Composition and Synthesis via Game Structures

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Introduction

Motivation

Example (Consider the following problems...)

- Conditional planning (even for temporally extended goals)
- Conditional planning in presence of (fully observable) exogenous events
- **Service/behavior/device composition**
- Agent planning programs, which mix planning and programming
- ...

There is a variety of behavior synthesis problems characterized by:

- Nondeterminism (of devilish nature!)
- Full observability

Key observation:

Sometimes we informally describe such problems as games between two players, where one player (the controller) tries to force that certain objectives no matter how other player (the environment) behave.

Introduction

Objectives:

- Take seriously the idea of modelling such synthesis problems as games among two contrasting agents.
- Develop a general framework for synthesis in AI based on two-player game structures.
- Develop reasoning/synthesis techniques leveraging on model-checking technologies.

In this lecture:

- Introduce two-players game structures (2GSs)
- Introduce μ-calculus variant for expressing the ability of the controller to force the game to satisfy desired temporal properties.
- Device reasoning and synthesis techniques based on model checking of 2GSs.
- Apply such tools to a variety of problem and reconstruct solutions, in an optimal way wrt computational complexity.

μ-calculus intermezzo: begin

- The actual techniques for 2GS-based synthesis are based on a variant of μ-calculus model checking.
- Hence before getting into the techniques we need to briefly look back at μ-calculus model checking.

μ-calculus overview

Transition system

Given a set $\mathcal{P}$ of propositions, and set $\mathcal{A}$ of atomic actions, a **transition system** is a triple $\mathcal{T} = (\mathcal{S}, \{\mathcal{R}_a | a \in \mathcal{A}\}, \Pi)$, with a set of states $\mathcal{S}$, a family of transition relations $\mathcal{R}_a \in \mathcal{S} \times \mathcal{S}$, and a mapping Π from $\mathcal{P}$ to subsets of $\mathcal{S}$.

The (modal) μ-calculus is a logic to talk about dynamic/temporal properties over TS. It is basically constituted by three kinds of components:

- **Propositions** to denote properties of the global store in a given configuration.
- **Modalities** to denote the capability of performing certain actions in a given configuration.
- **Least and greatest fixpoint constructs** to denote "temporal" properties of the system, typically defined by **induction** and **coinduction**.

μ-calculus

$$\Phi ::= A \mid true \mid false \mid \neg\Phi \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 \vee \Phi_2 \mid \langle a \rangle \Phi \mid [a]\Phi \mid \mu X.\Phi \mid \nu X.\Phi \mid X$$

μ-calculus semantics: extension function

Let $\mathcal{T} = (\mathcal{S}, \{\mathcal{R}_\alpha | \alpha \in 2^{\mathcal{A}}\}, \Pi)$ be a transition system, and $\mathcal{V}$ a valuation on $\mathcal{T}$. We assign meaning to μ-calculus formulae by associating to $\mathcal{T}$ and $\mathcal{V}$ an **extension function** $(\cdot)^{\mathcal{T}}_{\mathcal{V}}$, which maps μ-calculus formulae to subsets of $\mathcal{S}$.

The extension function $(\cdot)^{\mathcal{T}}_{\mathcal{V}}$ is defined inductively as follows:

