Transition Systems and Bisimulation

Giuseppe De Giacomo

Service Integration
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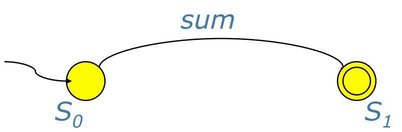
Transition Systems

Concentrating on behaviors: SUM two integers

- Consider a program for computing the sum of two integers.
- Such a program has essentially two states
  - the state S0 of the memory before the computation: including the two number to sum
  - the state S1 of the memory after the computation: including the result of the computation
- Only one action, i.e. "sum", can be performed

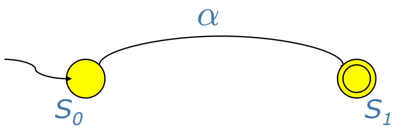


Concentrating on behaviors: CheckValidity

- Consider a program for computing the validity of a FOL formula:
- Also such a program has essentially two states
  - the state S_1 of the memory before the computation: including the formula to be checked
  - the state S_2 of the memory after the computation: including "yes", "no", "time-out"
- Only one action, i.e. "*checkValidity*", can be performed

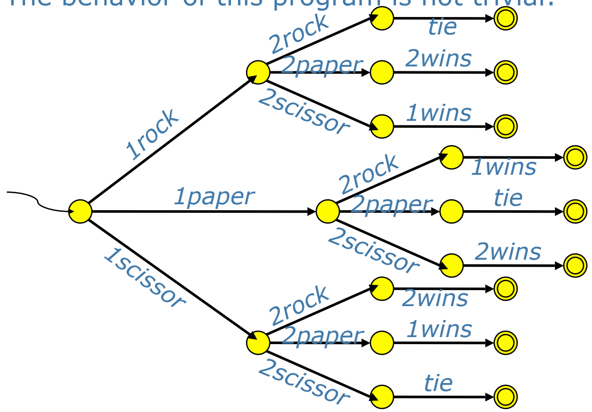
Concentrating on behaviors

- The programs SUM and CheckValidity are very different from a computational point of view.
  - SUM is trivial
  - CheckValidity is a theorem prover hence very complex
- However they are equally trivial from a behavioral point of view:
  - two states S_1 and S_2
  - a single action α causing the transition



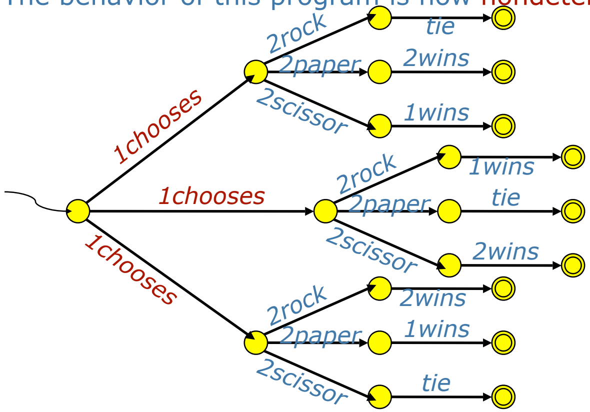
Concentrating on behaviors: RockPaperScissor

- Consider the program RockPaperScissor that allows to play two players the the well-known game.
- The behavior of this program is not trivial:



Concentrating on behaviors: RockPaperScissor (automatic)

- Consider a variant of the program RockPaperScissor that allows one players to play against the computer.
- The behavior of this program is now nondeterministic:

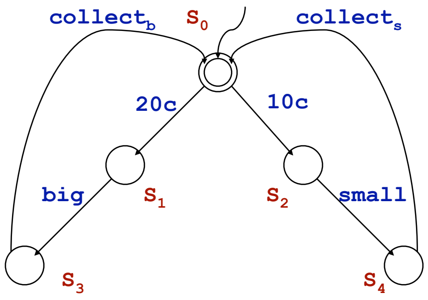


Concentrating on behaviors: WebPage

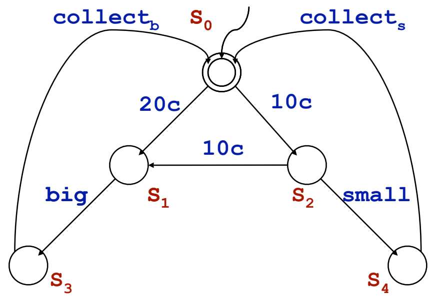
http://www.informatik.uni-trier.de/~ley/db/

A web page can have a complex behavior!

