relation R is a **bisimulation** iff:

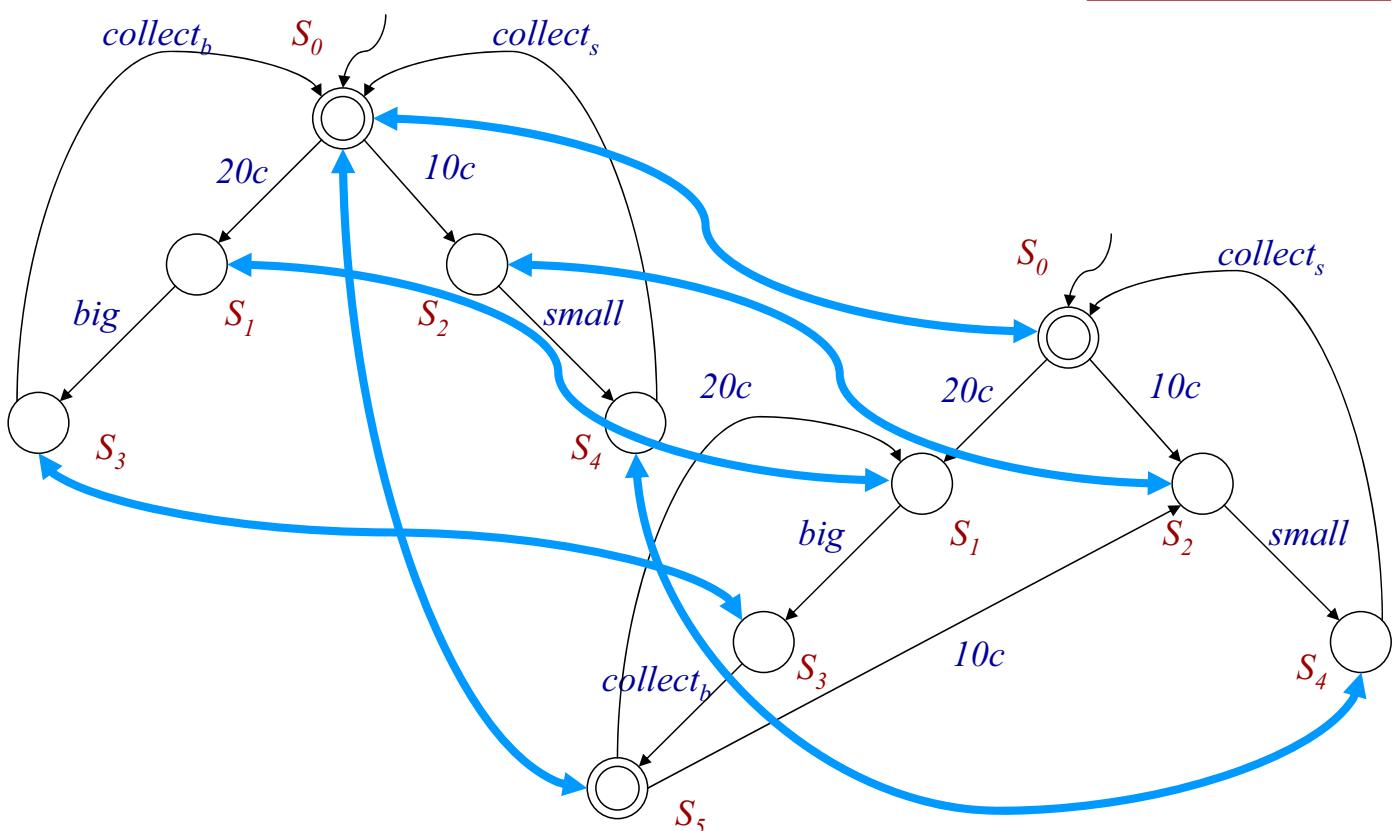
$(s,t) \in R$ implies that

- s is *final* iff t is *final*
- for all actions a
 - if $s \xrightarrow{a} s'$ then $\exists t'. t \xrightarrow{a} t'$ and $(s',t') \in R$
 - if $t \xrightarrow{a} t'$ then $\exists s'. s \xrightarrow{a} s'$ and $(s',t') \in R$

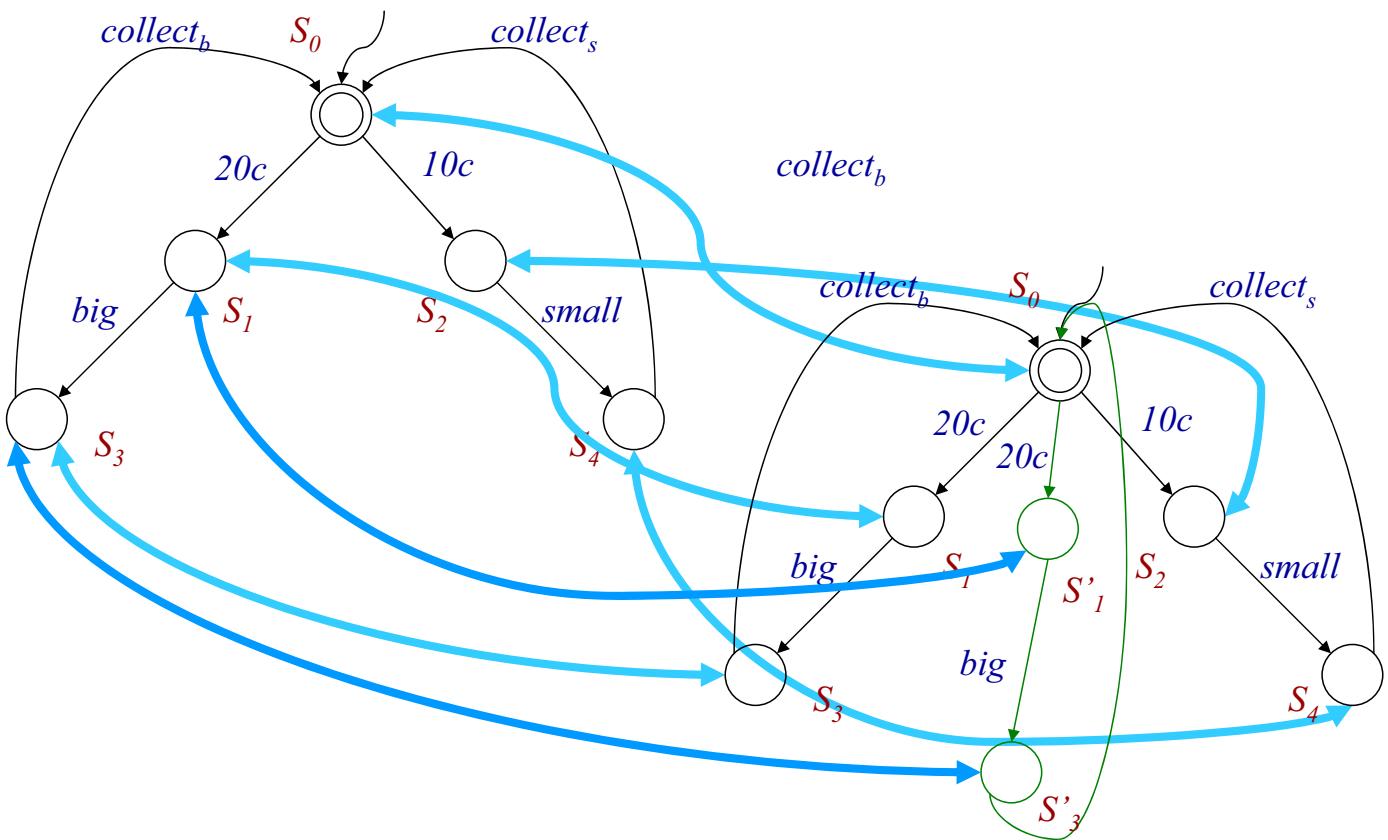
Note it is a co-inductive definition!

- A state s_0 of transition system S is **equivalent** to a state t_0 of transition system T iff there **exists** a **bisimulation** between the initial states s_0 and t_0 .