μ-calculus semantics

$$\begin{array}{lcl}
(A)^{\mathcal{T}}_{\mathcal{V}} & = & \Pi(A) \subseteq \mathcal{S} \\
(X)^{\mathcal{T}}_{\mathcal{V}} & = & \mathcal{V}(X) \subseteq \mathcal{S} \\
(true)^{\mathcal{T}}_{\mathcal{V}} & = & \mathcal{S} \\
(false)^{\mathcal{T}}_{\mathcal{V}} & = & \emptyset \\
(\neg\Phi)^{\mathcal{T}}_{\mathcal{V}} & = & \mathcal{S} - (\Phi)^{\mathcal{T}}_{\mathcal{V}} \\
(\Phi_1 \wedge \Phi_2)^{\mathcal{T}}_{\mathcal{V}} & = & (\Phi_1)^{\mathcal{T}}_{\mathcal{V}} \cap (\Phi_2)^{\mathcal{T}}_{\mathcal{V}} \\
(\Phi_1 \vee \Phi_2)^{\mathcal{T}}_{\mathcal{V}} & = & (\Phi_1)^{\mathcal{T}}_{\mathcal{V}} \cup (\Phi_2)^{\mathcal{T}}_{\mathcal{V}} \\
(\langle a \rangle \Phi)^{\mathcal{T}}_{\mathcal{V}} & = & \{s \in \mathcal{S} \mid \exists s'.\ (s, s') \in \mathcal{R}_a \text{ and } s' \in (\Phi)^{\mathcal{T}}_{\mathcal{V}}\} \\
([a]\Phi)^{\mathcal{T}}_{\mathcal{V}} & = & \{s \in \mathcal{S} \mid \forall s'.\ (s, s') \in \mathcal{R}_a \text{ implies } s' \in (\Phi)^{\mathcal{T}}_{\mathcal{V}}\} \\
(\mu X.\Phi)^{\mathcal{T}}_{\mathcal{V}} & = & \bigcap\{\mathcal{E} \subseteq \mathcal{S} \mid (\Phi)^{\mathcal{T}}_{\mathcal{V}[X \leftarrow \mathcal{E}]} \subseteq \mathcal{E}\} \\
(\nu X.\Phi)^{\mathcal{T}}_{\mathcal{V}} & = & \bigcup\{\mathcal{E} \subseteq \mathcal{S} \mid \mathcal{E} \subseteq (\Phi)^{\mathcal{T}}_{\mathcal{V}[X \leftarrow \mathcal{E}]}\}
\end{array}$$

μ-calculus semantics: intuition

For the fixpoint constructs we have:

Intuition on $(\mu X.\Phi)^{\mathcal{T}}_{\mathcal{V}}$ and $(\nu X.\Phi)^{\mathcal{T}}_{\mathcal{V}}$

- The extension of $\mu X.\Phi$ is the **smallest subset** $\mathcal{E}_\mu$ of $\mathcal{S}$ such that, assigning to X the extension $\mathcal{E}_\mu$, the resulting extension of Φ is contained in $\mathcal{E}_\mu$. That is, the extension of $\mu X.\Phi$ is the **least fixpoint** of the operator $\lambda\mathcal{E}.(\Phi)^{\mathcal{T}}_{\mathcal{V}[X\leftarrow\mathcal{E}]}$.
- Similarly, the extension of $\nu X.\Phi$ is the **greatest subset** $\mathcal{E}_\nu$ of $\mathcal{S}$ such that, assigning to X the extension $\mathcal{E}_\nu$, the resulting extension of Φ contains $\mathcal{E}_\nu$. That is, the extension of $\nu X.\Phi$ is the **greatest fixpoint** of the operator $\lambda\mathcal{E}.(\Phi)^{\mathcal{T}}_{\mathcal{V}[X\leftarrow\mathcal{E}]}$.

The syntactic monotonicity of Φ wrt X guarantees the monotonicity of the operator $\lambda\mathcal{E}.(\Phi)^{\mathcal{T}}_{\mathcal{V}[X\leftarrow\mathcal{E}]}$ and hence, by Tarski-Knaster Theorem, the unique existence of the least fixpoint.

μ-calculus: examples

Example

$$\mu X.P \vee \langle next\rangle X$$

expresses that there **exists an evolution** of the system such that P **eventually** holds. Indeed, its extension $\mathcal{E}_\mu$ is the smallest set that includes (1) the states in the extension of Φ; and (2) the states that can execute a transition leading to a successive state that is in $\mathcal{E}_\mu$. In other words, the extension $\mathcal{E}_\mu$ includes each state s such that there exists a run from s leading eventually (i.e. in a finite number of steps) to a state in the extension of P. Note the inductive nature of this property.

Example

$$\nu X.P \wedge [next]X$$

i.e. $\neg(\mu X.\neg P \vee \langle next\rangle X)$ – expresses the **invariance** of P under all of the evolutions of the system. Indeed, its extension $\mathcal{E}_\nu$ is the largest set of states in the extension of P from which every transition leads to a successive state which is still in $\mathcal{E}_\nu$. In other words, the extension $\mathcal{E}_\nu$ includes each state s such that every state along every run from s is in the extension of P. Note the coinductive nature of this property.

μ-calculus: model checking

The reasoning problem we are interested in is **model checking**:

Definition

Let $\mathcal{T} = (\mathcal{S}, \{\mathcal{R}_a \mid a \in \mathcal{A}\}, \Pi)$ be a transition system, let $s \in \mathcal{S}$ be one of its states, and let Φ be a closed (no free variables are present) μ-calculus formula. The related **model checking** problem is to verify whether

$$s \in (\Phi)^{\mathcal{T}}_{\mathcal{V}}$$

where $\mathcal{V}$ is any valuation, since Φ is closed.

