Control Systems

Introduction

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course main topics

- analysis (time and frequency domain)
- general feedback control system
- controller design in the frequency domain (loop shaping)
- performance and limitations of a control system
- analysis and design using root locus
- state space design
- stability theory

goal

fundamental objective of the control systems course

- analysis
- control

of dynamical systems

dynamical system:
a system whose state variables evolve over time

state:
variables whose time evolution univocally characterize the system

e.g. in mechanical engineering, rigid body dynamics studies forces/ torques that produce motion, i.e., that make positions and velocities vary over time

dynamics

- we need to describe how a quantity $x(t)$ varies in time
- how do we represent such a variation?

variation in time of the quantity x

depending on the nature of the time variable

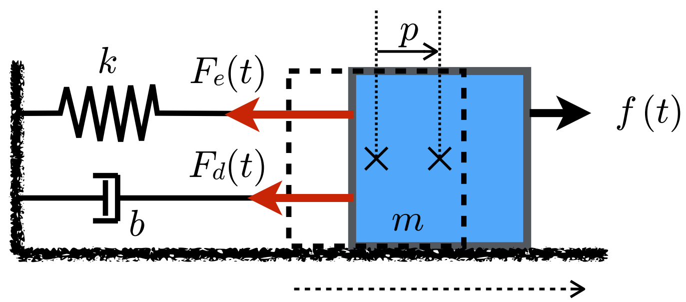
$t \in \mathbf{R}$	$t \in \mathbf{Z}$
in continuous time (C.T.)	in discrete time (D.T.)
derivative	difference
$\dfrac{dx(t)}{dt} = \dot{x}(t) = \dot{x}$	$x(t+1) - x(t)$

dynamics

examples of known relationships including time derivatives:

- mass-spring-damper system

  acceleration (a), velocity (v), position (p), force (f)



$$m\,a(t) + b\,v(t) + k\,p(t) = f(t)$$

$$m\,\ddot{p}(t) + b\,\dot{p}(t) + k\,p(t) = f(t)$$

- capacitor: voltage (v) - current (i)

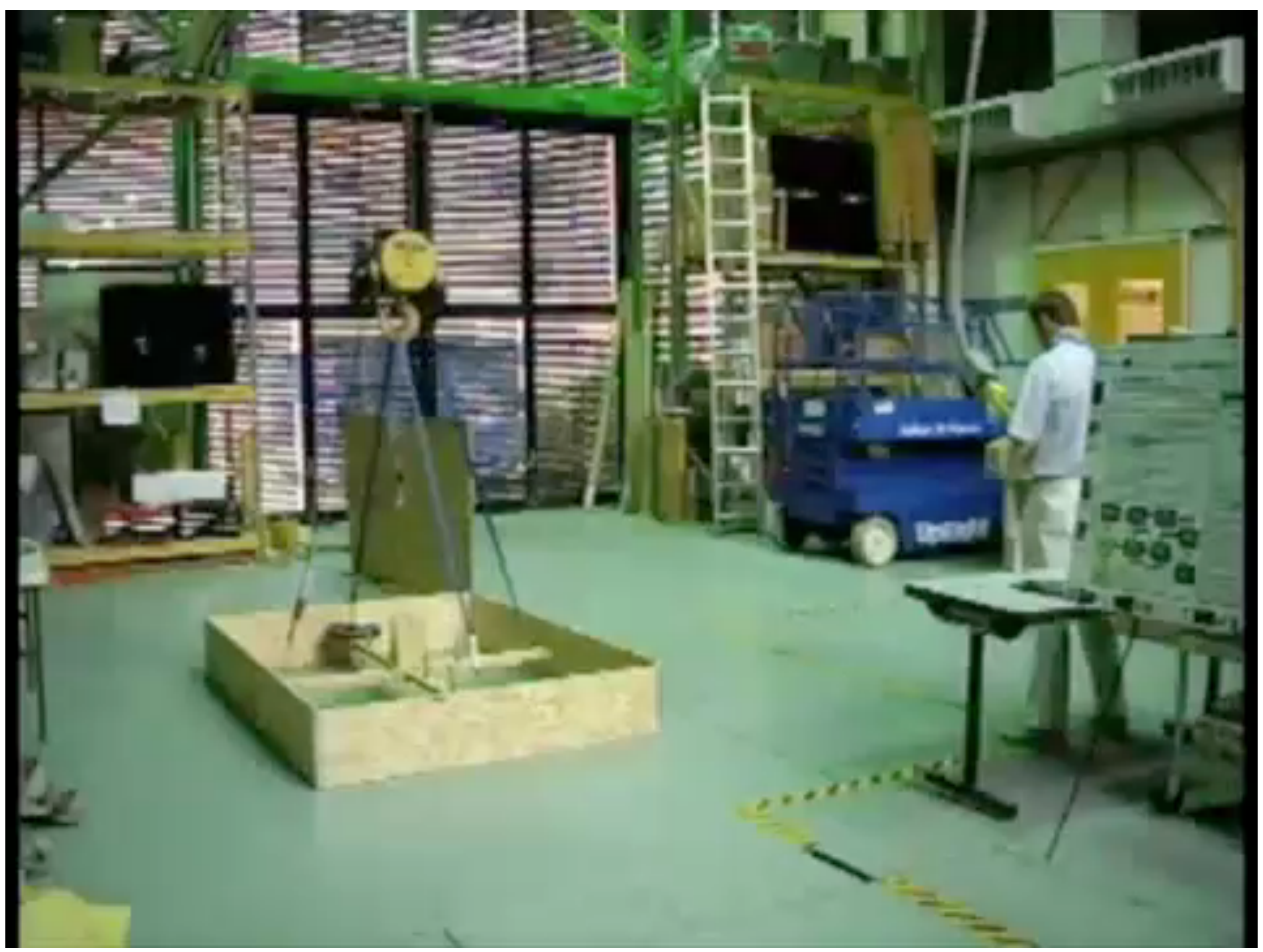
$$\frac{d\,v(t)}{dt} = \frac{1}{C} i(t)$$

