# Formal Models of Service Behaviors

# Transition Systems and Bisimulation

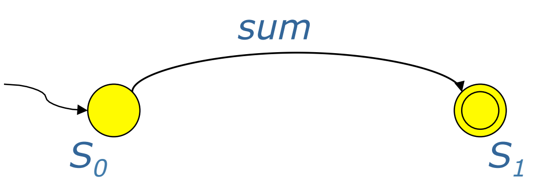
Giuseppe De Giacomo

Service Integration

# *Transition Systems*

## Concentrating on behaviors: SUM two integers

- Consider a program for computing the sum of two integers.
- Such a program has essentially two states
  - the state S0 of the memory before the computation: including the two number to sum
  - the state S1 of the memory after the computation: including the result of the computation
- Only one action, i.e. "sum", can be performed



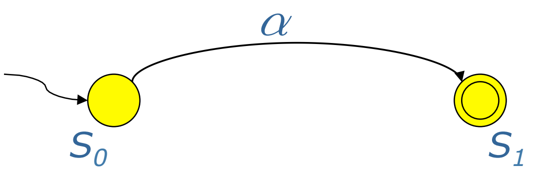



## Concentrating on behaviors: CheckValidity

- Consider a program for computing the validity of a FOL formula:
- Also such a program has essentially two states
  - the state $S_1$ of the memory before the computation: including the formula to be checked
  - the state $S_2$ of the memory after the computation: including "yes", "no", "time-out"
- Only one action, i.e. "*checkValidity*", can be performed

## Concentrating on behaviors

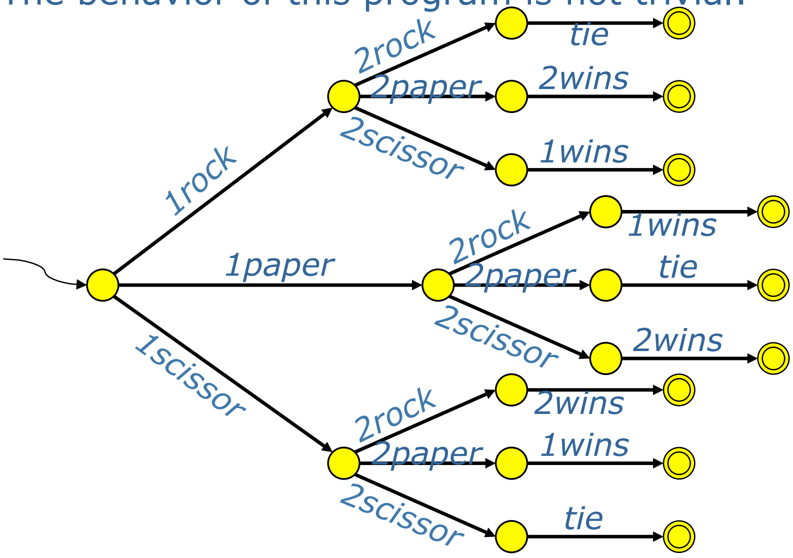
- The programs SUM and CheckValidity are very different from a computational point of view.
  - SUM is trivial
  - CheckValidity is a theorem prover hence very complex
- However they are equally trivial from a behavioral point of view:
  - two states $S_1$ and $S_2$
  - a single action $\alpha$ causing the transition





