Robotics 2

November 7, 2024

Exercise 1

Consider the robot in Fig. 1, with two prismatic and a third revolute joint, moving in a vertical plane. Friction at the joints can be neglected.

a) Derive the dynamic model of the robot in the Lagrangian form $\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}})+\boldsymbol{g}(\boldsymbol{q})=\boldsymbol{\tau}$. If needed, introduce any missing kinematic or dynamic parameter.

b) Find a linear parametrization $\boldsymbol{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\,\boldsymbol{a}=\boldsymbol{\tau}$ of the robot dynamics in terms of a vector $\boldsymbol{a}\in\mathbb{R}^r$ of dynamic coefficients and a $3\times r$ regressor matrix $\boldsymbol{Y}$. Discuss the minimality of r.

c) Provide an upper bound $\alpha>0$ for the norm of the matrix $\dfrac{\partial \boldsymbol{g}}{\partial \boldsymbol{q}}$. Where is this information used?

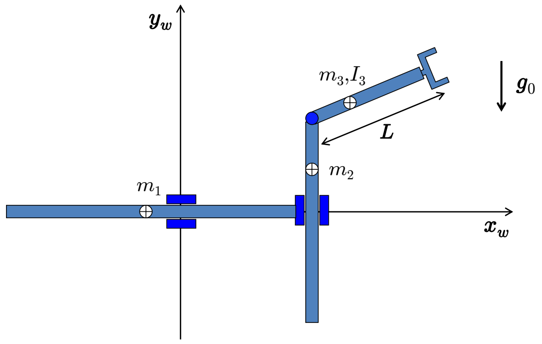

Figure 1: A PPR planar robot.

Exercise 2

For the PPR robot of Fig. 1, consider the following desired task trajectory involving position and orientation of the end-effector in the plane:

$$\boldsymbol{r}_d(t)=\begin{pmatrix} p_{x,d}(t)\\ p_{y,d}(t)\\ \phi_d(t)\end{pmatrix}=\begin{pmatrix} x_0+R\cos\phi_d(t)\\ y_0+R\sin\phi_d(t)\\ \omega\, t\end{pmatrix}\qquad t\in[0,T], \tag{1}$$

for given x_0, y_0, $R>0$ (and $R\neq L$), $\omega>0$, and $T>0$. Compute the inverse dynamics torque $\boldsymbol{\tau}_d(t)$ and the initial robot state $(\boldsymbol{q}(0),\dot{\boldsymbol{q}}(0))$ needed to perfectly execute this task trajectory. How would you modify in the simplest way the control torque $\boldsymbol{\tau}$ to handle possible task errors?

Exercise 3

Consider the writing task in Fig. 2. Assign a task frame and define the natural and artificial constraints for the execution of this task in nominal conditions.

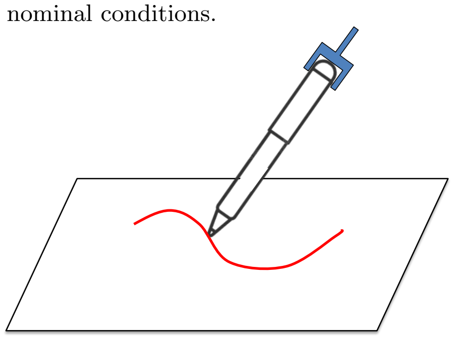

Figure 2: A robot rigidly holding a ball pen for writing on a flat sheet.

[180 minutes (3 hours); open books]