- inductor: current (i) - voltage (v)

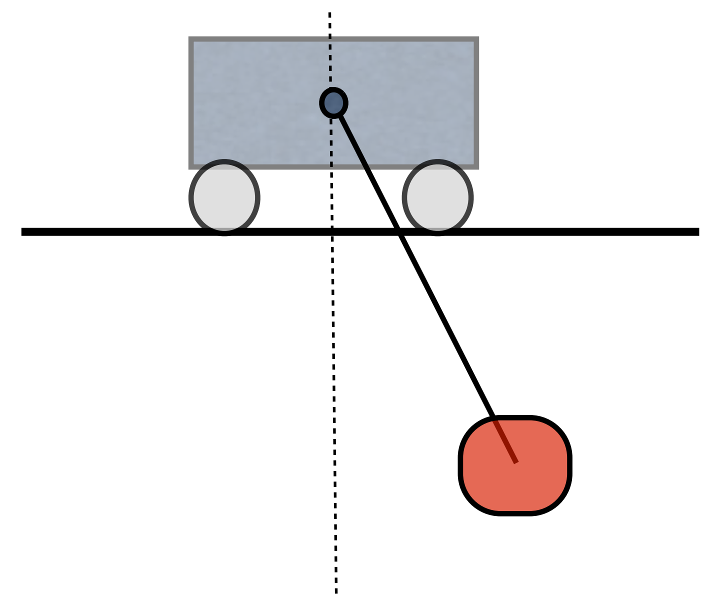
$$\frac{d\,i(t)}{dt} = \frac{1}{L} v(t)$$

importance of dynamics

- description of the motion (eg. satellite trajectory)
- simulation models (system behavior wrt to inputs)