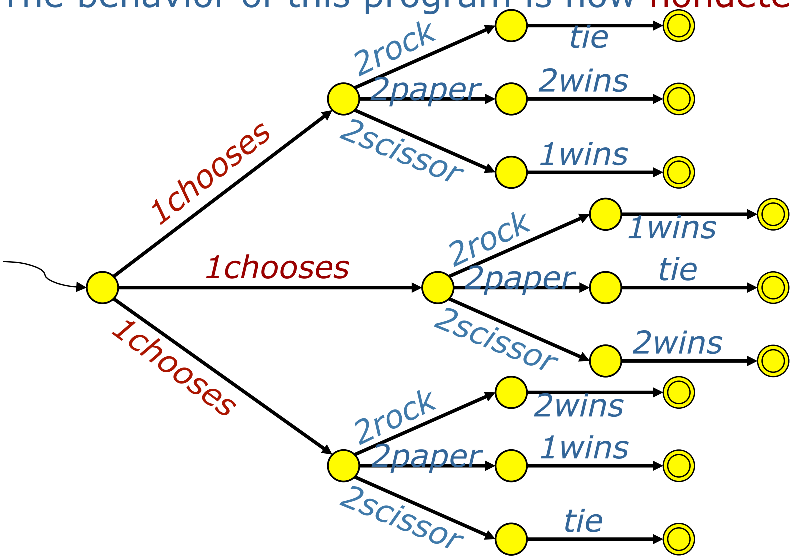

## Concentrating on behaviors: RockPaperScissor

- Consider the program RockPaperScissor that allows to play two players the the well-known game.
- The behavior of this program is not trivial:



# Concentrating on behaviors: RockPaperScissor (automatic)

- Consider a variant of the program RockPaperScissor that allows one players to play against the computer.
- The behavior of this program is now nondeterministic:






# Concentrating on behaviors: WebPage

http://www.informatik.uni-trier.de/~ley/db/

A web page can have a complex behavior!


dblp.uni-trier.de

COMPUTER SCIENCE BIBLIOGRAPHY

UNIVERSITÄT TRIER

maintained by Michael Ley - Welcome - FAQ

Mirrors: ACM SIGMOD - VLDB Endow. - SunSITE Central Europe

## Search

- Author - Title - Advanced - New: Faceted search (L3S Research Center, U. Hannover)

## Bibliographies

- Conferences: SIGMOD, VLDB, PODS, ER, EDBT, ICDE, POPL, ...
- Journals: CACM, TODS, TOIS, TOPLAS, DKE, VLDB J., Inf. Systems, TPLP, TCS, ...
- Series: LNCS/LNAI, IFIP
- Books: Collections - DB Textbooks
- By Subject: Database Systems, Logic Prog., IR, ...

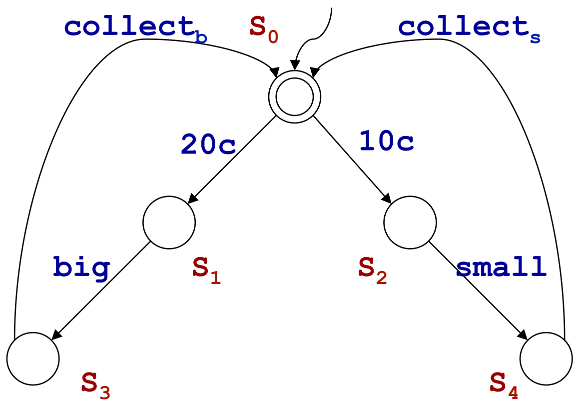
Full Text: ACM SIGMOD Anthology

## Links

- Computer Science Organizations: ACM ( DL / SIGMOD / SIGIR), IEEE Computer Society (DL), IEEE Xplore, IFIP, ...
- Related Services: CiteSeer, CS BibTeX, io-port.net, CoRR, NZ-DL, Zentralblatt MATH, MathSciNet, Erdös Number Proj., Math Genealogy Proj., BibSonomy, ...

## Concentrating on behaviors: Vending Machine

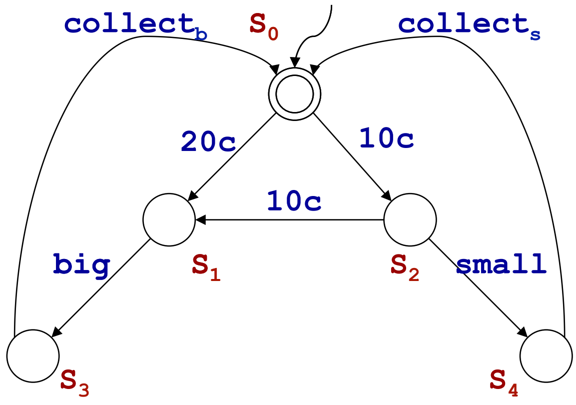
Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
SAPIENZA Università di Roma