Often we abbreviate $s \in (\Phi)^{\mathcal{T}}_{\mathcal{V}}$ by $\mathcal{T}, s \models \Phi$ or simply by $s \models \Phi$ referring to $\mathcal{T}$ only implicitly.

μ-calculus: complexity of reasoning

Theorem

Checking (closed) a μ-calculus formula Φ over a transition system $\mathcal{T} = (\mathcal{S}, \{\mathcal{R}_a \mid a \in \mathcal{A}\}, \Pi)$ can be done in time

$$O((|\mathcal{T}| \cdot |\Phi|)^k)$$

where $|\mathcal{T}| = |\mathcal{S}| + \Sigma_{a \in \mathcal{A}} |R_a|$, i.e., the number of states plus the number of transitions of $\mathcal{T}$, $|\Phi|$ is the size of formula Φ (in fact, considering propositional formulas as atomic), and k is the number of nested fixpoints, i.e., fixpoints whose variables are one within the scope of the other.

Also, in general model checking is in $NP \cap coNP$.

Theorem

Checking satifiability/validity/logical implication in μ-calculus is decidable and more precisely EXPTIME-complete.

μ-calculus: model checking algorithm

Given a μ-calculus formula Φ over a transition system $\mathcal{T} = (\mathcal{S}, \{\mathcal{R}_a \mid a \in \mathcal{A}\}, \Pi)$ and a valuation $\mathcal{V}$, the **model checking algorithm** is based on recursively **labeling the states** of the transition systems with the formulas that are true in them, following closely the semantics.

μ-calculus model checking algorithm

$$
\begin{array}{rcl}
[[A]]^{\mathcal{T}}_{\mathcal{V}} & = & \Pi(A) \\
[[X]]^{\mathcal{T}}_{\mathcal{V}} & = & \mathcal{V}(X) \\
[[true]]^{\mathcal{T}}_{\mathcal{V}} & = & \mathcal{S} \\
[[false]]^{\mathcal{T}}_{\mathcal{V}} & = & \emptyset \\
[[\neg\Phi]]^{\mathcal{T}}_{\mathcal{V}} & = & \mathcal{S} - [[\Phi]]^{\mathcal{T}}_{\mathcal{V}} \\
[[\Phi_1 \wedge \Phi_2]]^{\mathcal{T}}_{\mathcal{V}} & = & [[\Phi_1]]^{\mathcal{T}}_{\mathcal{V}} \cap [[\Phi_2]]^{\mathcal{T}}_{\mathcal{V}} \\
[[\Phi_1 \vee \Phi_2]]^{\mathcal{T}}_{\mathcal{V}} & = & [[\Phi_1]]^{\mathcal{T}}_{\mathcal{V}} \cup [[\Phi_2]]^{\mathcal{T}}_{\mathcal{V}}
\end{array}
\qquad
\begin{array}{rcl}
[[\langle a\rangle\Phi]]^{\mathcal{T}}_{\mathcal{V}} & = & \textsc{PreE}(a, [[\Phi]]^{\mathcal{T}}_{\mathcal{V}}) \\
[[[a]\Phi]]^{\mathcal{T}}_{\mathcal{V}} & = & \textsc{PreA}(a, [[\Phi]]^{\mathcal{T}}_{\mathcal{V}}) \\
[[\mu X.\Phi]]^{\mathcal{T}}_{\mathcal{V}} & = & \textsc{lfp}X.[[\Phi]]^{\mathcal{T}}_{\mathcal{V}} \\
[[\nu X.\Phi]]^{\mathcal{T}}_{\mathcal{V}} & = & \textsc{gfp}X.[[\Phi]]^{\mathcal{T}}_{\mathcal{V}}
\end{array}
$$

where $\textsc{PreE}, \textsc{PreA}, \textsc{gfp}, \textsc{lfp}$ are defined below.

For the atomic propositions, variables and propositional operator the labeling works in an obvious way.

μ-calculus: model checking algorithm

Let $\mathcal{E} \subseteq \mathcal{S}$ be a set of state and $a \in \mathcal{A}$ an action. Then $\textsc{PreE}$ and $\textsc{PreA}$ label the existential and universal a-preimage of $\mathcal{E}$ respectively.