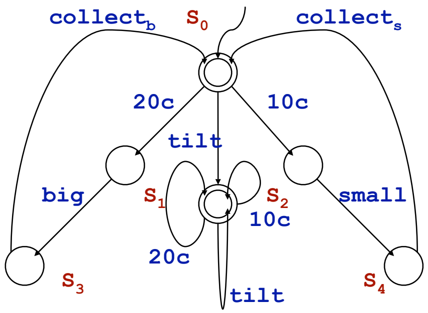
Concentrating on behaviors: Vending Machine



Concentrating on behaviors: Another Vending Machine



Concentrating on behaviors: Vending Machine with Tilt



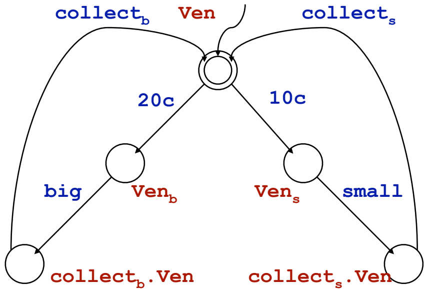
Transition Systems

- A transition system TS is a tuple T = < A, S, S^0, δ, F> where:
  - A is the set of actions
  - S is the set of states
  - $S^0 \subseteq S$ is the set of initial states
  - $\delta \subseteq S \times A \times S$ is the transition relation
  - $F \subseteq S$ is the set of final states
- Variants:
  - No initial states
  - Single initial state
  - Deterministic actions
  - States labeled by propositions other than Final/¬Final

(c.f. Kripke Structure)

Process Algebras are Formalisms for Describing TS

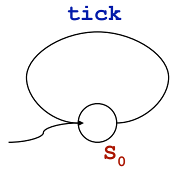
- Trans (a la CCS)
  - $Ven = 20c.Ven_b + 10c.Ven_s$
  - $Ven_b = big.collect_b.Ven$
  - $Ven_l = small.collect_s.Ven$
- Final
  - $\surd$ Ven



- *TS may have infinite states* - *e.g., this happens when generated by process algebras involving iterated concurrency*
- *However we have good formal tools to deal only with finite states TS*

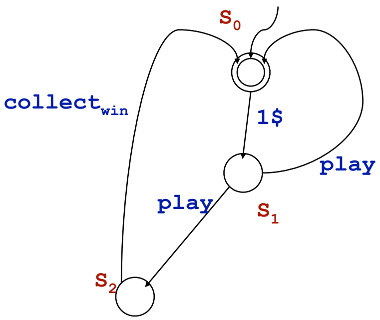
Example (Clock)

TS may describe (legal) nonterminating processes

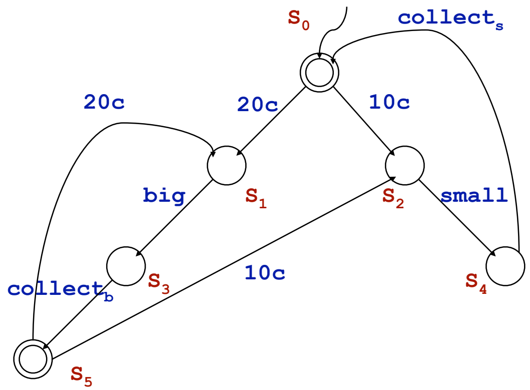


Example (Slot Machine)

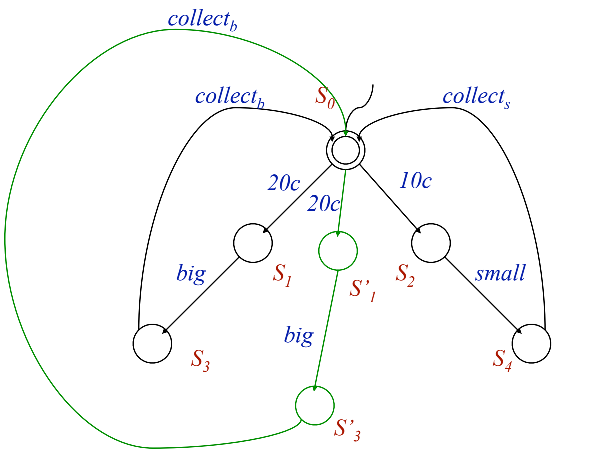
Nondereminisic transitions express
choice that is not under the control of clients