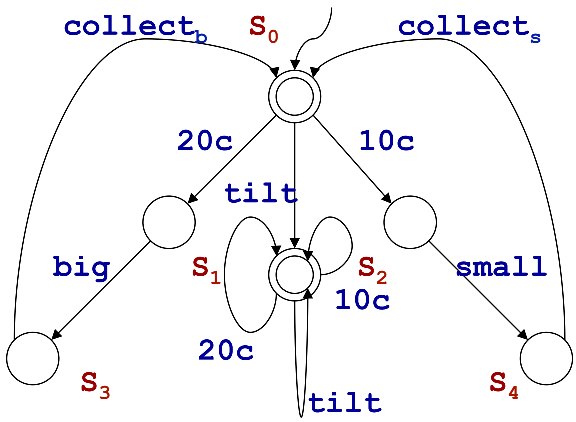

## Concentrating on behaviors: Another Vending Machine

Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
SAPIENZA Università di Roma



# Concentrating on behaviors: Vending Machine with Tilt

Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
SAPIENZA Università di Roma

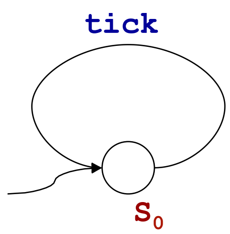





# Example (Clock)

Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
SAPIENZA Università di Roma

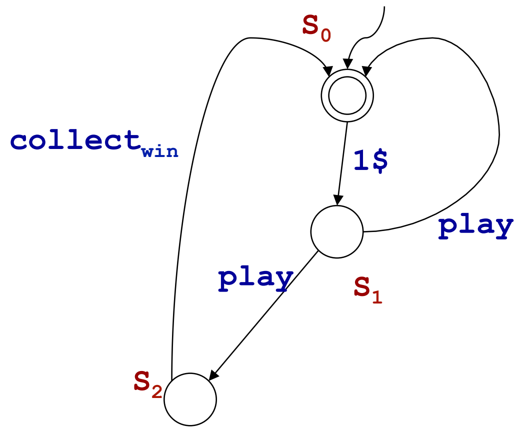
TS may describe (legal) nonterminating processes



# Example (Slot Machine)

Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
SAPIENZA Università di Roma

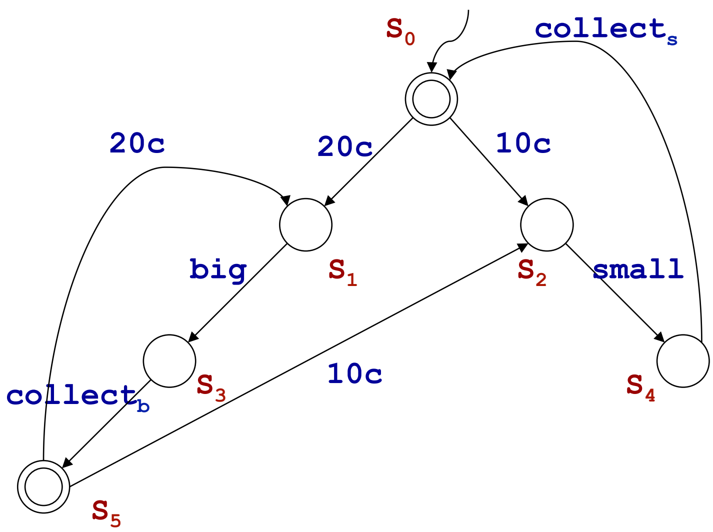
Nondereminisic transitions express
choice that is not under the control of clients