Existential a-preimage of $\mathcal{E}$

$\textsc{PreE}(a, \mathcal{E})$, i.e., the **existential** a**-preimage of** $\mathcal{E}$, is defined as follows:

$$\textsc{PreE}(a, \mathcal{E}) = \{s \in \mathcal{S} \mid \exists s'.\ (s, s') \in \mathcal{R}_a \text{ and } s' \in \mathcal{E}\}$$

Universal a-preimage of $\mathcal{E}$

$\textsc{PreA}(a, \mathcal{E})$, i.e., the **universal** a**-preimage of** $\mathcal{E}$, is defined as follows:

$$\textsc{PreA}(a, \mathcal{E}) = \{s \in \mathcal{S} \mid \forall s'.\ (s, s') \in \mathcal{R}_a \text{ implies } s' \in \mathcal{E}\}$$

Notice the preimage operators follow the semantics of the $\langle a\rangle\cdot$ and $[a]\cdot$ very closely.

μ-calculus: model checking algorithm

Procedures $\text{LFP}X.[[\Phi]]^{\mathcal{T}}_{\mathcal{V}}$ and $\text{GFP}X.[[\Phi]]^{\mathcal{T}}_{\mathcal{V}}$ apply Tarski-Knaster approximates theorem to compute **least fixpoint** and **greatest fixpoint** of operator $[[\Phi]]^{\mathcal{T}}_{\mathcal{V}}$:

Procedure $\text{LFP}X.[[\Phi]]^{\mathcal{T}}_{\mathcal{V}}$

$\mathcal{X}_{old} := [[False]]^{\mathcal{T}}_{\mathcal{V}}$;
$\mathcal{X} := [[\Phi]]^{\mathcal{T}}_{\mathcal{V}[X \leftarrow \mathcal{X}_{old}]}$;
while $(\mathcal{X} \neq \mathcal{X}_{old})$ {
$\mathcal{X}_{old} := \mathcal{X}$;
$\mathcal{X} := [[\Phi]]^{\mathcal{T}}_{\mathcal{V}[X \leftarrow \mathcal{X}_{old}]}$;
}
return $\mathcal{X}$;

Procedure $\text{GFP}X.[[\Phi]]^{\mathcal{T}}_{\mathcal{V}}$

$\mathcal{X}_{old} := [[True]]^{\mathcal{T}}_{\mathcal{V}}$;
$\mathcal{X} := [[\Phi]]^{\mathcal{T}}_{\mathcal{V}[X \leftarrow \mathcal{X}_{old}]}$;
while $(\mathcal{X} \neq \mathcal{X}_{old})$ {
$\mathcal{X}_{old} := \mathcal{X}$;
$\mathcal{X} := [[\Phi]]^{\mathcal{T}}_{\mathcal{V}[X \leftarrow \mathcal{X}_{old}]}$;
}
return $\mathcal{X}$;

Notice the number of interations of the **while** *is at most equal to the number of states $\mathcal{S}$ of the transition system $\mathcal{T}$.*

μ-calculus intermezzo: end

Now we are ready to switch back to 2 player game structure and synthesis.

Two-player Game Structures

Inspired by Pnueli's work on LTL synthesis by model checking (and aslo ATL).

- 2GS's are akin to transition systems used to describe the systems to be checked in Verification ...
- ... but with a substantial difference:

Two-player Game Structures

Inspired by Pnueli's work on LTL synthesis by model checking (and aslo ATL).

- 2GS's are akin to transition systems used to describe the systems to be checked in Verification ...
- ... but with a substantial difference:

while a transition system describes the evolution of a system...

Two-player Game Structures

- A 2GS describes the joint evolution of two autonomous systems—the environment and the controller—running together and interacting at each step, as if engaged in a sort of game.

