Epistemic First-Order Queries over Description Logic Knowledge Bases

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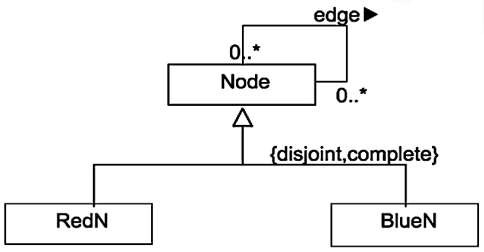
2006 International Workshop on Description Logics
Lake District, U.K., May 30 -- June 1, 2006

Motivation

- Good techniques for doing instance checking are known
  - But DLs are poor query languages [Lenzerini-Schaerf-AAAI'91]
  - Also for most DLs, coNP-hard in data complexity
- Techniques for answering CQs and UCQs are known
  - High complexity for expressive DLs [cf. next talk]
  - In LOGSPACE (as for SQL in DBs) for DL-lite
- What about going beyond UCQs?
  - FOL/SQL queries over KB are undecidable

But, often users expect to have SQL-like query capabilities!!!

Example

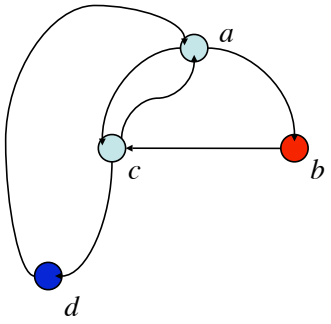


- TBox:

  $\exists\, edge^- \sqsubseteq Node$
  $\exists\, edge \sqsubseteq Node$
  $RedN \sqsubseteq Node$
  $BlueN \sqsubseteq Node$
  $RedN \sqsubseteq \neg BlueN$
  $Node \sqsubseteq RedN \sqcup BlueN$

- ABox:

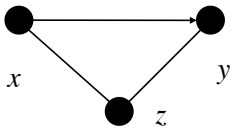
  $edge(a,b)$
  $edge(b,c)$
  $edge(c,a)$
  $edge(c,d)$
  $edge(d,a)$
  $RedN(b)$
  $BlueN(d)$



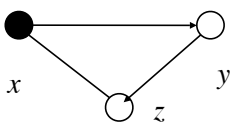
Queries

$q(x) \text{ :- } \exists\, y, z, w.\ edge(x,y) \wedge edge(y,z) \wedge edge(z,w)$

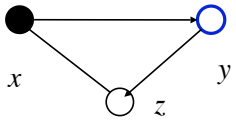
$q(x,y,z) \text{ :- } edge(x,y) \wedge edge(y,z) \wedge edge(z,x)$

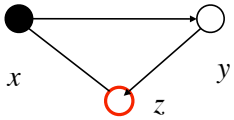


$q(x) \text{ :- } \exists\, y, z.\ edge(x,y) \wedge edge(y,z) \wedge edge(z,x)$



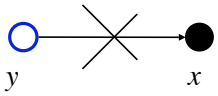
$q(x) \text{ :- } \exists\, y, z.\ edge(x,y) \wedge edge(y,z) \wedge edge(z,x) \wedge (BlueN(y) \vee RedN(z))$



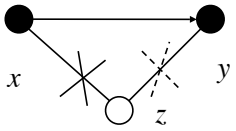


Queries

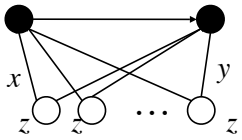
$q(x) :- \exists y.\ BlueN(y) \wedge \neg edge(x,y)$



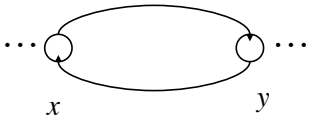
$q(x,y) :- edge(x,y) \wedge \neg\exists z.\ (edge\ (z,x) \wedge edge(z,y))$



$q(x,y) :- edge(x,y) \wedge \forall z.\ (edge\ (z,x) \Rightarrow edge(z,y))$



$q() :- \forall x,y.\ (edge\ (x,y) \Rightarrow edge(y,x))$



An experiment on relational databases

Person

name	birthdate
john	1940
paul	1942
george	1943
richard	null

SQL query:

$q(x) :- \exists b.(Person(x,b) \wedge b = 1940) \vee$
$\exists b.(Person(x,b) \wedge b \neq 1940)$

Answer:

$\{john, paul, george\}$

*What about richard? Since the DBMS **doesn't know** his birthdate, the DBMS can't extablish whether it is equal to 1940 or different from 1940, hence the DBMS skips it!*

Epistemic Query Language (EQL)

- Let KB be a DL KB, intepreted over fixed domain Δ and standard names
- EQL = FOL + epistemic operator (minimal knowledge) over KB

$$\varphi ::= A(t) \mid P(t_1,...,t_n) \mid t_1 = t_2 \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \exists x.\varphi \mid \mathbf{K}\varphi$$

A : concept name in *KB*
P : role/relation name in *KB*
t : constant in *KB* or variable

Cf:

- Levesque's "*Foundations of a Functional Approach to KR*" [– AIJ'84]
- Reiter's "*What should a DB know?*" [– JLP'92]
- Levesque & Lakemeyer's "*The Logic of KBs*" [- Book'01]
- Epistemic operator and DLs [Donini-Lenzerini-Nardi-Schaerf-Nutt-KR'92]

EQL: semantics

- Let KB a KB and φ a EQL formula
- Epistemic interpretation E,w
  - E is the set of **all** models of KB
  - w is **one** such model
- φ true in E,w, written $E,w \models \varphi$

$E,w \models A(c)$	iff	$c \in A^w$
$E,w \models P(c_1,...,c_n)$	iff	$(c_1,...,c_n) \in P^w$
$E,w \models c_1=c_2$	iff	$c_1=c_2$
$E,w \models \neg\varphi$	iff	$E,w \not\models \varphi$
$E,w \models \varphi_1 \wedge \varphi_2$	iff	$E,w \models \varphi_1$ and $E,w \models \varphi_2$
$E,w \models \exists x.\varphi(x)$	iff	$E,w \models \varphi(c)$ for some c
$E,w \models \mathbf{K}\varphi$	iff	$E,v \models \varphi$ for all $v \in E$

EQL: objective and subjective formulas

- Objective formulas
  - no occurrence of ***K***
  - talk about what is true in the world
  - example: $\exists x, y.\ edge(x,y)$
  - $E,w \models \varphi$ reduces to $w \models \varphi$
- Subjective formulas
  - all atoms under the scope of ***K***
  - talk about what is known by the KB
  - example: $\exists x, y.\ \boldsymbol{K}\ edge(x,y)$
  - $E,w \models \varphi$ reduces to $E \models \varphi$
- Non objective and non subjective formulas
  - talk about what is true in world in relation to what is known by the KB
  - example: $\exists x, y.\ edge(x,y) \wedge \boldsymbol{K}\ edge(x,y)$

EQL: knowledge & logical implication

Fundamental property of EQL: minimal knowledge

$$KB \models \varphi \quad \text{iff} \quad KB \models \boldsymbol{K}\varphi$$
$$KB \not\models \varphi \quad \text{iff} \quad KB \models \neg\boldsymbol{K}\varphi$$

In other words:

- $\boldsymbol{K}\varphi$ can be read as φ is logically implied
- $\neg\boldsymbol{K}\varphi$ can be read as φ is not logically implied
  ie $\neg\varphi$ is satisfiable

Example:

$$\boldsymbol{K}edge(a,b) \wedge \boldsymbol{K}edge(b,c) \wedge \boldsymbol{K}edge(c,a)$$

can be read:

- edges $(a,b), (b,c), (c,d)$ are known
- edges $(a,b), (b,c), (c,d)$ are logically implied

EQL: queries

- EQL query:

$$q(x_1,\ldots,x_n) \text{ :- } \varphi(x_1,\ldots,x_n)$$

- Answer:

$$ans(q, KB) = \{ (c_1,\ldots,c_n) \mid KB \models \varphi(c_1,\ldots,c_n), \quad c_i \in \Delta \}$$

EQL: queries - CQs without existential variables

Example [cf.LUBM, Ralph's talk, Bijan's talk]

$$q(x,y,z) \text{ :- } edge(x,y) \wedge edge(y,z) \wedge edge(z,x)$$

is equivalent to *(since* $KB \models \varphi$ *iff* $KB \models \mathbf{K}\varphi$ *)*

$$q(x,y,z) \text{ :- } \mathbf{K}(edge(x,y) \wedge edge(y,z) \wedge edge(z,x))$$

is equivalent to *(since* **K** *distributes over ANDs)*

$$q(x,y,z) \text{ :- } \mathbf{K}edge(x,y) \wedge \mathbf{K}edge(y,z) \wedge \mathbf{K}edge(z,x)$$