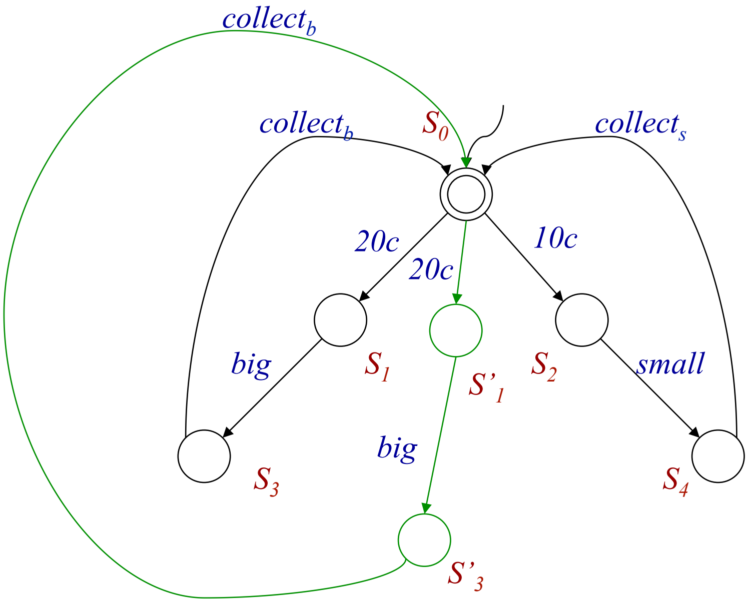



# Example (Vending Machine - Variant 1)

Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
SAPIENZA Università di Roma



# Example (Vending Machine - Variant 2)





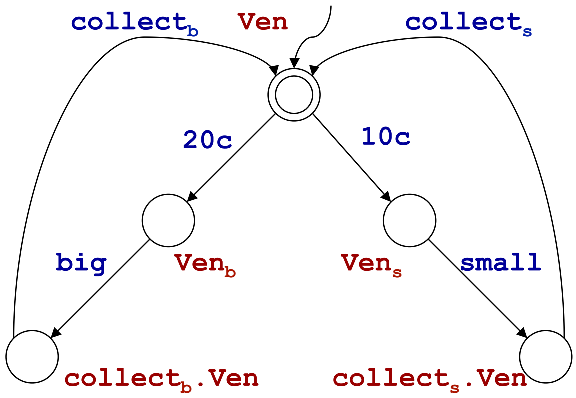

# Transition Systems

- A transition system TS is a tuple T = < A, S, S$^0$, $\delta$, F> where:
  - A is the set of actions
  - S is the set of states
  - S$^0$ $\subseteq$ S is the set of initial states
  - $\delta \subseteq S \times A \times S$ is the transition relation
  - F $\subseteq$ S is the set of final states
- Variants:
  - No initial states (c.f. Kripke Structure)
  - Single initial state
  - Deterministic actions
  - States labeled by propositions other than Final/$\neg$Final

# Process Algebras are Formalisms for Describing TS

Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
SAPIENZA Università di Roma

- Trans (a la CCS)
  - $Ven = 20c.Ven_b + 10c.Ven_s$
  - $Ven_b = big.collect_b.Ven$
  - $Ven_l = small.collect_s.Ven$
- Final
  - $\surd$ Ven




- *TS may have infinite states - e.g., this happens when generated by process algebras involving iterated concurrency*
- *However we have good formal tools to deal only with finite states TS*



Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
SAPIENZA Università di Roma

# Inductive vs Coinductive Definitions: Reachability, Bisimilarity, …

# Reachability

- A binary relation R is a **reachability-like relation** iff:
  - $(s,s) \in R$
  - if $\exists$ a, s'. $s \rightarrow_a s' \wedge (s',s'') \in R$ then $(s,s'') \in R$

- A state $s_0$ of transition system S **reaches** a state $s_f$ iff for **all** a **reachability-like relations** R we have $(s_0, s_f) \in R$.