Example of Bisimulation

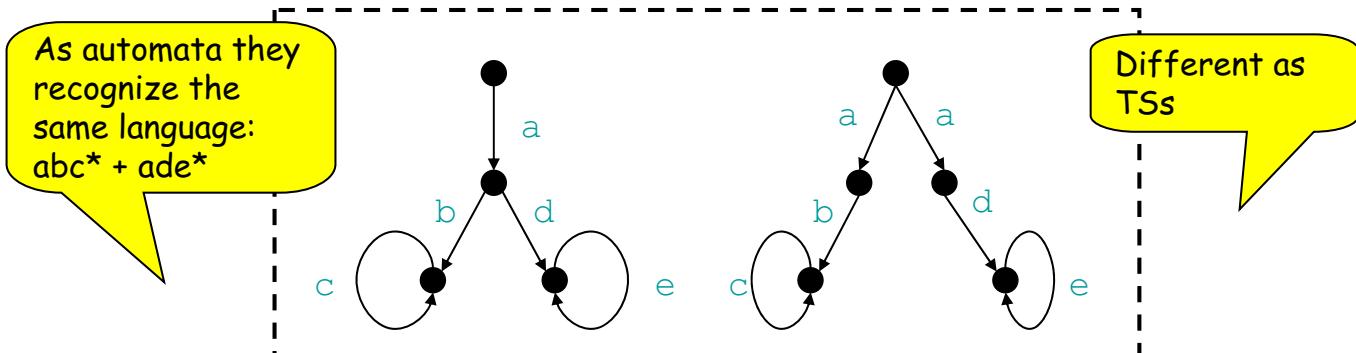


Example of Bisimulation



Automata vs. Transition Systems

- Automata
 - define sets of runs (or traces or strings): (finite) length sequences of actions
- TSs
 - ... but I can be interested also in the alternatives "encountered" during runs, as they represent client's "choice points"



Logics of Programs



- Are modal logics that allow to describe properties of transition systems
- Examples:
 - HennessyMilner Logic
 - Propositional Dynamic Logics
 - Modal (Propositional) Mu-calculus
- Perfectly suited for describing transition systems: they can tell apart transition systems modulo bisimulation

HennessyMilner Logic



- $\Phi := P \mid \neg \Phi \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 \vee \Phi_2 \mid [a]\Phi \mid <a>\Phi$
 - (atomic propositions)
 - (closed under boolean operators)
 - (modal operators)
- Propositions are used to denote final states
- $<a>\Phi$ means there exists an a-transition that leads to a state where Φ holds; i.e., expresses the capability of executing action a bringing about Φ
- $[a]\Phi$ means that all a-transitions lead to states where Φ holds; i.e., express that executing action a brings about Φ

Logics of Programs: Examples

- Useful abbreviation:
 - $\langle \text{any} \rangle \Phi$ stands for $\langle a_1 \rangle \Phi \vee \dots \vee \langle a_n \rangle \Phi$
 - $[\text{any}] \Phi$ stands for $[a_1] \Phi \wedge \dots \wedge [a_n] \Phi$
 - $\langle \text{any} - a_1 \rangle \Phi$ stands for $\langle a_2 \rangle \Phi \vee \dots \vee \langle a_v \rangle \Phi$
 - $[\text{any} - a_1] \Phi$ stands for $[a_2] \Phi \wedge \dots \wedge [a_v] \Phi$
- Examples:
 - $\langle a \rangle \text{true}$ *cabability of performing action a*
 - $[a] \text{false}$ *inability of performing action a*
 - $\neg \text{Final} \wedge \langle \text{any} \rangle \text{true} \wedge [\text{any}-a] \text{false}$
*necessity/inevitability of performing action a
i.e., action a is the only action possible*
 - $\neg \text{Final} \wedge [\text{any}] \text{false}$ *deadlock!*

Propositional Dynamic Logic

- $\Phi := P \mid \neg \Phi \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 \vee \Phi_2 \mid [r]\Phi \mid \langle r \rangle \Phi$ *(atomic propositions)
(closed under boolean operators)
(modal operators)*
- $r := a \mid r_1 + r_2 \mid r_1; r_2 \mid r^* \mid P?$ *(complex actions as regular expressions)*
- Essentially add the capability of expressing partial correctness assertions via formulas of the form
 - $\Phi_1 \rightarrow [r]\Phi_2$ *under the conditions Φ , all possible executions of r that terminate reach a state of the TS where Φ holds*
- Also add the ability of asserting that a property holds in all nodes of the transition system
 - $[(a_1 + \dots + a_v)^*]\Phi$ *in every reachable state of the TS Φ holds*
- Useful abbreviations:
 - any stands for $(a_1 + \dots + a_v)^*$ *- observe that + can be expressed in HM Logic*
 - u stands for any* *- this is the so called master/universal modality*

Modal Mu-Calculus



- $\Phi := P \mid \neg \Phi \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 \vee \Phi_2 \mid [r]\Phi \mid <r>\Phi$ (atomic propositions)
 $\quad \quad \quad$ (closed under boolean operators)
 $\quad \quad \quad$ (modal operators)
- $\mu X.\Phi(X) \mid \nu X.\Phi(X)$ (fixpoint operators)
- It is the most expressive logic of the family of logics of programs.
- It subsumes
 - PDL (modalities involving complex actions are translated into formulas involving fixpoints)
 - LTL (linear time temporal logic),
 - CTS, CTS* (branching time temporal logics)
- Examples:
 - $[\text{any}^*]\Phi$ can be expressed as $\nu X. \Phi \wedge [\text{any}]X$
 - $\mu X. \Phi \vee [\text{any}]X$ along all runs eventually Φ
 - $\mu X. \Phi \vee <\text{any}>X$ along some run eventually Φ
 - $\nu X. [a](\mu Y. <\text{any}>\text{true} \wedge [\text{any}-b]Y) \wedge X$ every run that that contains a contains later b

Model Checking



- Model checking is polynomial in the size of the TS for
 - HennessyMilner Logic
 - PDL
 - Mu-Calculus
- Also model checking is wrt the formula
 - Polynomial for HennessyMiner Logic
 - Polynomial for PDL
 - Polynomial for Mu-Calculus with bounded alternation of fixpoints and NP \cap coNP in general

Model Checking

- Given a TS T , one of its states s , and a formula Φ verify whether the formula holds in s . Formally:

$$T, s \models \Phi$$

- Examples (TS is our vending machine):

- $S_0 \models \text{Final}$
- $S_0 \models \langle 10c \rangle \text{true}$ capability of performing action 10c
- $S_2 \models [\text{big}] \text{false}$ inability of performing action big
- $S_0 \models [10c][\text{big}] \text{false}$ after 10c cannot execute big
- $S_i \models \mu X. \text{Final} \vee [\text{any}] X$ eventually a final state is reached
- $S_0 \models \forall Z. (\mu X. \text{Final} \vee [\text{any}] X) \wedge [\text{any}] Z$ or equivalently
 $S_0 \models [\text{any}^*](\mu X. \text{Final} \vee [\text{any}] X)$ from everywhere eventually final

Planning as Model Checking

- Build the TS of the domain:

- Consider the set of states formed all possible truth value of the propositions (this works only for propositional setting).
 - Use Pre's and Post of actions for determining the transitions
- Note: the TS is exponential in the size od the description.

- Write the goal in a logic of program

- typically a single least fixpoint formula of Mu-Calculus

- Planning:

- model check the formula on the TS starting from the given initial state.
- use the path (paths) used in the above model checking for returning the plan.