EQL-lite($\mathcal{Q}$)

- Restriction on EQL, parametric wrt an objective query language $\mathcal{Q}$

- EQL-lite($\mathcal{Q}$) queries have the form (with α in $\mathcal{Q}$)

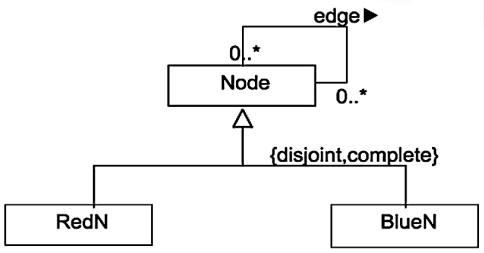
$$\varphi ::= \boldsymbol{K}\alpha \mid t_1 = t_2 \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \exists\, x.\varphi$$

and are domain independent (cf. relational algebra)

Example

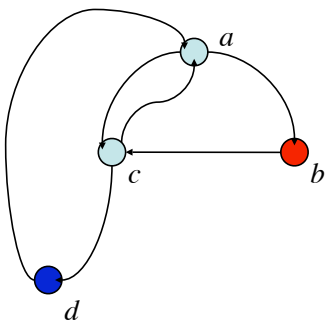
- TBox:

$\exists\, edge^- \sqsubseteq Node$
$\exists\, edge \sqsubseteq Node$
$RedN \sqsubseteq Node$
$BlueN \sqsubseteq Node$
$RedN \sqsubseteq \neg BlueN$
$Node \sqsubseteq RedN \sqcup BlueN$



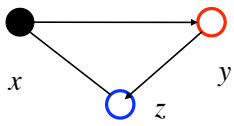
- ABox:

edge(a,b)
edge(b,c)
edge(c,a)
edge(c,d)
edge(d,a)
RedN(b)
BlueN(d)

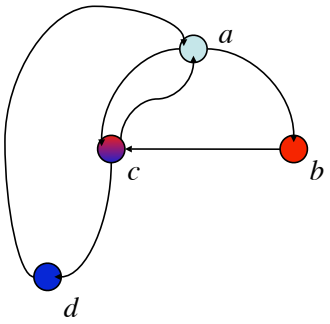


Example

- Query:
  $q(x) :- \exists y, z.\ edge(x,y) \wedge RedN(y) \wedge edge(y,z) \wedge BlueN(z) \wedge edge(z,x)$

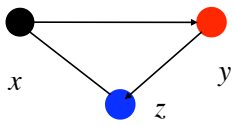


- Answer: $\{a\}$

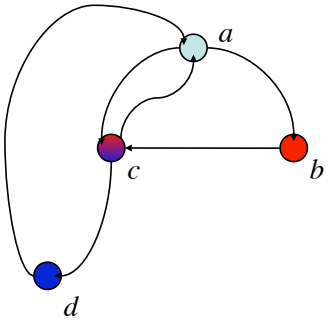


Example

- Query:
  $q(x) :- \exists y, z.\ \mathbf{K}\, edge(x,y) \wedge \mathbf{K}\, RedN(y) \wedge \mathbf{K}\, edge(y,z) \wedge \mathbf{K}\, BlueB(z) \wedge \mathbf{K}\, edge(z,x)$

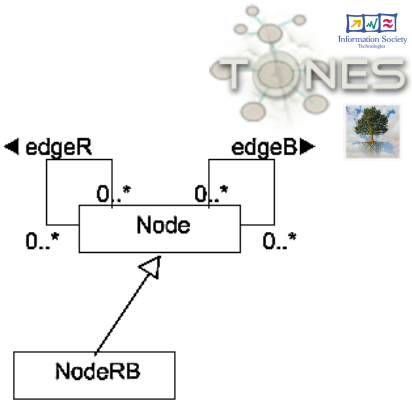


- Answer: $\{\}$

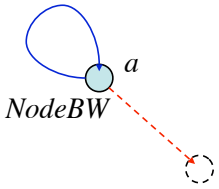


Example

- TBox:
  $\exists\, edgeR^- \sqsubseteq Node$
  $\exists\, edgeR \sqsubseteq Node$
  $\exists\, edgeB^- \sqsubseteq Node$
  $\exists\, edgeB \sqsubseteq Node$
  $NodeRB \sqsubseteq \exists\, edgeR$
  $NodeRB \sqsubseteq \exists\, edgeB$