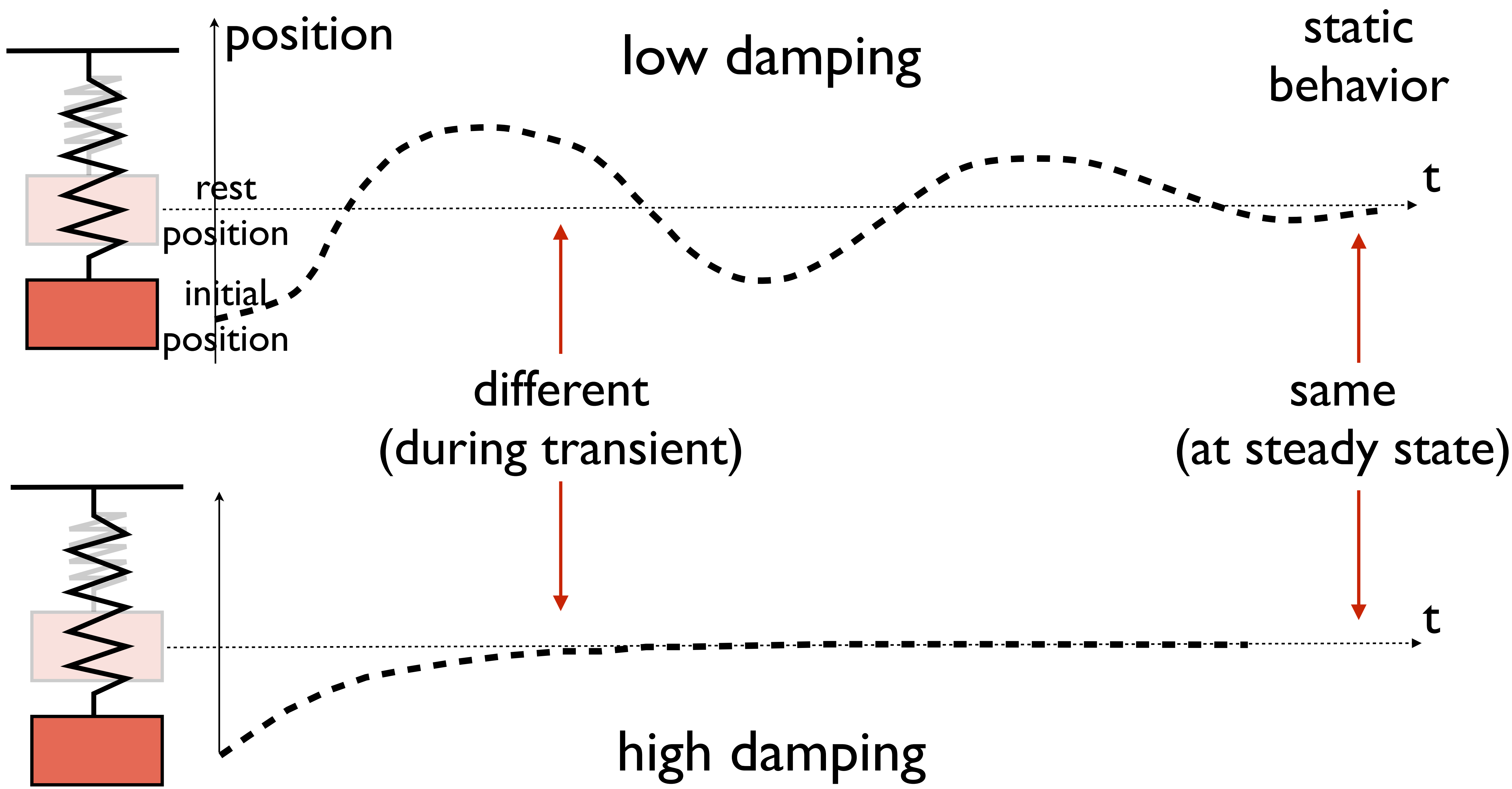
simplified model



crane control:
input shaping technique
(Georgia Tech)

importance of dynamics

- same static behavior, different dynamic one
  two similar systems with different damping or friction coefficient and similar spring elastic coefficient, starting from same initial position



analysis of dynamic properties

- infer important properties from few basic quantities
  - e.g. stability (dynamic matrix eigenvalues)
  - characterization of the dynamic behavior as the transient or the steady-state (bandwidth, overshoot, poles, …)
  - study the possibility to influence the dynamics through the input (controllability analysis)
  - understand the internal dynamics through the observation of the output (observability analysis)
- these will allow a clear formulation of specifications for the control system design
- other uses: forecast, prediction

qualitative analysis of systems of differential equations

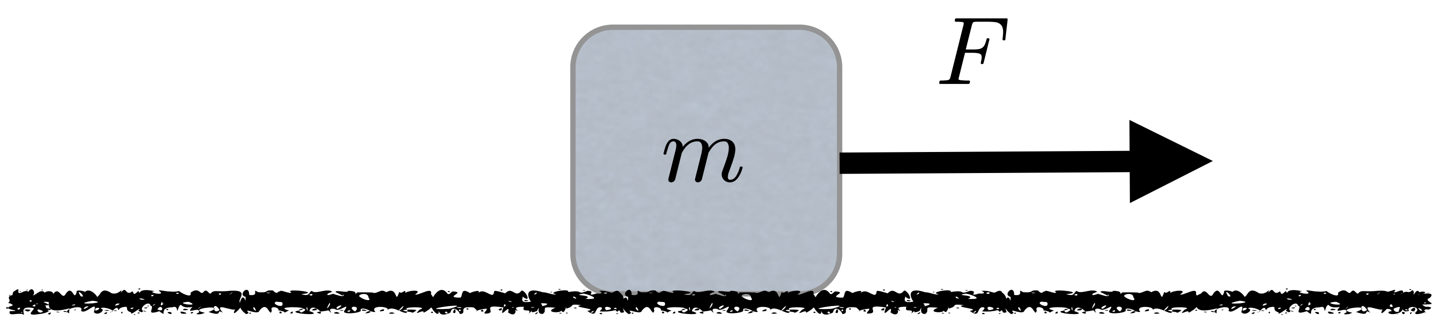
analysis

- model based approach:

  representation of the real system through a **model**

  (usually includes approximations)
- in particular we consider dynamical systems whose **mathematical model** is a set of differential equations
- the analysis consists in the study of some characteristics of the system's mathematical description with particular emphasis on quantities that characterize the system motion

example



- mass m moving on a line (one-dimensional motion) under the action of a force F
- hyp: no friction

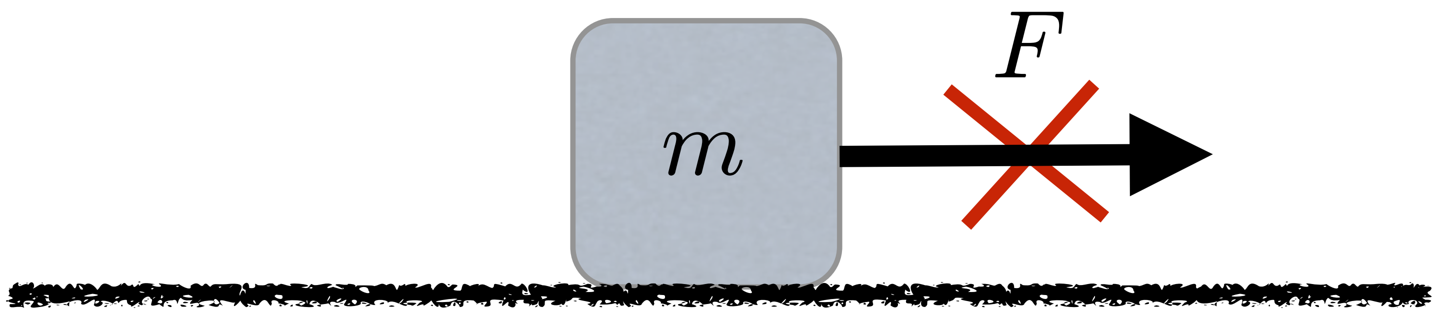
$$m\,\dot{v} = F$$

mathematical model

this mathematical relationship tells us how the variation of the mass velocity is related to the applied force under the assumed hypothesis: it is our **model**

+ other tacit hypothesis
(ex. m constant otherwise linear momentum)

example (cont.)



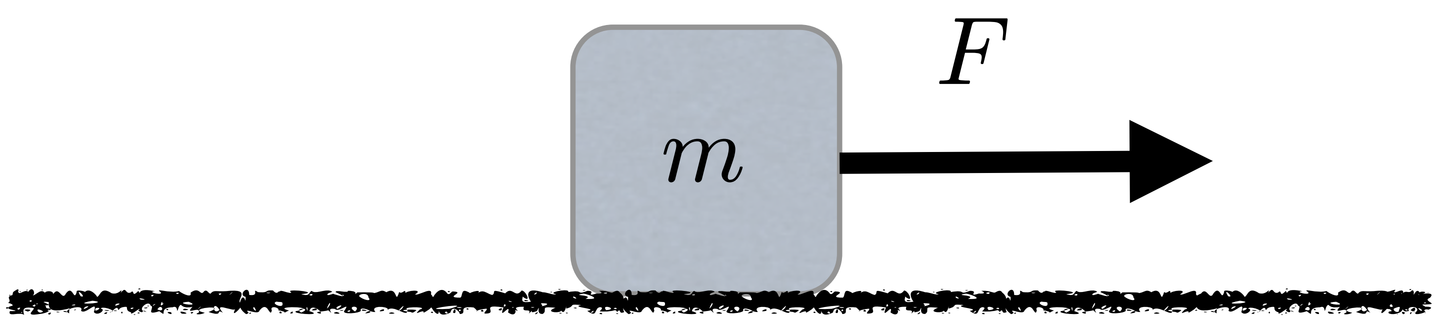
- if $F = 0$ do we still have motion?

model becomes $\dot{v} = 0$ and the solution is $v(t) = v(0)$

  - if we have a non-zero initial velocity $v(0)$, the mass moves (at constant speed)

we need to learn how to read the information hidden in the mathematical model

example (cont.)



