Self assessment - 01

30/10/2014 - Updated 31/01/2020

1 Exercise

Let the input $u(t)$ and output $y(t)$ of a system satisfy the following linear differential equation

$$y^{(5)}(t) + 4y^{(4)}(t) + 3y^{(3)}(t) - 2y^{(2)}(t) + y^{(1)}(t) + y(t) - u(t) = 0$$

where $y^{(i)}(t)$ denotes the i-th time derivative of $y(t)$. For this system:

1. find a state space representation
2. compute the transfer function and say if there exists any uncontrollable or unobservable mode
3. say if the system is asymptotically stable or not.

2 Exercise

Let the system S respond, from zero initial conditions, with

$$y(t) = \left(1 - t + \frac{t^2}{2} - e^{-t}\right)\delta_{-1}(t)$$

to the input

$$u(t) = \delta(t) - 2e^{-3t}\delta_{-1}(t)$$

Find the impulse response $w(t)$ of S.

3 Exercise

Find the output forced response (output zero-state response) $y(t)$ of the system represented by

$$F(s) = \frac{50}{s^2 + 15s + 50}$$

to the input $u(t)$ shown in Fig. 1

4 Exercise

For each system having the dynamics matrix A_i discuss the stability property

$$A_1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 1 & 3 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -1 & 4 & -2 \\ 0 & -3 & 1 \\ 0 & 0 & 3 \end{pmatrix}, \quad A_3 = \begin{pmatrix} -1 & 0 & 0 \\ -3 & -3 & 0 \\ -3 & 1 & 3 \end{pmatrix},$$

$$A_4 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{pmatrix}, \quad A_5 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -1 & -12 \end{pmatrix}, \quad A_6 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

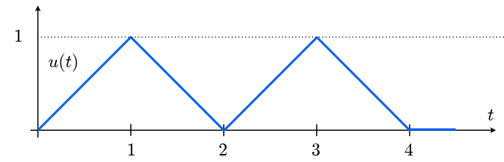

Figure 1: Ex. 3, input $u(t)$

5 Exercise

Assuming the coincidence of poles and eigenvalues, study the stability property of the following systems.

$$P_1(s) = \frac{s-1}{s^2}, \quad P_2(s) = \frac{s-1}{s(s+1)}, \quad P_3(s) = \frac{s+1}{s^3+12s^2+3s}, \quad P_4(s) = \frac{s+1}{s^3+12s^2+s+10}$$

$$P_5(s) = \frac{s^2-18}{s^3+12s^2+s-12}, \quad P_6(s) = \frac{-1}{s^3+2s^2+s+1}, \quad P_7(s) = \frac{s-10}{s^5+s^4+2s^3+s^2+3s+4}$$

6 Exercise

For the system having dynamics matrix

$$A = \begin{pmatrix} k & 1 \\ 0 & 0 \end{pmatrix}$$

determine, depending upon the values of $k \in R$, the natural modes and study stability.

7 Exercise

Find the forced response of the system

$$P(s) = \frac{s-1}{s+1}$$

to the input $u(t) = e^t \delta_{-1}(t) - 2t\delta_{-1}(t)$.

8 Exercise

For the system

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y &= \begin{pmatrix} 1 & -1 \end{pmatrix} x \end{aligned}$$

find the forced zero-state response to the input $u(t)$ shown in Fig. 2 using

$$\mathcal{L}\left[\sin(\omega t)\, \delta_{-1}(t)\right] = \frac{\omega}{s^2+\omega^2}$$

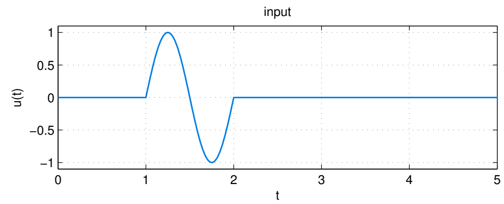

Figure 2: Ex. 8, input $u(t)$

9 Exercise

Find the natural modes of the system having dynamics matrix

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

10 Exercise

Compute the free state and output response of the system

$$\begin{aligned} \dot{x}(t) &= \begin{pmatrix} -2 & -1 \\ -1 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u(t) \\ y(t) &= \begin{pmatrix} 2 & 1 \end{pmatrix} x(t) \end{aligned}$$

from the initial condition

$$x(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

11 Exercise

Determine the initial conditions of the system

$$\begin{aligned} \dot{x}(t) &= \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u(t) \\ y &= \begin{pmatrix} 1 & -1 \end{pmatrix} x(t) \end{aligned}$$

for which we obtain a non-diverging free output.

12 Exercise

For the system given by

$$\dot{x}(t) = \begin{pmatrix} 6 & -3 \\ 2 & -1 \end{pmatrix} x(t)$$

determine the initial conditions, if any, such that the zero-input output response remains constant.