- Notably that
  - **reaches** is a reachability-like relation itself
  - **reaches** is the smallest reachability-like relation

*Note it is a inductive definition!*



# Computing Reachability on Finite Transition Systems

**Algorithm** ComputingReachability

**Input:** transition system TS
**Output:** the **reachable-from** relation (the smallest reachability-like relation)

**Body**
  $R = \emptyset$
  $R' = \{(s,s) \mid s \in S\}$
  while $(R \neq R')$ {
    $R := R'$
    $R' := R' \cup \{(s,s'') \mid \exists s',a.\ s \rightarrow_a s' \wedge (s',s'') \in R\}$
  }
  return R'
**YdoB**

# Bisimulation

- A binary relation $R$ is a **bisimulation** iff:

  $(s,t) \in R$ implies that
  - s is *final* iff t is *final*
  - for all actions a
    - if $s \to_a s'$ then $\exists\, t' .\ t \to_a t'$ and $(s',t') \in R$
    - if $t \to_a t'$ then $\exists\, s' .\ s \to_a s'$ and $(s',t') \in R$

- A state $s_0$ of transition system S is **bisimilar**, or simply **equivalent,** to a state $t_0$ of transition system T iff there **exists** a **bisimulation** between the initial states $s_0$ and $t_0$.

- Notably
  - **bisimilarity** is a bisimulation
  - **bisimilarity** is the largest bisimulation

*Note it is a co-inductive definition!*



# Computing Bisimilarity on Finite Transition Systems

**Algorithm** ComputingBisimulation
**Input:** transition system $TS_S = \langle A, S, S^0, \delta_S, F_S \rangle$ and
transition system $TS_T = \langle A, T, T^0, \delta_T, F_T \rangle$
**Output:** the **bisimilarity** relation (the largest bisimulation)

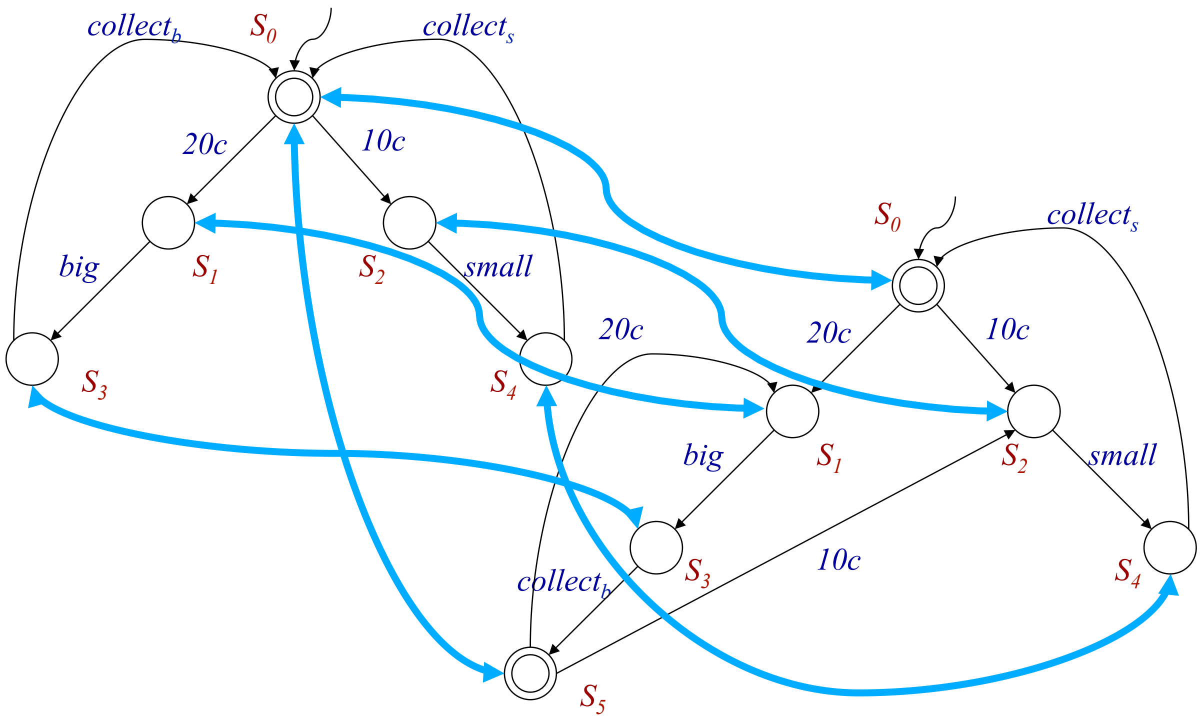
**Body**

$R = S \times T$
$R' = S \times T - \{(s,t) \mid \neg(s \in F_S \equiv t \in F_T)\}$
while $(R \neq R')$ {
  $R := R'$
  $R' := R' - (\{(s,t) \mid \exists s',a.\ s \to_a s' \wedge \neg\exists t' .\ t \to_a t' \wedge (s',t') \in R' \}$
    $\{(s,t) \mid \exists t',a.\ t \to_a t' \wedge \neg\exists s' .\ s \to_a s' \wedge (s',t') \in R' \})$
}
return R'