model
(linear differential equation)

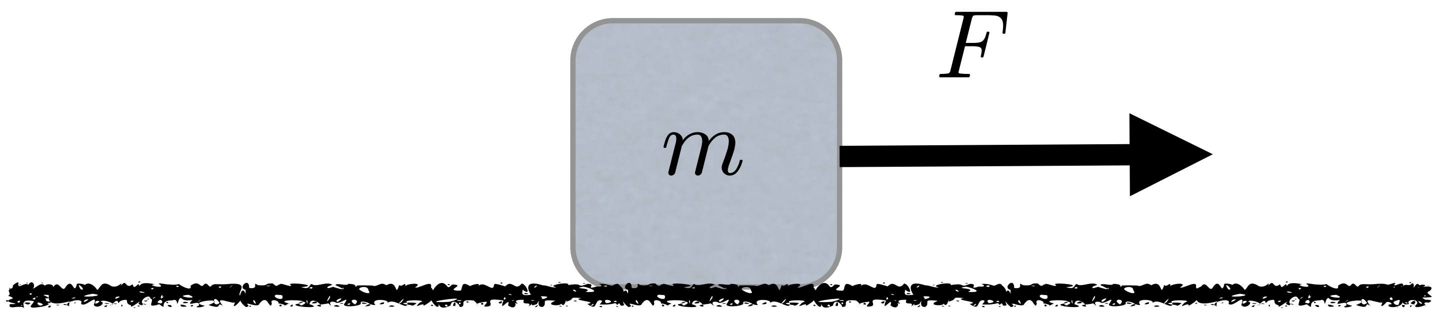
$$\begin{aligned} \dot{v} &= \frac{F}{m} \\ v(0) &= v_0 \end{aligned}$$

we have noticed that the motion is generated by

- forcing term $F(t)$ (will be called input to the system)
- initial condition v_0

here (v_0, F) represents the cause (of motion)
and $v(t)$ is the effect (motion) of such causes

example (cont.)



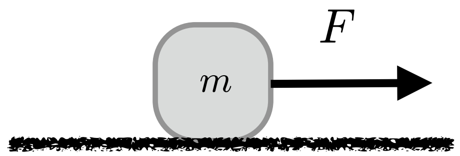
solution of the differential equation (model)

$$v(t) = v_0 + \frac{1}{m} \int_0^t F(\tau) d\tau$$

tells us how the velocity depends upon the initial condition **and** the applied force. Knowing the applied force and the initial velocity we know how the velocity of the point mass behaves in the future

new capacity: **analysis** & **prediction**

example (cont.) - linearity



- with initial condition $v_0 = 0$ and $F \neq 0$ we have velocity v

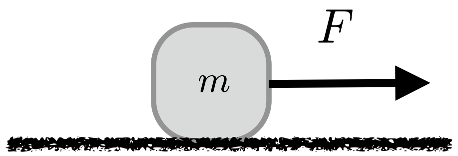
  if we apply $2F$ instead of F what happens to velocity?

$$\tilde{v}(t) = \cancel{v_0} + \frac{1}{m}\int_0^t {\color{red}2}F(\tau)d\tau$$

  the velocity will also double to $2\,v$

linear behavior wrt to F

example (cont.) - linearity



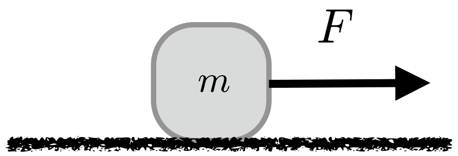
- if we apply no force $F = 0$ and start with non-zero $v_0 \neq 0$, the velocity will be $v = v_0$

  clearly, if the initial velocity changes to $3v_0$ the velocity will also triple

$$\tilde{v}(t) = 3v_0 + \cancel{\frac{1}{m}\int_0^t F(\tau)d\tau}$$

linear behavior wrt to the initial condition v_0

example (cont.) - linearity



- Att. $F \neq 0$ and $v_0 \neq 0$ simultaneously

if $F \longrightarrow 2F$

and $v_0 \longrightarrow 3v_0$

what happens to velocity?

previous examples

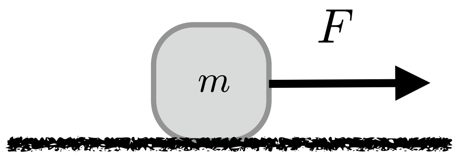
if $(v_0, 0) \longrightarrow v_1(t)$ and $(0, F) \longrightarrow v_2(t)$ then $(3v_0, 0) \longrightarrow 3v_1(t)$ and $(0, 2F) \longrightarrow 2v_2(t)$