Two-player Game Structures

Formally, a two-player game structure (2GS) is a tuple:

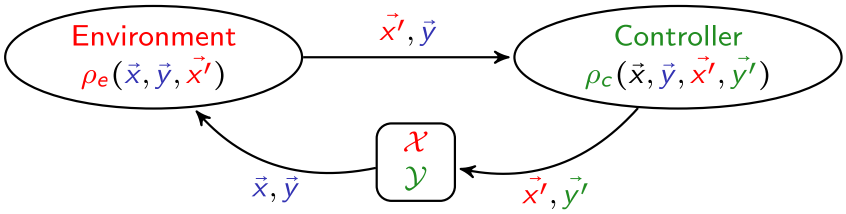
Definition (2GS)

$G = \langle \mathcal{X}, \mathcal{Y}, start, \rho_e, \rho_c \rangle$, where:

- $\mathcal{X} = \{x_1, \ldots, x_m\}$ is the set of environment (uncontrolled) variables ranging over finite domains;
- $\mathcal{Y} = \{y_1, \ldots, y_n\}$ are set of controller (controlled) variables ranging over finite domains;
- $start = \langle \vec{x}_o, \vec{y}_o \rangle$ is the initial state of the game.
- $\rho_e \subseteq \vec{X} \times \vec{Y} \times \vec{X}$ is the environment transition relation, which relates each game state to its possible successor environment states (or *moves*).
- $\rho_c \subseteq \vec{X} \times \vec{Y} \times \vec{X} \times \vec{Y}$ is the controller transition relation, which relates each game state and environment move to the possible controller replies.

2GS Transitions

2GS transitions:



- Uncontrolled ($\mathcal{X} = \{x_1, \ldots, x_n\}$) and controlled ($\mathcal{Y} = \{y_1, \ldots, y_m\}$) vars
- Environment assigns $\mathcal{X}$ vars (moves first),
- Controller sees results of environment'move and assigns $\mathcal{Y}$ vars
- Both have their own structural assumptions (constraints on execution)

Nondeterministic Planning Domains as a 2GS's

Example

Nondeterministic planning domain

$\mathcal{D} = \langle P, A, S_0, \rho \rangle$:

- $P = \{p_1, \ldots, p_n\}$ is a finite set of *domain propositions*;
- $A = \{a_1, \ldots, a_r\}$ is the finite set of *domain actions*;
- $S_0 \in 2^P$ is the *initial state*;
- $\rho \subseteq 2^P \times A \times 2^P$ is the *domain transition relation*.

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Corresponding 2GS

$G_{\mathcal{D}} = \langle \mathcal{X}, \mathcal{Y}, start, \rho_e, \rho_c \rangle$:

- $\mathcal{X} = P$;
- $\mathcal{Y} = \{act\}$, with act ranging over $A \cup \{a_{init}\}$;
- $start = \langle S_0, a_{init} \rangle$;
- $\rho_e(S, a, S')$ iff $\rho(S, a, S') + \rho_e(S_0, a_{init}, S_0)$;
- $\rho_c(S, a, S', a')$ iff action a' is executable in S'

(i.e., for some $S'' \in 2^P$, $\rho(S', a', S'')$).

Goal Formulas

To express winning condition for the controller in 2GS's we introduce goal formulas.
For goal formulas, we use a variant of the μ-calculus interpreted over 2GS's.

Definition

Goal formulas

$$\Psi \leftarrow \varphi \mid Z \mid \Psi_1 \wedge \Psi_2 \mid \Psi_1 \vee \Psi_2 \mid \neg\Psi \mid \odot\Psi \mid \mu Z.\Psi \mid \nu Z.\Psi$$

Ingredients

- Atomic formulas φ of the form $(x_i = \bar{x}_i)$ and $(y_i = \bar{y}_i)$;
- Boolean operators;
- Special operator $\odot\Psi$ that expresses that the controller can force Ψ next.
- Least and greatest fixpoint constructs to capture sophisticated dynamic/temporal properties, defined by **induction** or **coinduction**.

Operator $\odot\Psi$

Definition ($\odot\Psi$ formal interpretation)

$$\langle\vec{x},\vec{y}\rangle \models \odot\Psi \text{ iff}$$
$$\exists\vec{x}'.\rho_e(\vec{x},\vec{y},\vec{x}') \wedge$$
$$\forall\vec{x}'.\rho_e(\vec{x},\vec{y},\vec{x}') \rightarrow \exists\vec{y}'.\rho_c(\vec{x},\vec{y},\vec{x}',\vec{y}') \text{ s.t. } \langle\vec{x}',\vec{y}'\rangle \models \Psi.$$

$\odot\Psi$ intuitive meaning

For every move $\vec{x}$ of the environment from the game state $\langle\vec{x},\vec{y}\rangle$, there is a move $\vec{y}'$ of controller such that in the resulting state of the game $\langle\vec{x}',\vec{y}'\rangle$ the property Ψ holds.