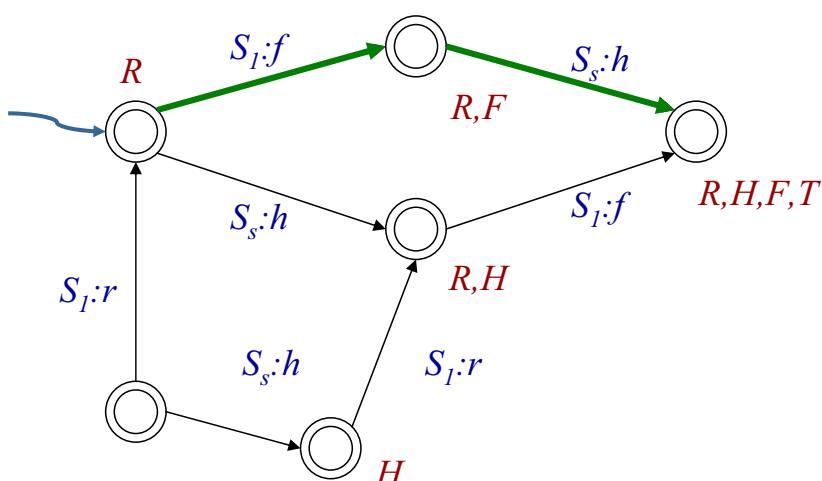
- This basic technique works only when we have complete information (or at least total observability on state):

- Sequential plans if initial state known and actions are deterministic
- Conditional plans if many possible initial states and/or actions are nondeterministic

Example

- Operators (Services + Mappings)
 - $\text{Registered} \wedge \neg \text{FlightBooked} \rightarrow [S_1:\text{bookFlight}] \text{ FlightBooked}$
 - $\neg \text{Registered} \rightarrow [S_1:\text{register}] \text{ Registered}$
 - $\neg \text{HotelBooked} \rightarrow [S_2:\text{bookHotel}] \text{ HotelBooked}$
- Additional constraints (Community Ontology):
 - $\text{TravelSettledUp} \equiv \text{FlightBooked} \wedge \text{HotelBooked} \wedge \text{EventBooked}$
- Goals (Client Service Requests):
 - Starting from state
 $\text{Registered} \wedge \neg \text{FlightBooked} \wedge \neg \text{HotelBooked} \wedge \neg \text{EventBooked}$
 $\text{check } <\text{any}^*> \text{TravelSettledUp}$
 - Starting from all states such that
 $\neg \text{FlightBooked} \wedge \neg \text{HotelBooked} \wedge \neg \text{EventBooked}$
 $\text{check } <\text{any}^*> \text{TravelSettledUp}$

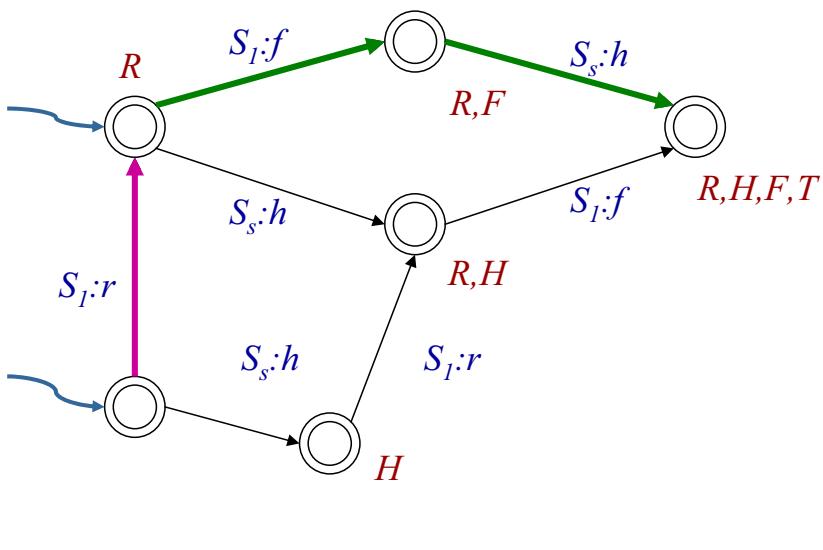
Example



Plan:
 $S_1:\text{bookFlight};$
 $S_2:\text{bookHotel}$

Starting from state
 $\text{Registered} \wedge \neg \text{FlightBooked} \wedge \neg \text{HotelBooked} \wedge \neg \text{EventBooked}$
 $\text{check } <\text{any}^*> \text{TravelSettledUp}$

Example



Plan:

```

if(¬Registered) {
  S1:register;
}
S1:bookFlight;
S2:bookHotel
  
```

Starting from states where
 $\neg \text{FlightBooked} \wedge \neg \text{HotelBooked} \wedge \neg \text{EventBooked}$
check
 $\langle \text{any}^* \rangle \text{TravelSettledUp}$

Satisfiability

- Observe that a formula Φ may be used to select among all TS T those such that for a given state s we have that $T,s \models \Phi$
- **SATISFIABILITY:** Given a formula Φ verify whether there exists a TS T and a state s such that. Formally:
 check whether exists T, s such that $T,s \models \Phi$
- Satisfiability is:
 - PSPACE for HennessyMilner Logic
 - EXPTIME for PDL
 - EXPTIME for Mu-Calculus

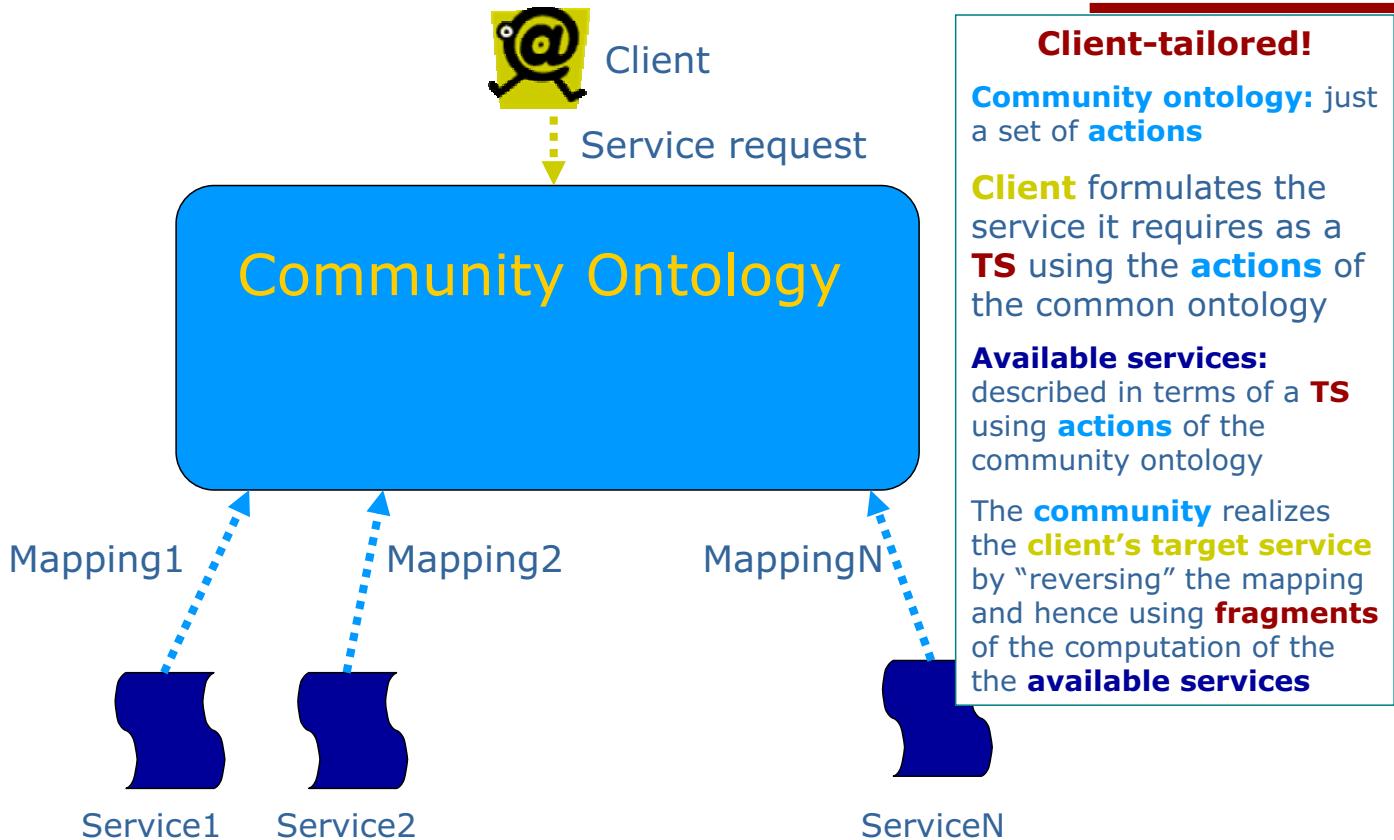
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Name by
Rick Hull

Composition: the "Roman" Approach

The Roman Approach

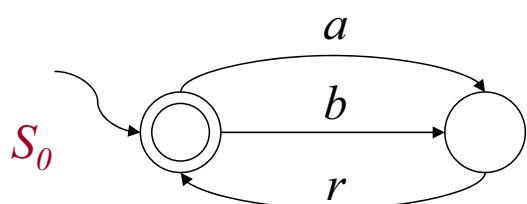


(Target & Available) Service TS

- We model services as finite TS $T = (\Sigma, S, s^0, \delta, F)$ with
 - single initial state (s^0)
 - deterministic transitions (i.e., δ is a partial function from $S \times \Sigma$ to S)