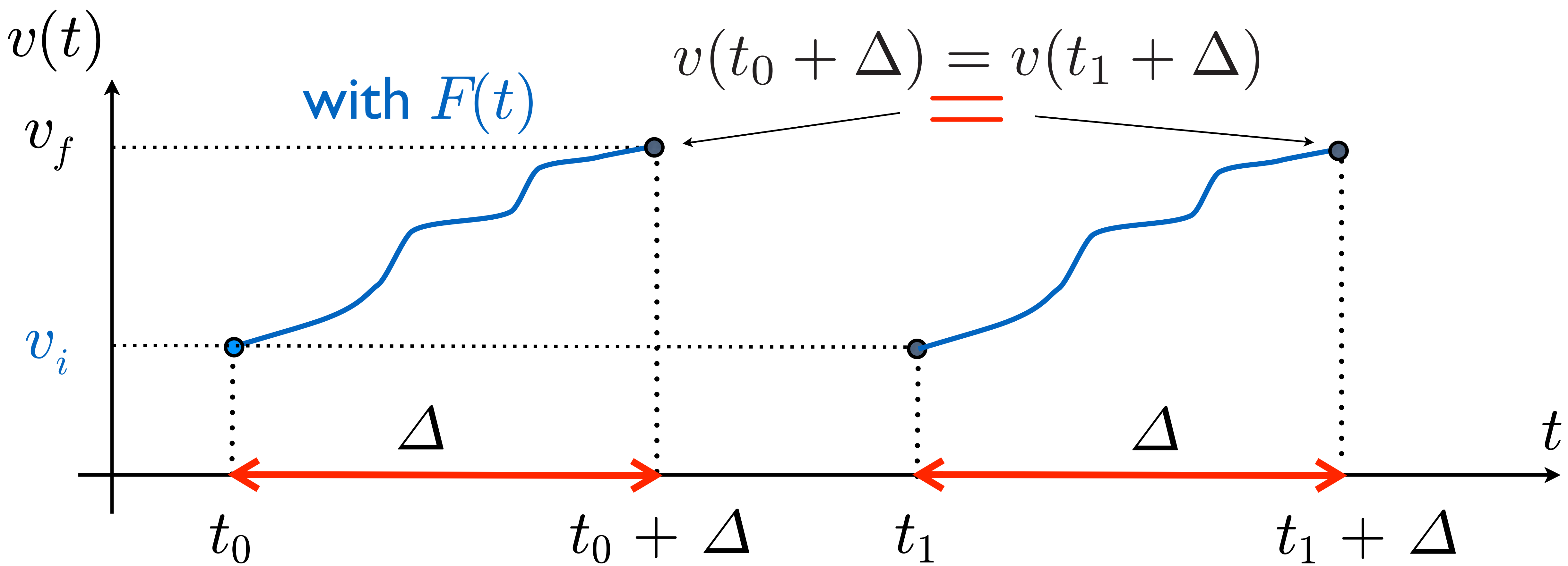
linear behavior wrt the motion causes (v_0, F)

Linear combination of causes	$\Rightarrow$	same linear combination of effects

this linearity comes from the differential equation being linear

example (cont.) - time invariance





same initial condition v_i and same input (force) $F(t)$ after same time interval Δ leads to the same state

- state evolution does not depend on the initial time t_0 but only on the elapsed time Δ
- this time invariance translates into the differential equation having **constant coefficients**

general mathematical model

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) + D\,u(t)$$
$$x(0) = x_0$$

Linear Time Invariant (LTI)
dynamical system
(Continuous Time)

