Robotics 1

September 19, 2025

Exercise 1

Consider the 2R planar robot in Fig. 1. Determine the three homogenous transformation matrices ${}^{w}\boldsymbol{T}_1$, ${}^{1}\boldsymbol{T}_2$, and ${}^{2}\boldsymbol{T}_e$ as functions of the constant geometric parameters L_1, L_2, and d_2, and of the two angles q_1 and q_2. Compute the position and orientation of the end-effector frame with respect to the world frame. Compare your result with the Denavit-Hartenberg homogeneous transformation matrix ${}^{w}\boldsymbol{T}_e(\theta_1, \theta_2)$, defining the associated table of parameters. What is your conclusion?

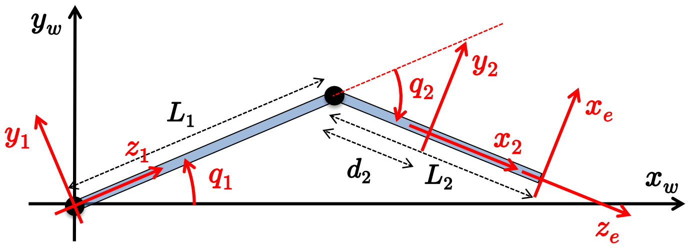

Figure 1: A 2R planar robot with world frame and generic frames attached to the links.

Exercise 2

For the 3-dof planar robot in Fig. 2, one can define a task vector $\boldsymbol{r} = (p_x, p_y, \alpha)$ containing the position of the robot tip and the angle $\alpha \in (-\pi, \pi]$ of the last link with respect to the axis x_w of the world frame. Using the joint variables defined in the figure, find all inverse kinematics solutions of this robot for a given task vector $\boldsymbol{r}_d$. Determine also the singular cases and explain what happens then. Evaluate your solution with the numerical data $L = 1$ m and $\boldsymbol{r}_d = (1, 0, -\pi/4)$ [m,m,rad]. Finally, compute the task Jacobian matrix $\boldsymbol{J}(\boldsymbol{q}) = \partial \boldsymbol{r}/\partial \boldsymbol{q}$ and find its singularities.

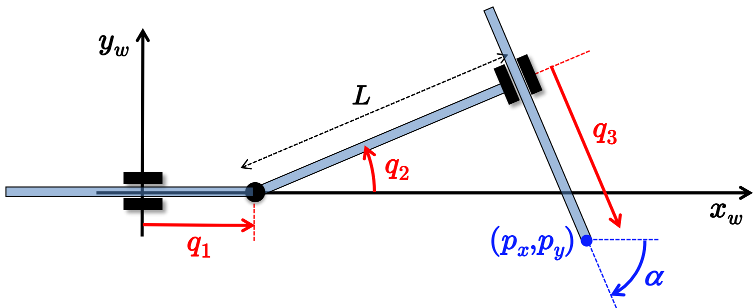

Figure 2: A 3-dof planar robot with the definition of joint and task variables.

Exercise 3

A robot joint should move from q_i at $t = 0$, with a generic initial velocity v_i, to q_f at $t = T$, using a trajectory $q(t)$ that has continuous acceleration in the open interval $(0, T)$. Choose the motion profile and determine analytically the value of the final velocity v_f to be attained at $t = T$ so that the resulting initial acceleration is $\ddot{q}(0) = 0$. Provide then the expression of the corresponding maximum values of $|\dot{q}(t)|$ and $|\ddot{q}(t)|$ in the closed interval $[0, T]$. Using the numerical data $q_i = -0.5$, $q_f = 1$ [rad] and $T = 3$ s, apply your results to the two cases *i)* $v_i = 0$ and *ii)* $v_i = 1$ [rad/s].

[150 minutes; open books]