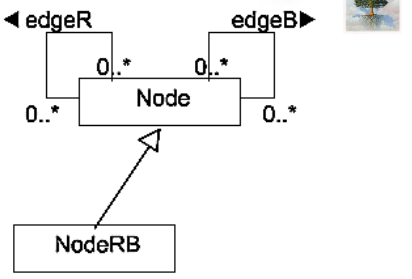


- ABox:
  $edgeB(a,a)$
  $NodeRB(a)$



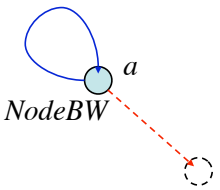
Queries

- Query:
  $q1(x) :- \exists\, y, z, w.\ edgeB(x,y) \wedge edgeR(x,z) \wedge edgeR(y,z)$
  Answer: {a}



- Query:
  $q2(x,y,z) :- edgeB(x,y) \wedge edgeR(x,z) \wedge edgeR(y,z)$
  Answer: {}

- Query:
  $q3(x) :- \exists\, y, z, w.\ \mathbf{K}\, edgeB(x,y) \wedge \mathbf{K}\, edgeR(x,z) \wedge \mathbf{K}\, edgeR(y,z)$
  Answer: {}



EQL-lite($\mathcal{Q}$): main result

- A $\mathcal{Q}$ query α is KB-range restricted iff ans(α,KB) is finite.
- An EQL-lite($\mathcal{Q}$) query is KB-range restricted iff all α appearing in it are KB-range restricted.
- **Thm:** if ans(α,KB) is finite, then it contains only constants occurring in $\mathcal{KB}$.
- **Thm:** Let $\mathcal{KB}$ be a KB expressed in the DL $\mathcal{L}$ and let ***C*** be the data complexity of answering queries in Q over KBs in $\mathcal{L}$, then, answering a $\mathcal{KB}$-range restricted EQL-lite($\mathcal{Q}$) is in LOGSPACE^{C} wrt data complexity.

EQL-lite on concepts/roles in $\mathcal{SHIQ}$ KB

[cf. Ralph's and Bijan's talks]

- $\mathcal{SHIQ}$ concepts and roles $\rightarrow \mathcal{Q}$
- $\mathcal{SHIQ} \rightarrow$ KB
  (or variants)
- Answering $\mathcal{Q}$ queries $\rightarrow$ instance checking which is coNP-complete in data complexity for $\mathcal{SHIQ}$
- Answering EQL-lite queries is LOGSPACE^{coNP}

EQL-lite on UCQ in $\mathcal{ALCQI}$ KB

- UCQs $\rightarrow \mathcal{Q}$
- $\mathcal{ALCQI} \rightarrow$ KB

- Query answering of UCQs in $\mathcal{ALCQI}$ KBs is coNP-complete in data complexity [cf. next talk]

- Answering EQL-lite queries is LOGSPACE$^{\text{coNP}}$

EQL-lite on concepts/roles in $\mathcal{EL}$ KB

- $\mathcal{EL}$ concepts and roles $\rightarrow \mathcal{Q}$
- $\mathcal{EL} \rightarrow$ KB
  (in fact any member of the $\mathcal{EL}$ family)

- Answering $\mathcal{Q}$ queries $\rightarrow$ instance checking, which is PTIME-complete in data complexity for $\mathcal{EL}$

- Answering EQL-lite queries is PTIME-complete

EQL-lite on UCQ in DL-lite KB

- UCQs $\rightarrow \mathcal{Q}$
- DL-lite $\rightarrow$ KB
  (in fact any member of the DL-lite family)
- Answering UCQs in DL-lite is LOGSPACE in data complexity, actually FOL reducible
- Answering EQL-lite queries is LOGSPACE, actually FOL reducible (rewritable in SQL)

Conclusions

- EQL-lite $\approx$ FOL queries for most users
- EQL-lite can be seen as a semantically well characterized approximation of FOL queries
- EQL-lite is based on a controlled use of the epistemic (minimal knowledge) operator
- Jumping from $\mathcal{Q}$ to EQL-lite($\mathcal{Q}$) is (almost) for free
- EQL-lite on UCQs over DL-lite is FOL-reducible (SQL!)
- EQL-lite is very interesting also for modeling constraints over ontologies