$x(t)$ **state** $x \in \mathbf{R}^n$

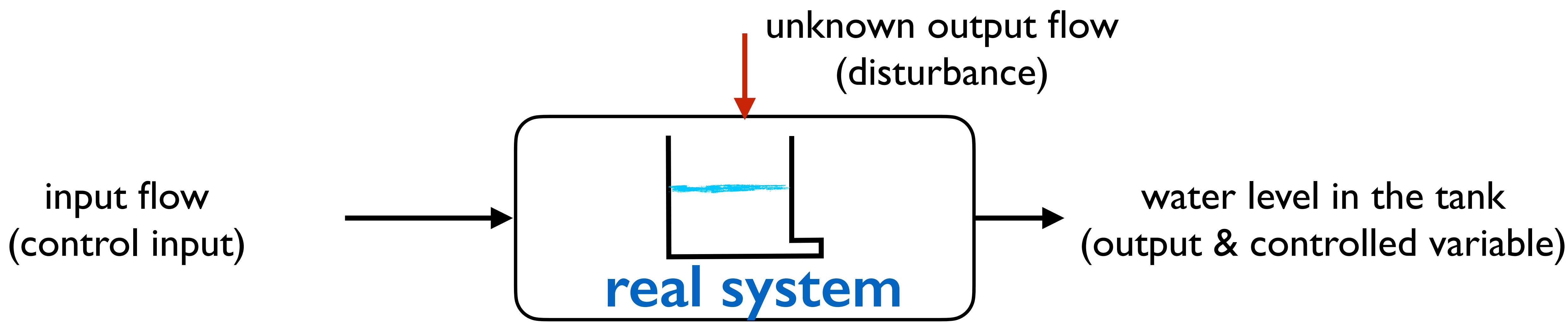
$u(t)$ **input** $u \in \mathbf{R}^m$ multi input (we consider $m = 1$, single input)

$y(t)$ **output** $y \in \mathbf{R}^p$ multi output (we consider $p = 1$, single output)

SISO (single input/single output) linear time-invariant system

state, input and output dimensions determine the 4 matrices dimensions

control example: water level in a tank



problem: we want to maintain the water level at a desired height regardless of the unknown output flow and any other disturbance

understand how to choose the (control) input in order to guarantee a **desired behavior** of the output

disturbance $d(t)$

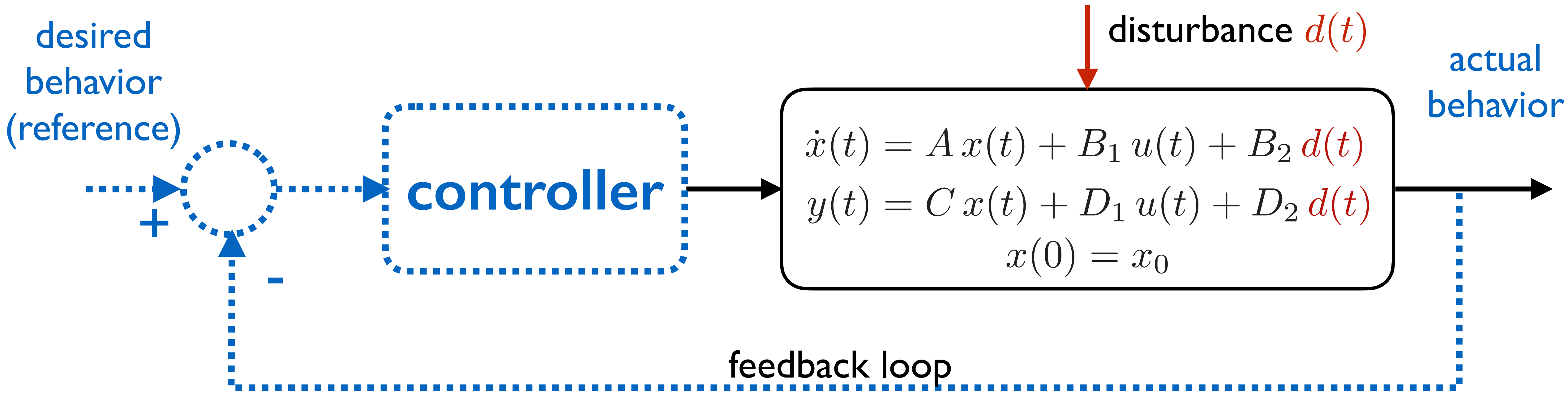
input $u(t)$
(control input)

$$\dot{x}(t) = A\,x(t) + B_1\,u(t) + B_2\,d(t)$$
$$y(t) = C\,x(t) + D_1\,u(t) + D_2\,d(t)$$
$$x(0) = x_0$$

output $y(t)$
(controlled variable)