Note: in μ-calculus such alternation of quantification (universal for the environment) and (existential for the controller) can be easily expressed!

Examples of Goal Formulas

Example (liveness: eventually goal)

A standard conditional planning goal: **reach** a desired state of affairs can be expressed as

$$\diamond\!\!\!\cdot\, goal \doteq \mu Z.\ goal \vee \odot Z.$$

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Example (safety: always *goal*)

Now assume to have a domain with exogenous actions then **maintaining** a property *goal* still in spite of environment moves can be expressed:

$$\boxdot goal \doteq \nu Z. goal \wedge \odot Z.$$

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$$\boxdot goal \doteq \nu Z.goal \wedge \odot Z.$$

Example (fairness: infinitely often *goal*)

In the same setting, we may be content with a strategy to force the game so that it is **always** the case that **eventually** a state where *goal* holds is reached.

$$\boxdot\diamond goal \doteq \nu Z_1.(\mu Z_2.((goal \wedge \odot Z_1) \vee \odot Z_2))$$

Service Composition

Example

Composition

Given a target service $\mathcal{S}_0$ and available service $\mathcal{S}_1, \ldots, \mathcal{S}_n$ with $\mathcal{S}_i = \langle A, S_i, s_{i0}, \delta_i, F_i \rangle$, check whether there exists a composition (and if so return it).

Service Composition

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Composition

Given a target service $\mathcal{S}_0$ and available service $\mathcal{S}_1, \ldots, \mathcal{S}_n$ with $\mathcal{S}_i = \langle A, S_i, s_{i0}, \delta_i, F_i \rangle$, check whether there exists a composition (and if so return it).

Simulation

Given a target service $\mathcal{S}_0$ and available service $\mathcal{S}_1, \ldots, \mathcal{S}_n$ with $\mathcal{S}_i = \langle A, S_i, s_{i0}, \delta_i, F_i \rangle$, check whether $\mathcal{S}_1, \ldots, \mathcal{S}_n$ can simulate (forever) $\mathcal{S}_0$ and (and if so return the "simulation strategy").

2GS for Service Composition

Example (2GS for Service Composition)

$G_{\mathcal{D}} = \langle \mathcal{X}, \mathcal{Y}, start, \rho_e, \rho_c \rangle$:

- $\mathcal{X} = \{st_0, st_1, \ldots, st_n, act, err\}$; with st_i ranging over S_i, act ranging over A and err over booleans;
- $\mathcal{Y} = \{srv\}$, with srv ranging over $1, \ldots, n$;
- $\rho_e(st_0, st_1, \ldots, st_n, act, err, srv, st_0', st_1', \ldots, st_n', act', err')$ for
  - $err = false$, $srv = i$ and for at least one st_i', we have $\delta_i(st_i, act, st_i')$:
    - $\delta_i(st_i, act, st_i')$;
    - $st_j' = st_j$ for $j = 1, \ldots, n$ and $j \neq i$;
    - $st_0' = \delta_0(st_0, act)$ (recall that the target service is deterministic);
    - act' s.t. $\delta_0(st_0', act')$ defined (NB: note this is the next step in the target!);
    - $err' = false$;
  - or $err = false$, $srv = i$ and for no st_i' $\delta_i(st_i, act, st_i')$; or if already $err = true$:
    - $st_j' = st_j$ and $j = 1, \ldots, n$
    - $st_0' = st_0$ (recall that the target service is deterministic);
    - $act' = act$;
    - $err' = true$
  - plus a suitable treatment of the starting state $start$.
- $\rho_c(st_0, \ldots, err, srv, s_0', \ldots, err', srv')$ for $srv' = 1, \ldots, n$.

Goal Formulas for Service Composition

Example (Goal Formulas for Service Composition)

The goal formula requires the to always maintain the following condition ϕ true:

$$\phi \doteq \neg err \wedge (F_0 \rightarrow F_1 \wedge \ldots \wedge F_n)$$

That is:

$$\boxdot\phi \doteq \nu Z.\phi \wedge \odot Z.$$

This is a so called safety formula.

Reasoning (Model Checking) on 2GS

Theorem

Checking a goal formula Ψ over a game structure $G = \langle \mathcal{X}, \mathcal{Y}, start, \rho_e, \rho_c \rangle$ can be done in time

$$O((|G| \cdot |\Psi|)^k)$$

where $|G|$ denotes the number of game states of G plus $|\rho_e| + |\rho_c|$, $|\Psi|$ is the size of formula Ψ (considering propositional formulas as atomic), and k is the number of nested fixpoints sharing the same free variables in Ψ.