Example (Vending Machine - Variant 1)



Example (Vending Machine - Variant 2)



Inductive vs Coinductive Definitions: Reachability, Bisimilarity, …

Reachability

- A binary relation R is a **reachability-like relation** iff:
  - (s,s) ∈ R
  - if ∃ a, s′. s $\rightarrow_a$ s′ ∧ (s′,s′′) ∈ R then (s,s′′)∈ R
- A state s_0 of transition system S **reaches** a state s_f iff for **all** a **reachability-like relations** R we have (s_0, s_f)∈ R.
- Notably that
  - **reaches** is a reachability-like relation itself
  - **reaches** is the smallest reachability-like relation

Note it is a inductive definition!

Computing Reachability on Finite Transition Systems

Algorithm ComputingReachability

Input: transition system TS
Output: the **reachable-from** relation (the smallest reachability-like relation)

Body
R = ∅
R′ = {(s,s) | s ∈ S}
while (R ≠ R′) {
  R := R′
  R′ := R′ ∪ {(s,s′′) | ∃ s′,a. s $\rightarrow_a$ s′ ∧ (s′,s′′)∈ R }
}
return R′
YdoB

Bisimulation

- A binary relation R is a **bisimulation** iff:

  $(s,t) \in R$ implies that
  - s is *final* iff t is *final*
  - for all actions a
    - if $s \rightarrow_a s'$ then $\exists\, t' .\ t \rightarrow_a t'$ and $(s',t') \in R$
    - if $t \rightarrow_a t'$ then $\exists\, s' .\ s \rightarrow_a s'$ and $(s',t') \in R$

- A state s_0 of transition system S is **bisimilar**, or simply **equivalent,** to a state t_0 of transition system T iff there **exists** a **bisimulation** between the initial states s_0 and t_0.

- Notably
  - **bisimilarity** is a bisimulation
  - **bisimilarity** is the largest bisimulation

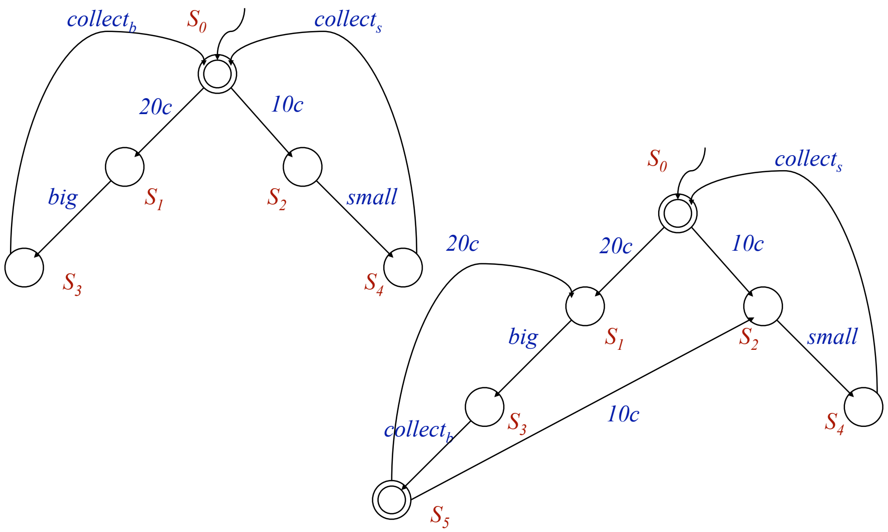
Note it is a co-inductive definition!