Note: In this way the client entirely controls/chooses the transition to execute

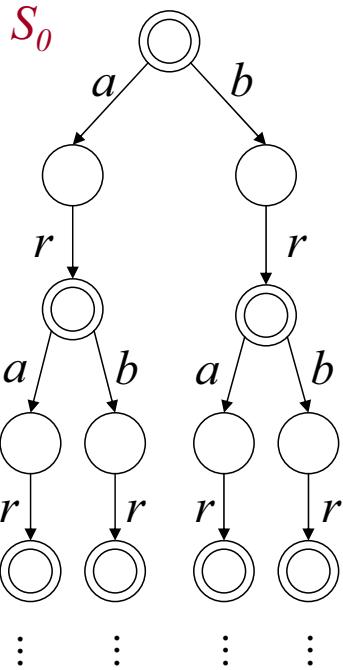
Example:



- a: "search by author (and select)"
- b: "search by title (and select)"
- r: "listen (the selected song)"

Service Execution Tree

By "unfolding" a (finite) TS one gets an (infinite) execution tree
-- yet another (infinite) TS which bisimilar to the original one)



- *Nodes: history (sequence) of actions executed so far*
- *Root: no action yet performed*
- *Successor node $x \cdot a$ of x : action a can be executed after the sequence of action x*
- *Final nodes: the service can terminate*

Formalizing Service Composition

Composition:

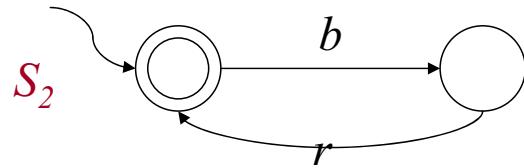
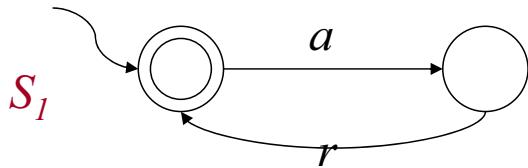
- coordinating program ...
- ... that realizes the target service ...
- ... by suitably coordinating available services

⇒ Composition can be formalized as:

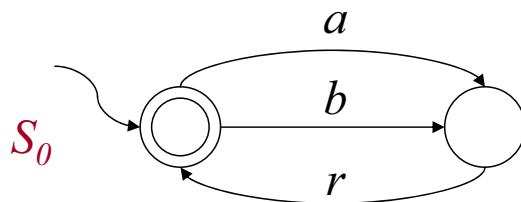
- a labeling of the execution tree of the target service such that ...
- ... each action in the execution tree is labeled by the available service that executes it ...
- ... and each possible sequence of actions on the target service execution tree corresponds to possible sequences of actions on the available service execution trees, suitably interleaved

Example of Composition (1)

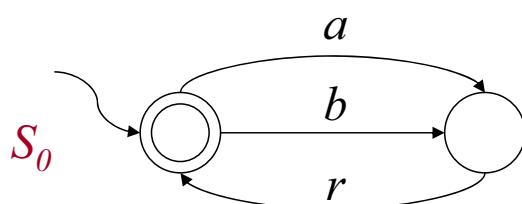
- Available services



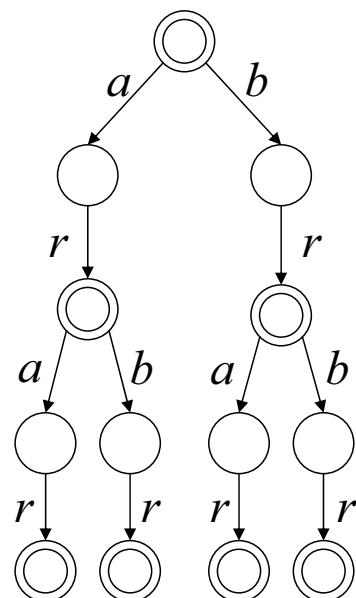
- Target service



Example of Composition (2)



Execution tree of S_0

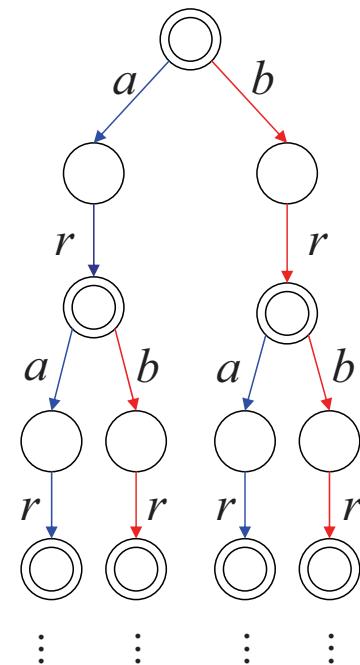
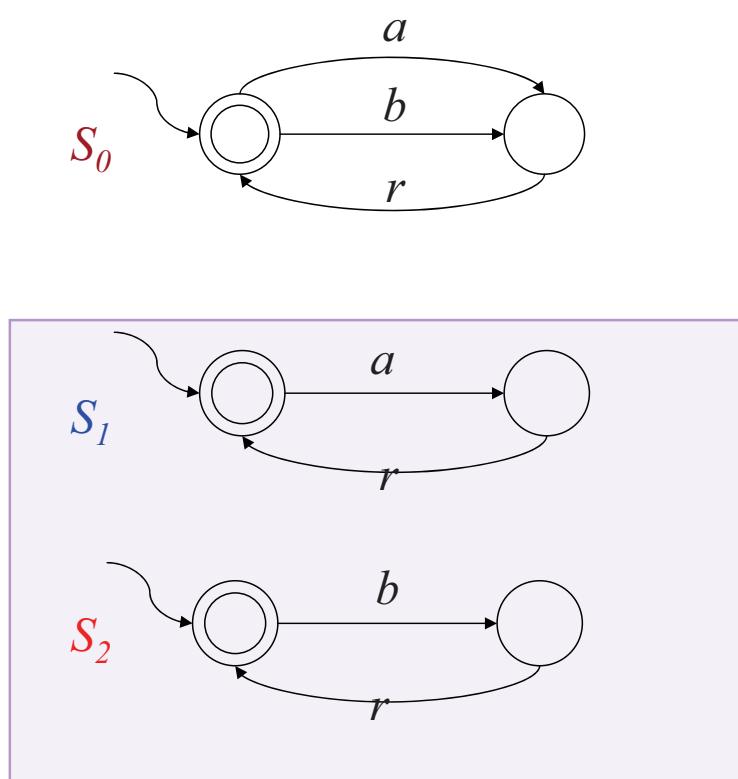


Note: we cannot label the target service TS directly ...

... we need to label the execution tree

Example of Composition (3)

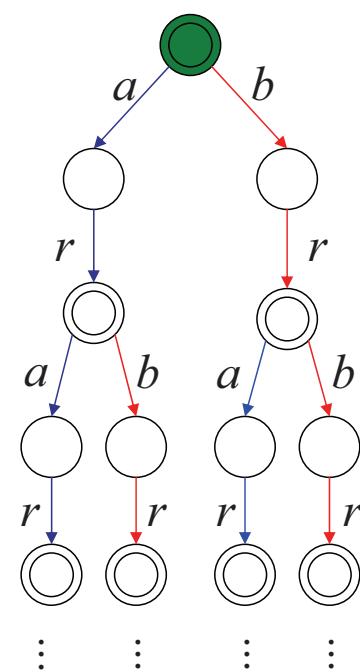
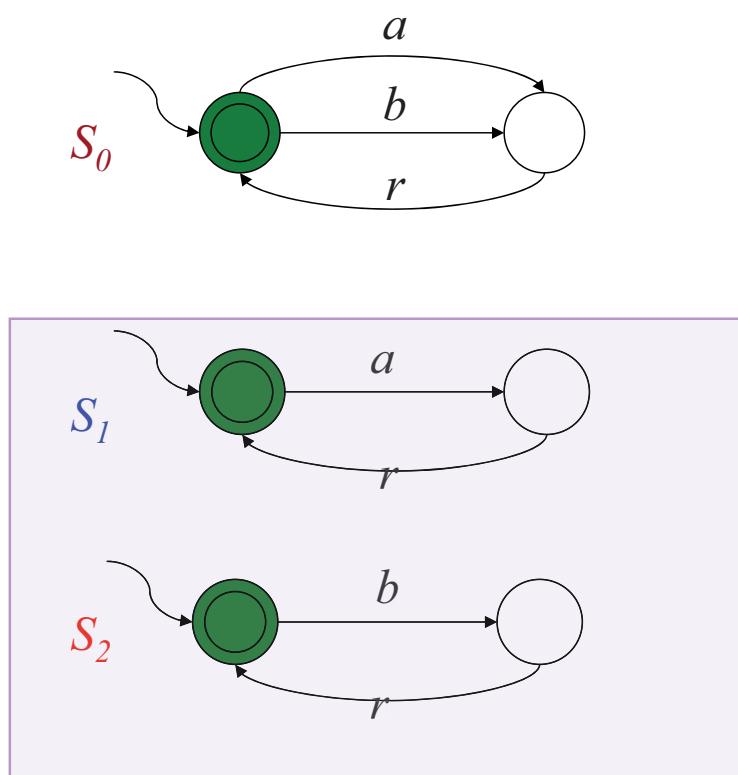
$$S_0 = \text{orch}(S_1 \parallel S_2)$$