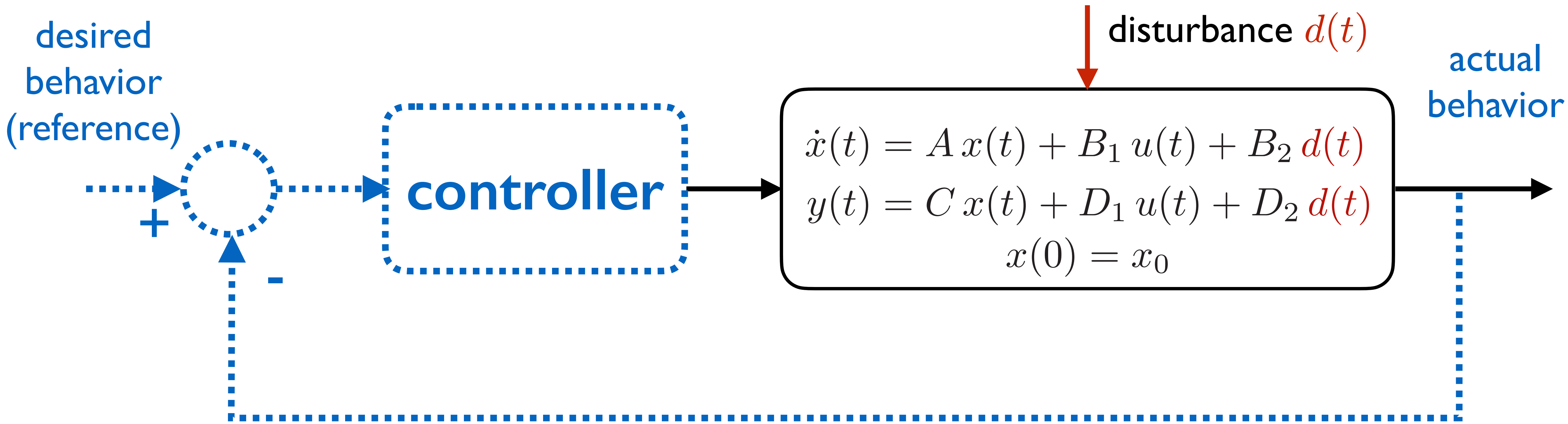
model

control example: water level in a tank

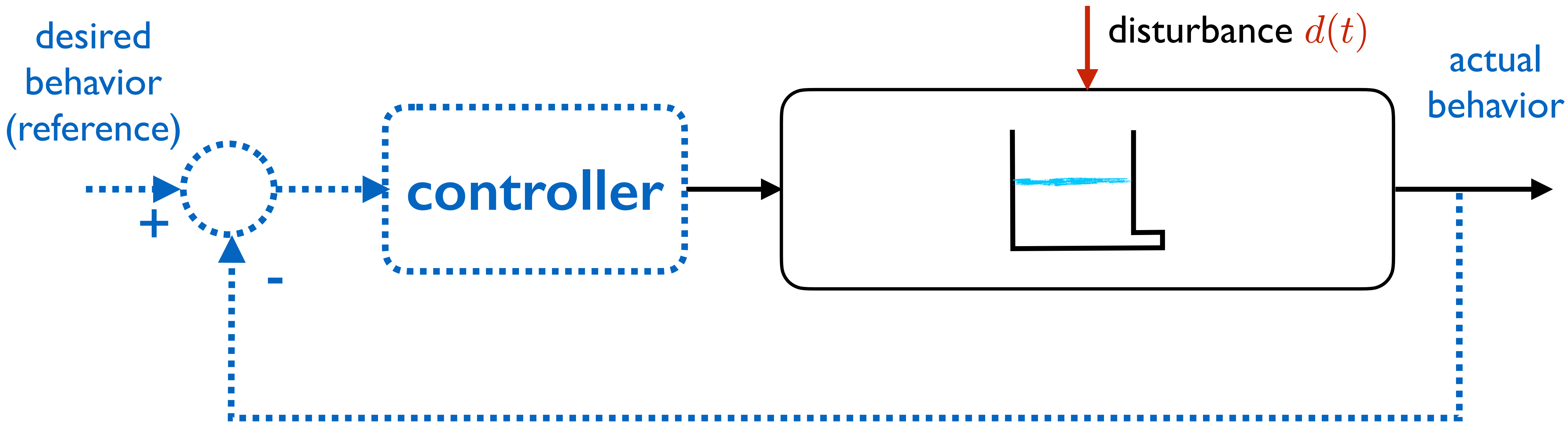


- schematic diagram of an **automatic control system** based on **feedback**
- the design of such a control system requires the determination (design) of the **controller**
- need a systematic procedure in order to design the controller
- design (and controller) will be based on the **plant model**

control example: water level in a tank



control scheme is **implemented** on the real system



typical flow

- problem definition (real system)
- mathematical model (+hypothesis & simplifications)
- real specifications translated into control system language
- design of the control system
- simulation on the more complete available model
- implementation