Computing Bisimilarity on Finite Transition Systems

Algorithm ComputingBisimulation
Input: transition system $TS_S = \langle A, S, S^0, \delta_S, F_S \rangle$ and
transition system $TS_T = \langle A, T, T^0, \delta_T, F_T \rangle$
Output: the **bisimilarity** relation (the largest bisimulation)

Body
$R = S \times T$
$R' = S \times T - \{(s,t) \mid \neg(s \in F_S \equiv t \in F_T)\}$
while $(R \neq R')$ {
$R := R'$
$R' := R' - (\{(s,t) \mid \exists s',a.\ s \rightarrow_a s' \wedge \neg\exists t' .\ t \rightarrow_a t' \wedge (s',t') \in R' \}$
$\{(s,t) \mid \exists t',a.\ t \rightarrow_a t' \wedge \neg\exists s' .\ s \rightarrow_a s' \wedge (s',t') \in R' \})$
}
return R'
Ydob

Example of Bisimulation



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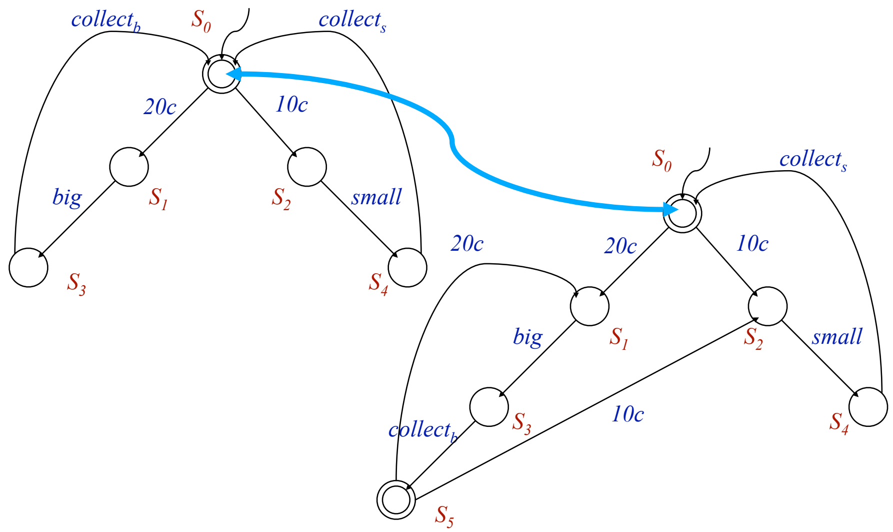
Example of Bisimulation



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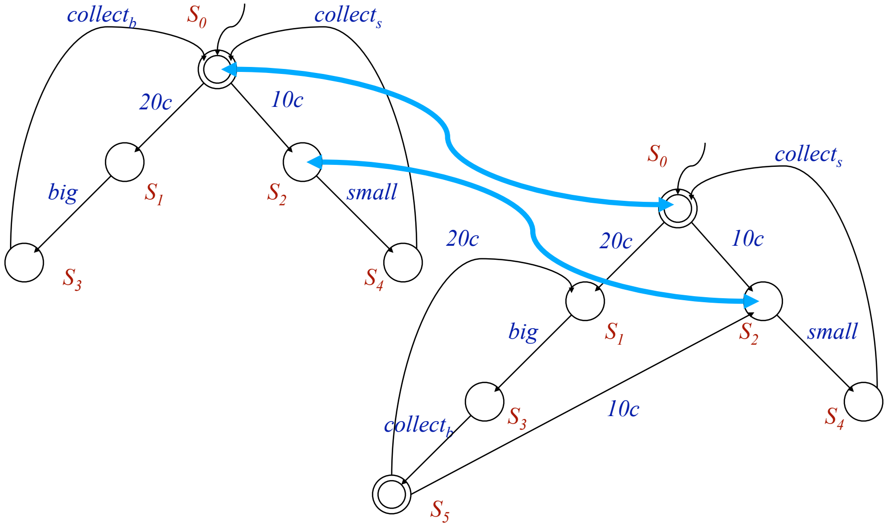