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Example of Composition (4)

$$S_0 = \text{orch}(S_1 \parallel S_2)$$

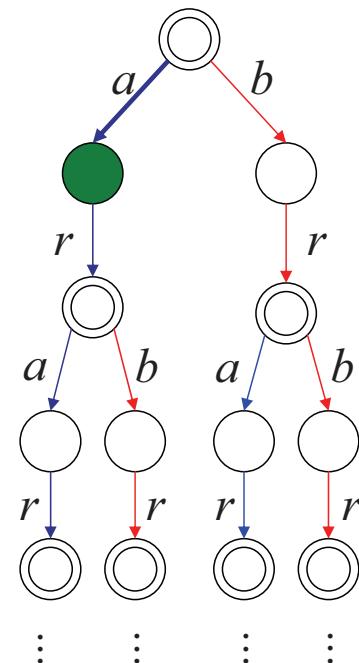
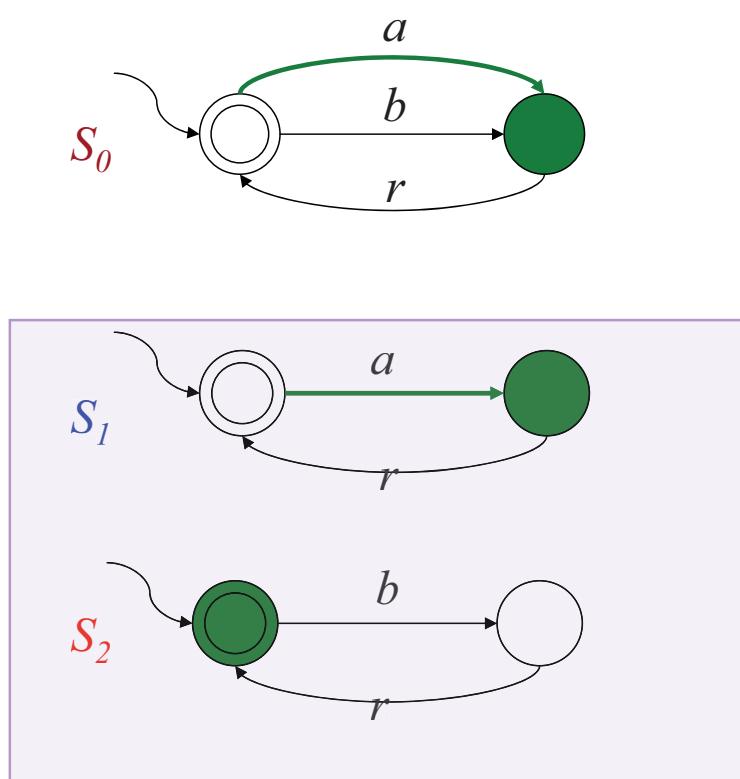


All services start from their starting state

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Example of Composition (5)

$$S_0 = \text{orch}(S_1 \parallel S_2)$$

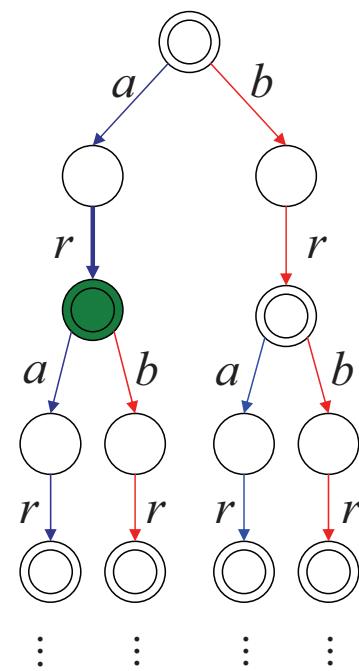
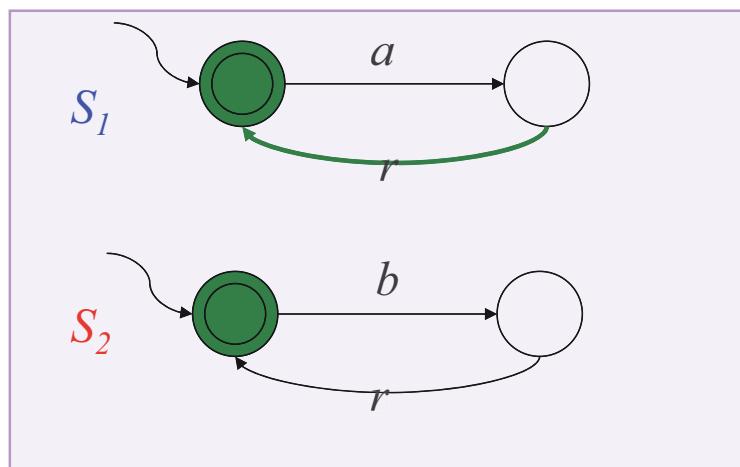


Each action of the target service is executed by at least one of the component services

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Example of composition (6)

$$S_0 = \text{orch}(S_1 \parallel S_2)$$

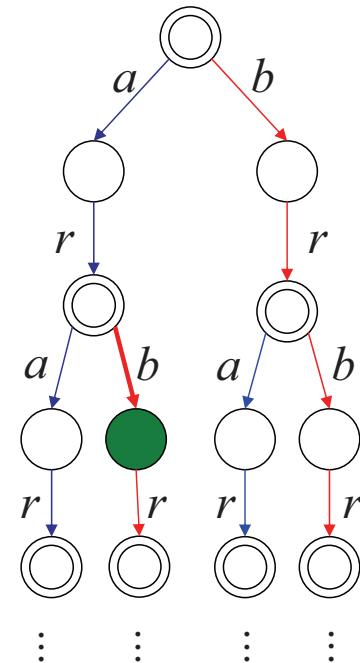
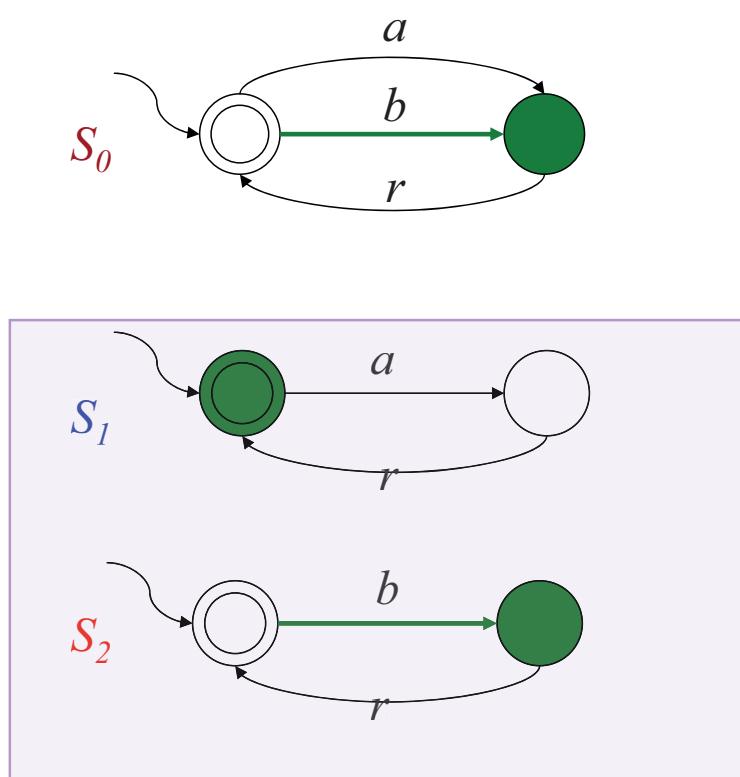