Observation

In fact we can easily adapt standard model checking algorithms for μ-calculus:

- *Note that while we use $\odot\Psi$ operator, which, though more sophisticated than in standard μ-calculus $\langle\Psi\rangle$, in order to evaluate it we only needs local checks.*

Examples (Cont.)

Example (liveness: eventually goal)

A standard conditional planning goal: $\Diamond goal \doteq \mu Z.\, goal \vee \odot Z.$

Can be done in linear time in the size of the 2GS G, i.e., $2^{|G|}$ wrt a compact representation of G (Problem is known to be EXPTIME-complete.)

Examples (Cont.)

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Can be done in linear time in the size of the 2GS G, i.e., $2^{|G|}$ wrt a compact representation of G (Problem is known to be EXPTIME-complete.)

Example (safety: always *goal*)

Maintaining a property *goal* in spite of environment moves:
$\boxdot goal \doteq \nu Z.goal \wedge \odot Z.$

Can be done in linear time in the size of the 2GS G, i.e., $2^{|G|}$ wrt a compact representation of G. (Problem also is known to be EXPTIME-complete.)

Examples (Cont.)

Example (liveness: eventually goal)

A standard conditional planning goal: $\diamond goal \doteq \mu Z.\ goal \lor \odot Z.$

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Example (safety: always *goal*)

Maintaining a property *goal* in spite of environment moves:
$\boxdot goal \doteq \nu Z.goal \land \odot Z.$

Can be done in linear time in the size of the 2GS G, i.e., $2^{|G|}$ wrt a compact representation of G. (Problem also is known to be EXPTIME-complete.)

Example (fairness: infinitely often *goal*)

Force the game so that it is always the case that eventually a state where *goal* holds is reached: $\boxdot\diamond goal \doteq \nu Z_1.(\mu Z_2.((goal \land \odot Z_1) \lor \odot Z_2))$

Can be done in linear time in the size of the 2GS G, i.e., $2^{|G|^2}$ wrt a compact representation of G. (Problem is EXPTIME-complete.)

Synthesis

Strategies

A controller strategy is a partial function

$$f : (\vec{X} \times \vec{Y})^+ \times \vec{X} \mapsto \vec{Y}$$

such that for every sequence $\lambda = \langle \vec{x}_0, \vec{y}_0 \rangle \cdots \langle \vec{x}_n, \vec{y}_n \rangle$ and every $\vec{x}' \in \vec{X}$ such that $\rho_e(\vec{x}_n, \vec{y}_n, \vec{x}')$ holds, it is the case that $\rho_c(\vec{x}_n, \vec{y}_n, \vec{x}', f(\lambda, \vec{x}'))$ applies.

Extracting winning strategy from model checking witness

- Model checking algorithms provide a witness of the checked property.
- The witness consists of a labeling of the game structure produced during the model checking process.
- From labelled game states, one can read how the controller is meant to react to the environment at each step in order to fulfill the formulas that label the state itself, and from this, define a strategy to fulfill the goal formula.

Implementation

What's available off-the-shelf

- There are a few model checker for μ-calculus – but none very optimized.
- Most of them do (symbolically) search backward (typical in model checking), but interestingly some work forward ("local model checking").
- For formulas without nested fixpoints one can use ATL model checkers such as MCMAS. But notice that, e.g., fairness cannot be expressed!
- For some of the most prominent 2-nested fixpoints properties one can use Pnueli's TLV also based on synbolic methods (used for GR(1) LTL –strong fairness constraints).

In general, more work has to be done, but quite promising: we can leverage on available model checking techniques!

Conclusion

Summary

- 2GS is a powerful framework to express and solve sophisticates synthesis problems ...
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Personal note

I'd like to thank Amir Pnueli [Apr. 22, 1941 Nov. 2, 2009].

- I met him in June 2005 at a Dagstuhl seminar *[Synthesis and Planning organized by Kautz, Thomas, Vardi]*.
- We talked about service/behavior composition, and he suggested me to look into LTL synthesis via model checking.
- In June 2006 he visited Rome and gave a PhD course on LTL synthesis, including synthesis by model checking *[Fabio Patrizi's PhD Thesis, 2009]*.
- It was an extremely fruitful and enjoyable visit, and an entire line of research was started.