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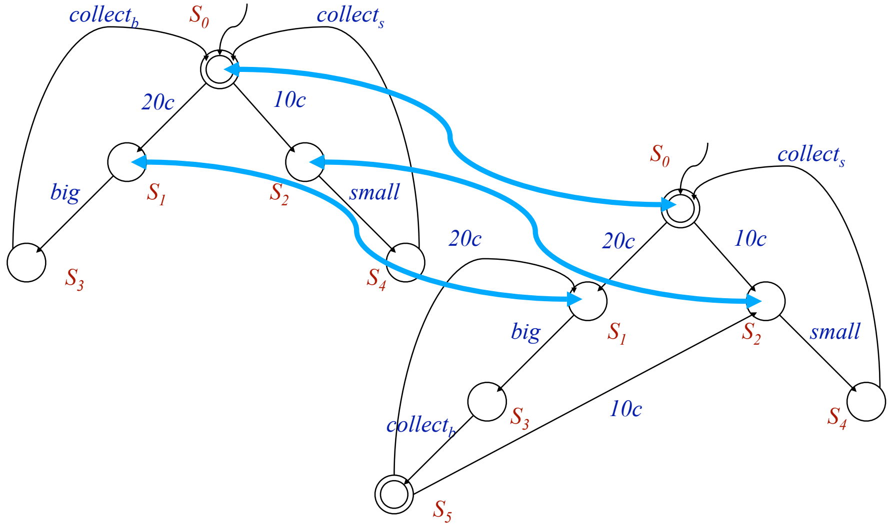

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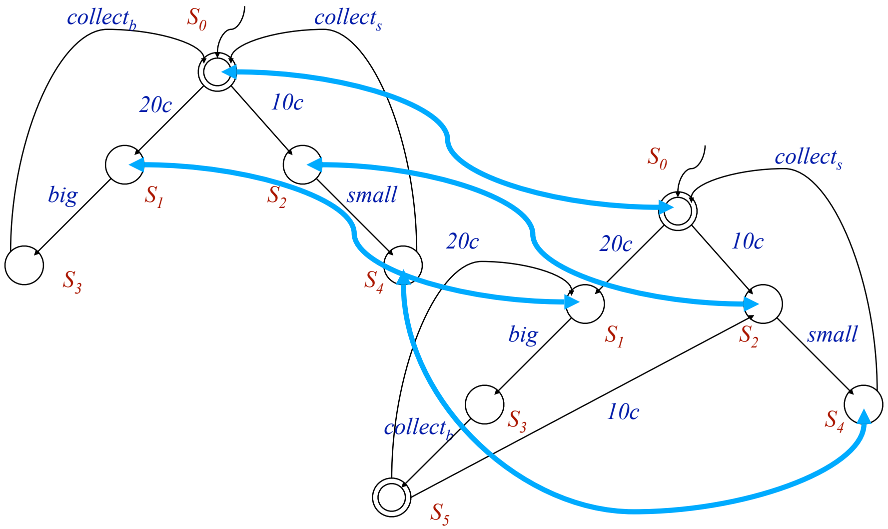

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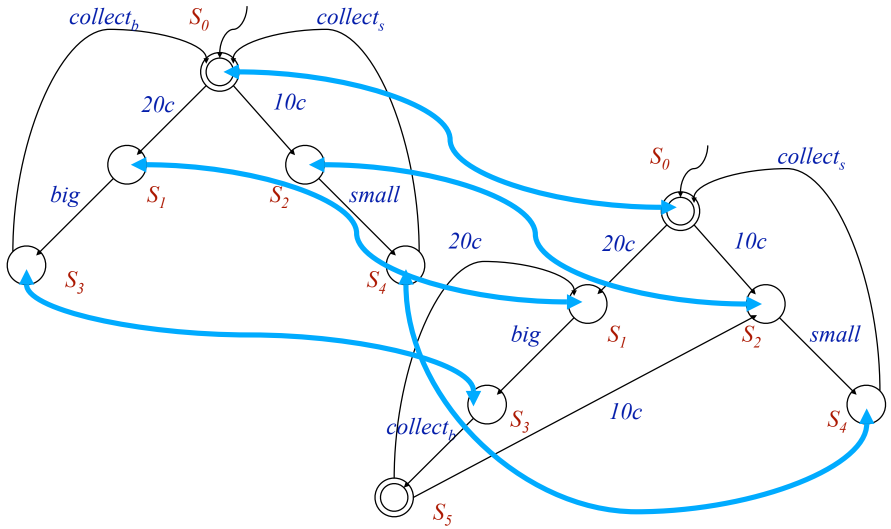