**Ydob**

Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
SAPIENZA Università di Roma

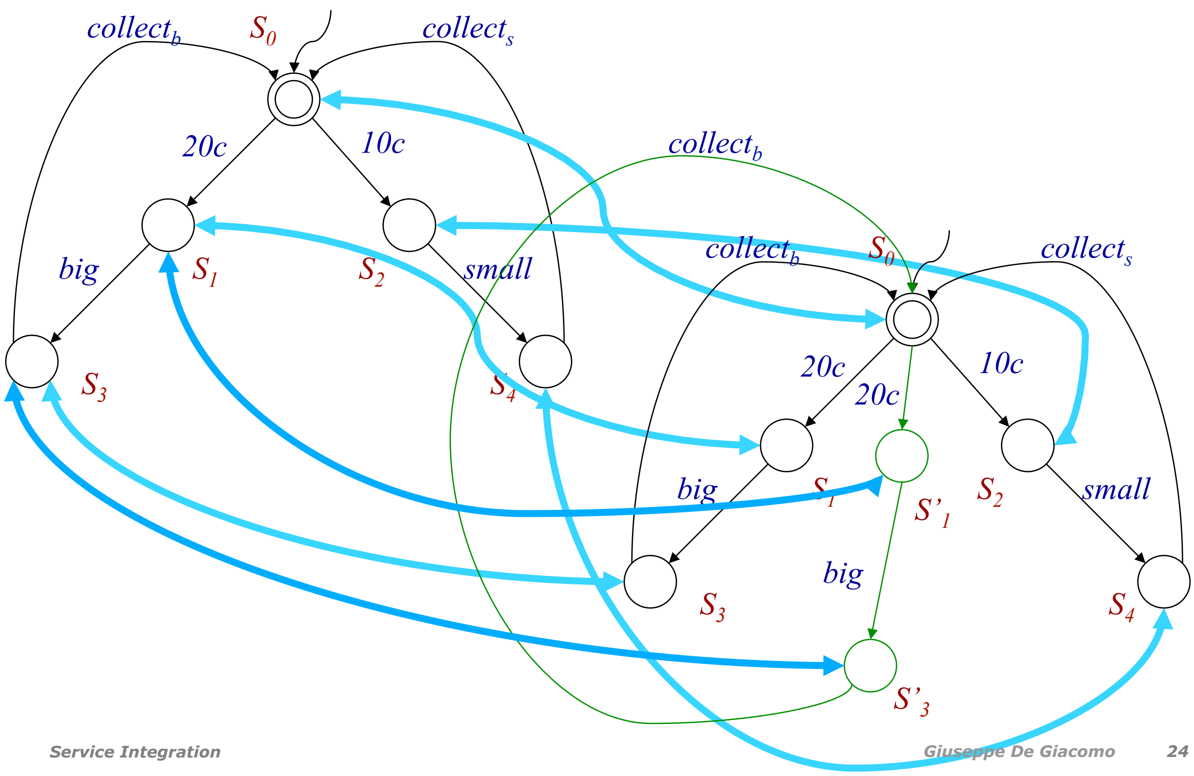
# Example of Bisimulation






Dipartimento di Informatica e Sistemistica "Antonio Ruberti"
SAPIENZA Università di Roma

# Example of Bisimulation



# Automata vs.Transition Systems

- Automata
  - define sets of runs (or traces or strings): (finite) length sequences of actions
- TSs
  - … but I can be interested also in the alternatives “encountered” during runs, as they represent client’s “choice points”