When the target service can be left, then all component services must be in a final state

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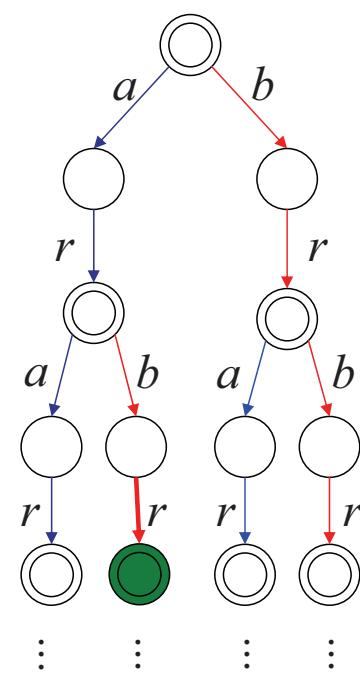
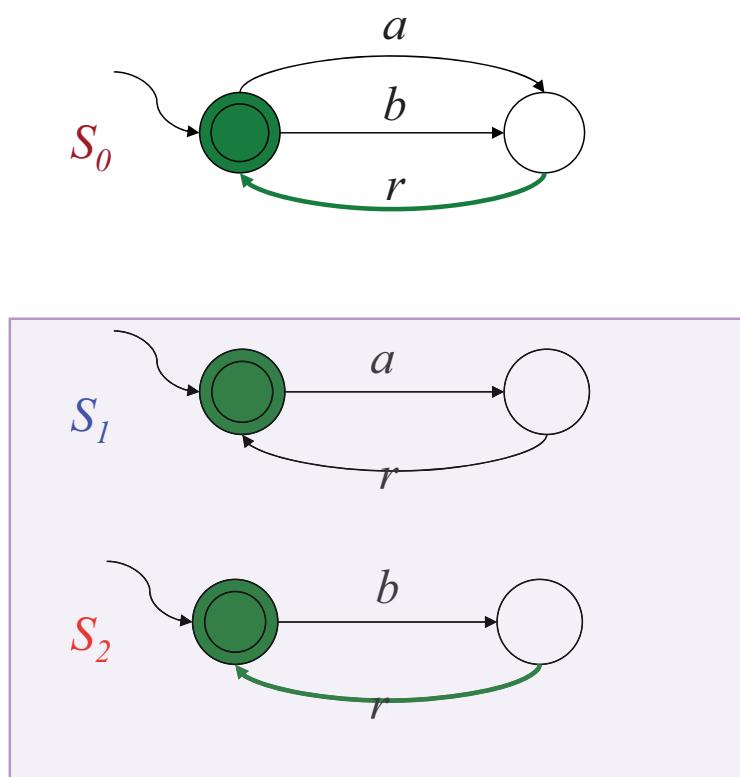
Example of composition (7)

$$S_0 = \text{orch}(S_1 \parallel S_2)$$



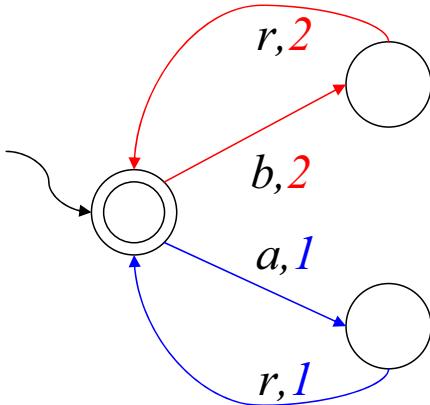
Example of composition (8)

$$S_0 = \text{orch}(S_1 \parallel S_2)$$



Observation

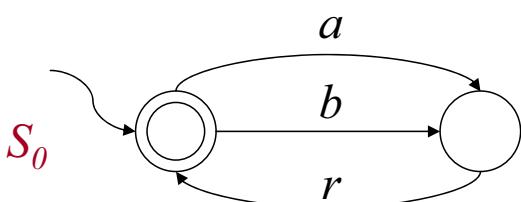
- This labeled execution tree has a finite representation as a **finite TS** ...
- ...with transitions labeled by an action and the **service** performing the action



Is this always the case when we deal with services expressible as finite TS? See later...

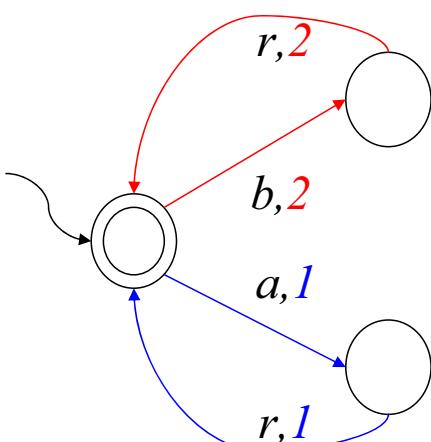
TS for Services and TS for Composition

Finite TS for services



- Deterministic
- Transitions labeled by actions
- Output on state to signal when final

Finite TS for composition



- Deterministic
- Transitions labeled by actions and services
- Output on transition to signal which service

Questions

Assume services of community and target service are finite TSs

- Can we always check composition existence?
- If a composition exists there exists one which is a finite TS?
- If yes, how can a finite TS composition be computed?

To answer we exploit PDL SAT

Answers

Reduce service composition synthesis to satisfiability in (deterministic) PDL

- Can we always check composition existence?
Yes, SAT in PDL is decidable in EXPTIME
- If a composition exists there exists one which is a finite TS?
Yes, by the small model property of PDL
- How can a finite TS composition be computed?
From a (small) model of the corresponding PDL formula

Structure of the PDL Encoding

$$\Phi = \text{Init} \wedge [u](\Phi_0 \wedge \bigwedge_{i=1, \dots, n} \Phi_i \wedge \Phi_{aux})$$

Initial states of all services

PDL encoding of i th component service

PDL encoding of target service

PDL additional domain-independent conditions

PDL encoding is polynomial in the size of the service TSs

PDL Encoding

- Target service $S_0 = (\Sigma, S_0, s^0_0, \delta_0, F_0)$ in PDL we define Φ_0 as the conjunction of:
 - $s \rightarrow \neg s'$ for all pairs of distinct states in S_0
service states are pair-wise disjoint
 - $s \rightarrow \langle a \rangle T \wedge [a]s'$ for each $s' = \delta_0(s, a)$
target service can do an a-transition going to state s'
 - $s \rightarrow [a] \perp$ for each $\delta_0(s, a)$ undef.
target service cannot do an a-transition
 - $F_0 \equiv \vee_{s \in F_0} s$
denotes target service final states
- ...

PDL Encoding (cont.d)

- available services $S_i = (\Sigma, S_i, s^0_i, \delta_i, F_i)$ in PDL we define Φ_i as the conjunction of:
 - $s \rightarrow \neg s'$ for all pairs of distinct states in S_i
Service states are pair-wise disjoint
 - $s \rightarrow [a](\text{moved}_i \wedge s' \vee \neg \text{moved}_i \wedge s)$ for each $s' = \delta_i(s, a)$
if service moved then new state, otherwise old state
 - $s \rightarrow [a](\neg \text{moved}_i \wedge s)$ for each $\delta_i(s, a)$ undef.
if service cannot do a, and a is performed then it did not move
 - $F_i \equiv \vee_{s \in F_i} s$ denotes available service final states
- ...