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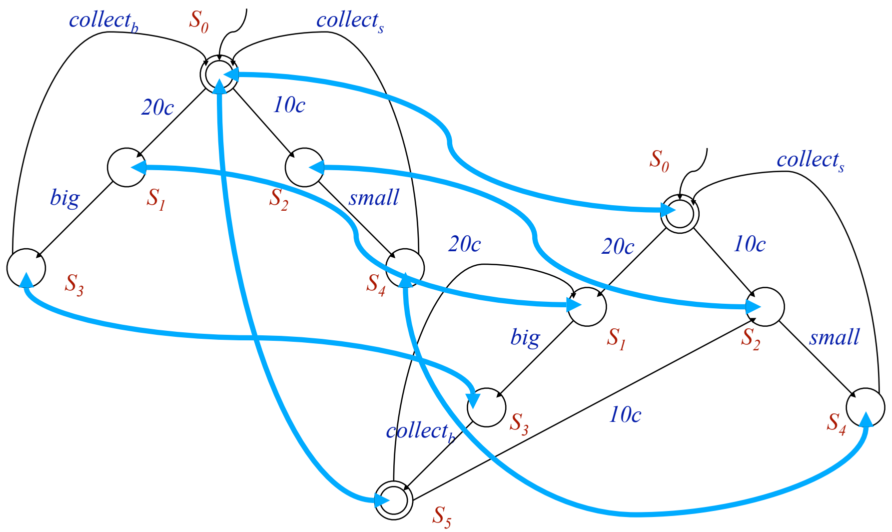

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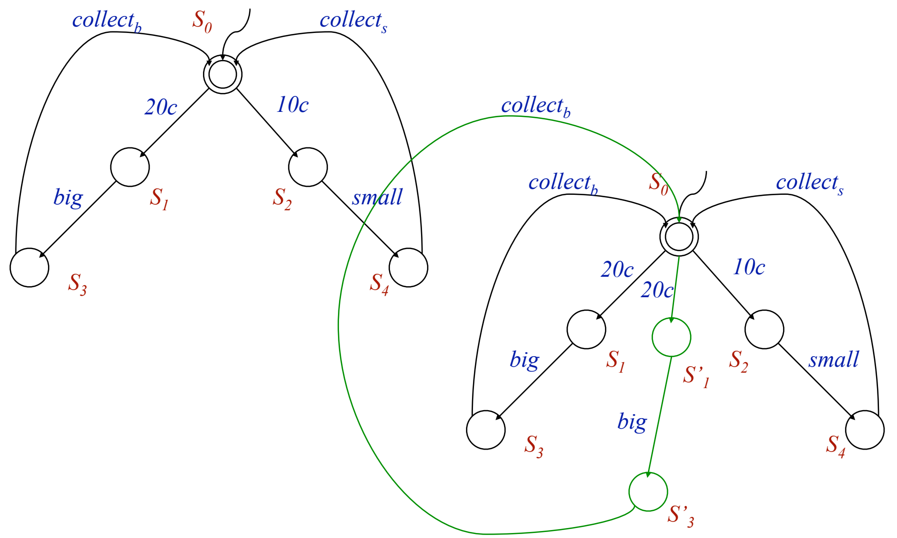

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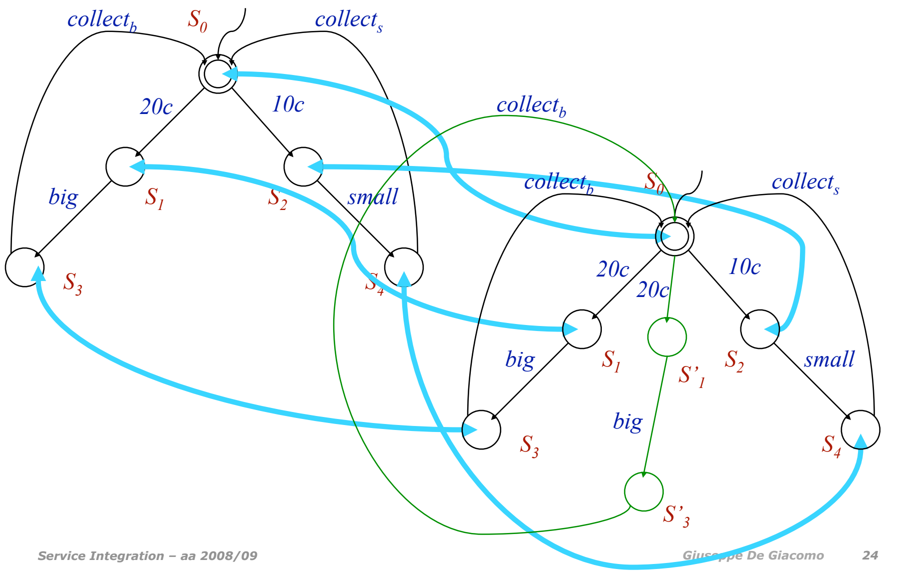