PDL Encoding (cont.d)

- Additional assertions Φ_{aux}
 - $< a > T \rightarrow [a] \vee_{i=1, \dots, n} \text{moved}_i$ for each action a
at least one of the available services must move at each step
 - $F_0 \rightarrow \wedge_{i=1, \dots, n} F_i$ when target service is final all comm. services are final
 - $\text{Init} \equiv s^0_0 \wedge_{i=1, \dots, n} s^0_i$ Initially all services are in their initial state

$$\text{PDL encoding: } \Phi = \text{Init} \wedge [u](\Phi_0 \wedge \bigwedge_{i=1, \dots, n} \Phi_i \wedge \Phi_{aux})$$

Results



Thm: Composition exists iff PDL formula Φ SAT

From composition labeling of the target service one can build a tree model of the PDL formula and viceversa

Information on the labeling is encoded in predicates moved,

⇒ Composition existence of services expressible as finite TS is decidable in EXPTIME

Results on TS Composition



Thm: If composition exists then finite TS composition exists.

*From a small model of the PDL formula Φ ,
one can build a finite TS machine*

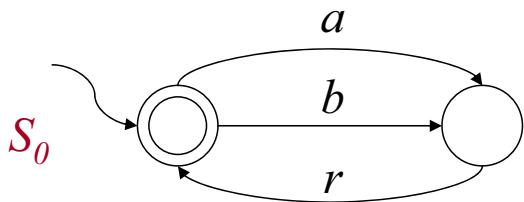
Information on the output function of the machine is encoded in predicates moved,

⇒ finite TS composition existence of services expressible as finite TS is decidable in EXPTIME

Example (1)

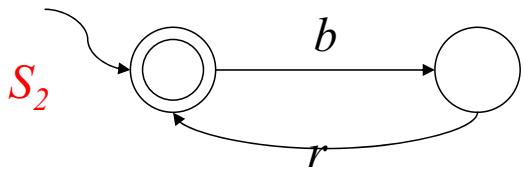
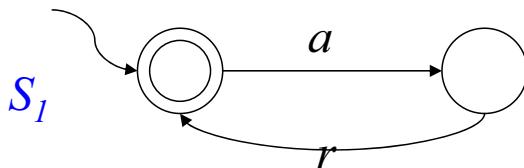


Target service



DPDL

Available services



...
...
...

$$s_0^0 \wedge s_1^0 \wedge s_2^0$$

$$\langle a \rangle T \rightarrow [a] (\text{moved}_1 \vee \text{moved}_2)$$

$$\langle b \rangle T \rightarrow [b] (\text{moved}_1 \vee \text{moved}_2)$$

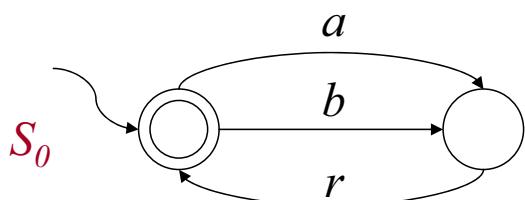
$$\langle r \rangle T \rightarrow [r] (\text{moved}_1 \vee \text{moved}_2)$$

$$F_0 \rightarrow F_1 \wedge F_2$$

Example (2)



Target service



$$s_0^0 \rightarrow \neg s_0^1$$

$$s_0^0 \rightarrow \langle a \rangle T \wedge [a] s_0^1$$

$$s_0^0 \rightarrow \langle b \rangle T \wedge [b] s_0^1$$

$$s_0^1 \rightarrow \langle r \rangle T \wedge [r] s_0^0$$

$$s_0^0 \rightarrow [r] \perp$$

$$s_0^1 \rightarrow [a] \perp$$

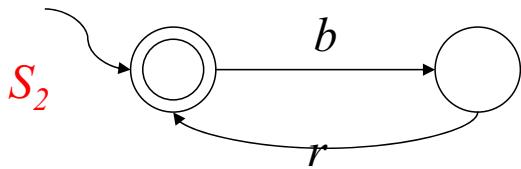
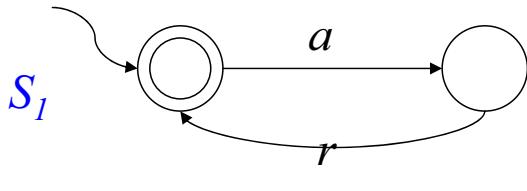
$$s_0^1 \rightarrow [b] \perp$$

$$F_0 \equiv s_0^0$$

...
...
...

Example (3)

Available services



...

$$\begin{aligned}
 s_1^0 &\rightarrow \neg s_1^1 \\
 s_1^0 &\rightarrow [a] (\text{moved}_1 \wedge s_1^1 \vee \neg \text{moved}_1 \wedge s_1^0) \\
 s_1^0 &\rightarrow [r] \neg \text{moved}_1 \wedge s_1^0 \\
 s_1^0 &\rightarrow [b] \neg \text{moved}_1 \wedge s_1^0 \\
 s_1^1 &\rightarrow [a] \neg \text{moved}_1 \wedge s_1^1 \\
 s_1^1 &\rightarrow [b] \neg \text{moved}_1 \wedge s_1^1 \\
 s_1^1 &\rightarrow [r] (\text{moved}_1 \wedge s_1^0 \vee \neg \text{moved}_1 \wedge s_1^0) \\
 F_1 &\equiv s_1^0
 \end{aligned}$$

$$\begin{aligned}
 s_2^0 &\rightarrow \neg s_2^1 \\
 s_2^0 &\rightarrow [b] (\text{moved}_2 \wedge s_2^1 \vee \neg \text{moved}_2 \wedge s_2^0) \\
 s_2^0 &\rightarrow [r] \neg \text{moved}_2 \wedge s_2^0 \\
 s_2^0 &\rightarrow [a] \neg \text{moved}_2 \wedge s_2^0 \\
 s_2^1 &\rightarrow [b] \neg \text{moved}_2 \wedge s_2^1 \\
 s_2^1 &\rightarrow [a] \neg \text{moved}_2 \wedge s_2^1 \\
 s_2^1 &\rightarrow [r] (\text{moved}_2 \wedge s_2^0 \vee \neg \text{moved}_2 \wedge s_2^0) \\
 F_2 &\equiv s_2^0
 \end{aligned}$$

...

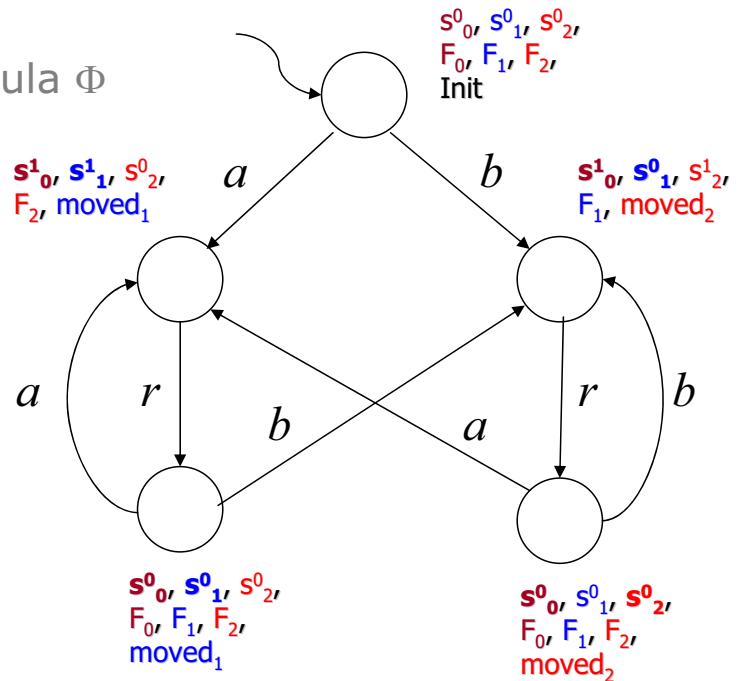
Example (4)

Check: run SAT on PDL formula Φ

Example

Check: run SAT on PDL formula Φ

Yes \Rightarrow (small) model

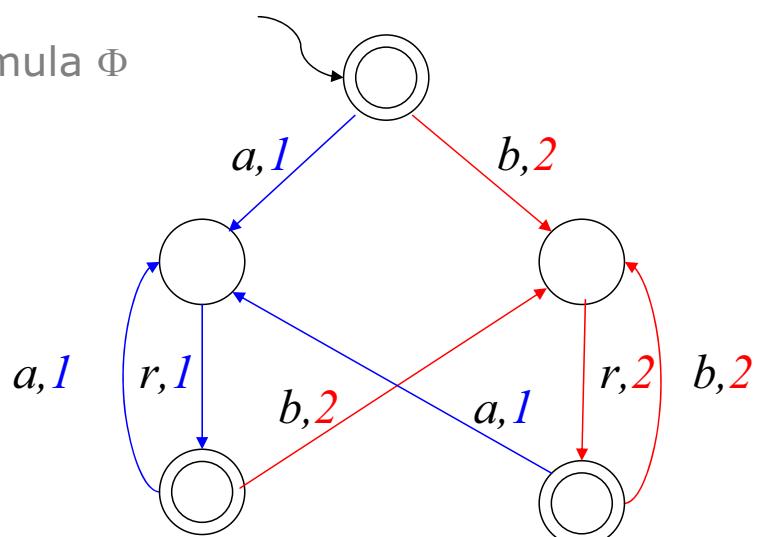


Example

Check: run SAT on PDL formula Φ

Yes \Rightarrow (small) model

\Rightarrow extract finite TS



Example

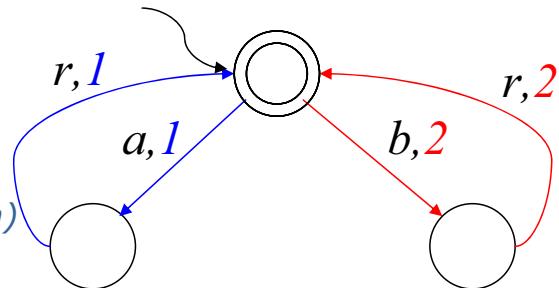
Check: run SAT on PDL formula Φ

Yes \Rightarrow (small) model

\Rightarrow extract finite TS

\Rightarrow minimize finite TS

(similar to Mealy machine minimization)



Results on Synthesizing Composition

- Using PDL reasoning algorithms based on model construction (cf. tableaux), build a (small) model
Exponential in the size of the PDL encoding/services finite TS

*Note: SitCalc, etc. can compactly represent finite TS,
PDL encoding can preserve compactness of representation*
- From this model extract a corresponding finite TS
Polynomial in the size of the model
- Minimize such a finite TS using standard techniques (opt.)
Polynomial in the size of the TS

*Note: finite TS extracted from the model is not minimal
because encodes output in properties of individuals/states*

Tools for Synthesizing Composition



- In fact we use only a fragment of PDL in particular we use fixpoint (transitive closure) only to get the universal modality ...
- ... thanks to a tight correspondence between PDLs and Description Logics (DLs), we can use current highly optimized DL reasoning systems to do synthesis ...
- ... when the ability of returning models will be added ...
- ... meanwhile we have developed a prototype tool on this idea (see ESC – E-Service Composer:
<http://sourceforge.net/projects/paride>)



END

Composition by Simulation

Simulation

- A binary relation R is a **simulation** iff:

$(s,t) \in R$ implies that

- if s is *final* then t is *final*
- for all actions a
 - if $s \xrightarrow{a} s'$ then $\exists t' . t \xrightarrow{a} t'$ and $(s',t') \in R$

Note it is a co-inductive definition!

Essentially is one direction of the bisimulation!

- A transition system S is **simulates** a transition system T iff there **exists** a **simulation** between the initial states s_0 and t_0 .

Potential behavior of the community

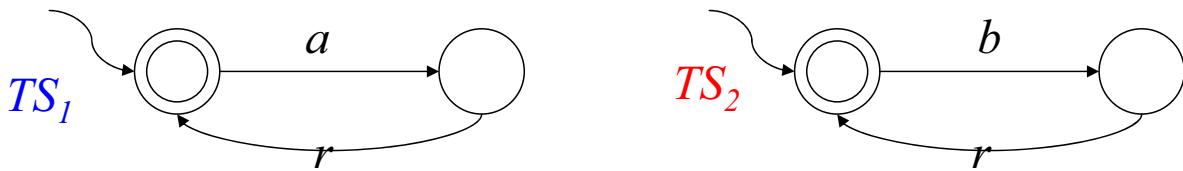
- Let TS_1, \dots, TS_n be the TSs of the component services.
- The Community Big TS is defined as
 $TS_c = \langle A, S_c, S_c^0, \delta_c, F_c \rangle$ where:
 - A is the set of actions
 - $S_c = S_1 \times \dots \times S_n$
 - $S_c^0 = \{(s_1^0, \dots, s_n^0)\}$
 - $F \subseteq F_1 \times \dots \times F_n$
 - $\delta_c \subseteq S_c \times A \times S_c$ is defined as follows:
 $(s_1 \times \dots \times s_n) \rightarrow_a (s'_1 \times \dots \times s'_n)$ iff
 - $\exists i. s_i \rightarrow_a s'_i \in \delta_i$
 - $\forall j. s_j \rightarrow_a s'_j \in \delta_j \vee s'_j = s_j$

Composition by Simulation

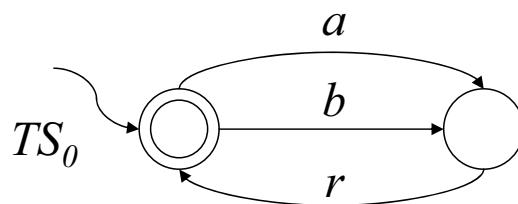
- Thm:** A composition exists iff there exists a simulation from (the initial state of) TS_c to (the initial state of) TS_t
- Given a simulation R from TS_c to TS_t (which include the initial states), we can build a finite composition as follows:
 $TS = \langle A_r, S_r, S_r^0, \delta_r, F_r \rangle$ with
 - $A_r = A \times 2^{[n]}$
 - $S_c = S_1 \times \dots \times S_n \times S_t$
 - $S_c^0 = \{ (s_1^0, \dots, s_n^0, s_t^0) \}$
 - $F \subseteq \{ (s_1 \times \dots \times s_n \times s) \mid s \in F_t \}$
 - $\delta_r \subseteq S_r \times A_r \times S_r$ is defined as follows:
 $(s_1 \times \dots \times s_n, s) \rightarrow_{a,I} (s'_1 \times \dots \times s'_n, s')$ iff
 - $s \rightarrow_a s'$
 - $(s_1 \times \dots \times s_n) \rightarrow_{a,I} (s'_1 \times \dots \times s'_n)$
 - $((s'_1 \times \dots \times s'_n), s) \in R$
 - $I = (j \mid s_j \rightarrow_a s'_j \in \delta_j)$

Example of Composition

- Available Services



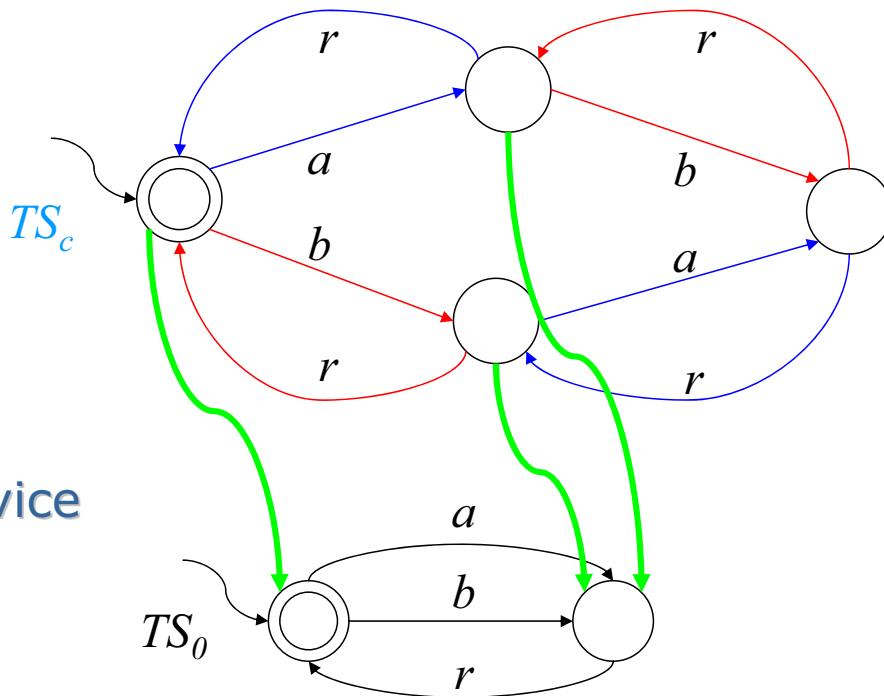
- Target Service



Example of Composition

Community Big Service

Target Service



Composition exists!