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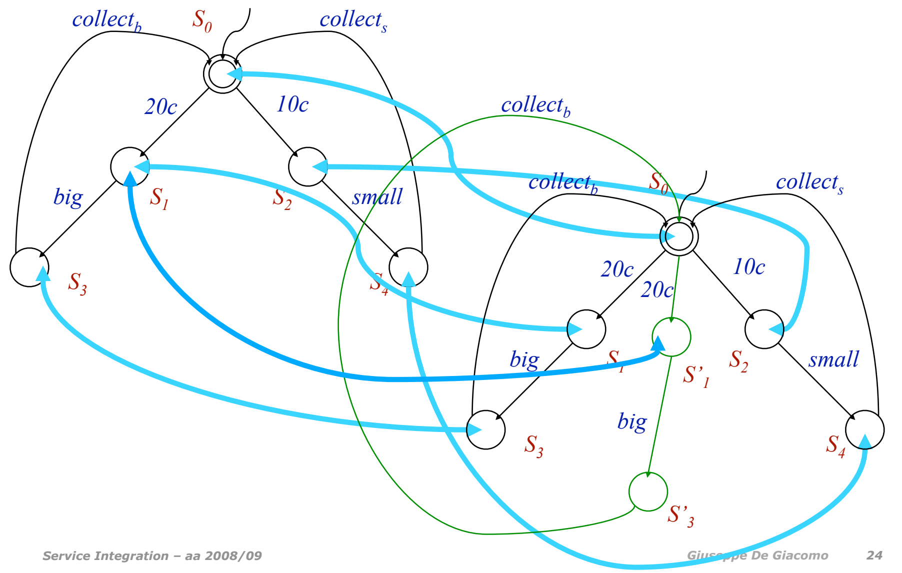
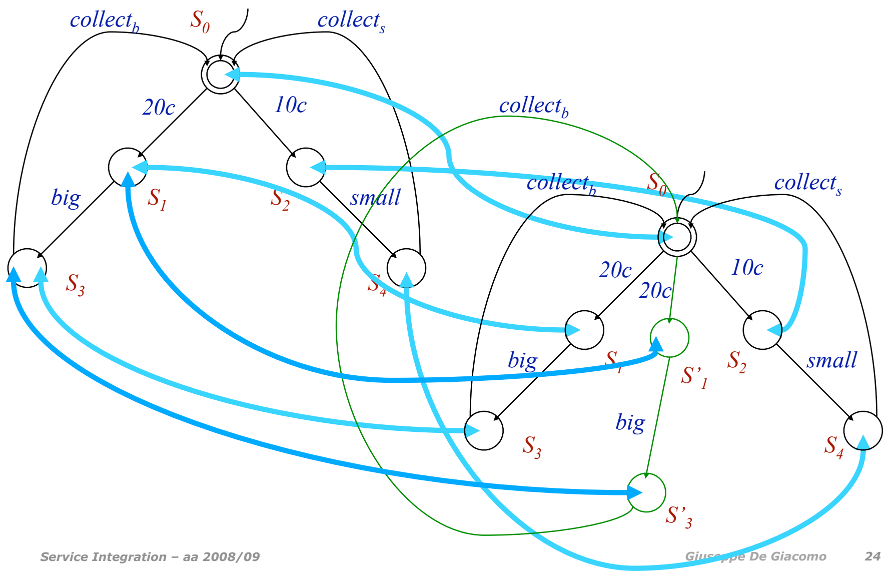


Example of Bisimulation



Automata vs.Transition Systems

- Automata
  - define sets of runs (or traces or strings): (finite) length sequences of actions
- TSs
  - … but I can be interested also in the alternatives "encountered" during runs, as they represent client's "choice points"