Self assessment - 02 - Solutions highlights

Updated 30/12/2019 - 03/01/2020 - 30/01/2020 - 03/01/2024

1 Exercise

Consider the plant

$$\begin{aligned}\dot{x}_1 &= x_1 + x_3 + u\\ \dot{x}_2 &= u\\ \dot{x}_3 &= -2x_3\\ y &= x_1 + x_2 + \alpha x_3\end{aligned}$$

with $\alpha \in \mathbb{R}$ a real parameter.

1. Study the controllability property and, if necessary, do a Kalman decomposition w.r.t. controllability.
2. Find the value(s) of α such that there exists an unobservable asymptotically stable subsystem. Decompose w.r.t. observability. From now on use this value of α.
3. Using the previous decomposition, is it possible to find an output stabilizing dynamic controller of dimension 2? Why?
4. Find the plant's transfer function and determine if the system is stabilizable with a simple static output feedback.
5. Determine an output dynamic controller which assigns the closed-loop poles in -1, -2 and -3.
6. How does the previous closed-loop system behaves at steady-state w.r.t. a constant reference and to an unknown constant output disturbance?

2 Exercise

Let the open-loop system be

$$F(s) = \frac{K(s+1)}{s(s+100)^2}$$

1. Study, as $K \in \mathbb{R}$ varies, the stability of the unit feedback closed-loop system both using the Nyquist criterion and the root-locus plot.
2. Determine if there is a closed-loop dominant pole and, if it exists, discuss its contribution as K increases (positive values only).

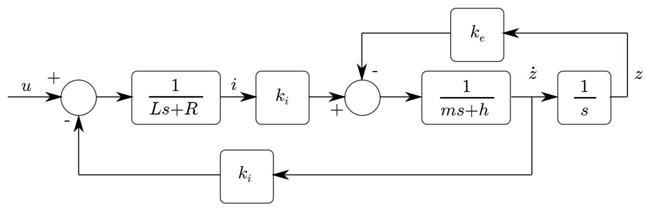

Figure 1: Ex. 5, loudspeaker block diagram

3 Exercise

Let the open-loop system be

$$F(s) = \frac{10}{s(s+11)}$$

Determine the frequency range (in rad/s or Hertz) where the closed-loop system guarantees an attenuation of at least 20 dB to sinusoidal disturbances acting on the feedback loop. An answer based on an approximate behavior is accepted.

4 Exercise

Let the open-loop system be

$$P(s) = \frac{K(s+z)}{(s+2)(s+3)}, \quad z > 0$$

Determine for the closed-loop system the different types of stability as both $K \in \mathbb{R}$ and $z > 0$ vary. Illustrate the different corresponding root-locus plots.

5 Exercise

In a magnetic loudspeaker, a cone of mass m and position $z(t)$ is kept in place by an elastic suspension characterized by an elastic constant k_e. During its movement, the cone is subject to some viscous damping (acoustic coupling with the air) which depends linearly, through the coefficient h, on the cone's velocity $\dot{z}$. The mobile coil is represented by an electrical circuit with a resistor R and an inductance L while the electroacoustic coupling due to the magnetic flux in the air gap is given by k_i. Let $i(t)$ be the current through the mobile coil and $u(t)$ the applied input voltage. The dynamic equations are

$$\begin{aligned} L\frac{di(t)}{dt} + Ri(t) + k_i\frac{dz(t)}{dt} &= u \qquad \text{electric components} \\ m\frac{d^2z(t)}{dt^2} + h\frac{dz(t)}{dt} + k_e z(t) &= k_i i(t) \qquad \text{forces equilibrium} \end{aligned}$$

- Show that the block diagram reported in Fig. 1 corresponds to the system under consideration.
- Find the transfer function $u(s) \to \dot{z}(s)$.
- Is there a physical interpretation of the particular numerator found in the previous question?

6 Exercise

Let the plant be modeled by the following transfer function

$$P(s) = \frac{10}{s(s+0.1)}$$

Design a control scheme which guarantees a steady-state error in magnitude smaller than 1% w.r.t. a unit reference ramp, a phase margin of at least 30° and a crossover frequency of 1 rad/s.

7 Exercise

Study, as $K \in \mathbb{R}$ varies, the stability of the unit feedback closed-loop system having $F(s)$ as open-loop. Use both the Nyquist criterion and the root-locus plot.

$$F(s) = \frac{K(s^2 + 20s + 100)}{s^2(s+1)}$$

Finally check with the Routh criterion.

8 Exercise

We want to control the temperature $T(t)$ inside a closed tank containing a fluid. Using the energy conservation principle we obtain the following differential equation which describes the temperature $T(t)$ time evolution

$$C\dot{T}(t) + qc_v\left[T(t) - T_i\right] + \frac{1}{R}\left[T(t) - T_a\right] = Q_{in}(t)$$

where $Q_{in}(t)$ can be manipulated.

Symbol	Units	Description
C	J/K	Thermal capacity
q	kg/s	Fluid flow in transit
c_v	J/(kg . K)	Fluid specific heat
T_i	K	Constant input fluid temperature
R	K . s/J	Thermal resistance due to the tank's wall
T_a	K	External constant temprature
$Q_{in}(t)$	J/s	Input heat flux

- Give a state-space representation of the system. Let $T(t)$ be measurable.
- How does the dynamic behavior change as C varies? Give a physical interpretation.
- Draw a control scheme to regulate the internal temperature $T(t)$.
- Discuss which specifications would you require and how to solve them.

9 Exercise

For the interconnected system in Fig. 2, find the transfer function $d_2 \to y$.

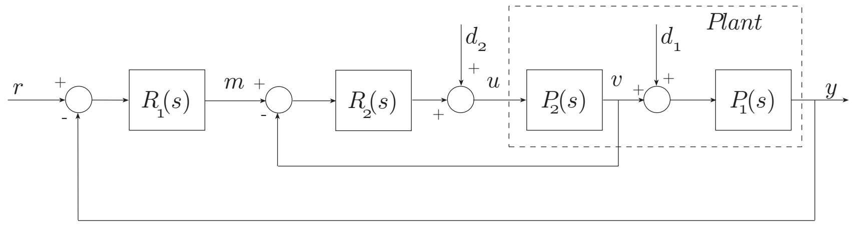

Figure 2: Ex. 9, interconnected system

10 Exercise

Let the plant be

$$P(s) = \frac{1}{s + 0.1}$$

Design a control system such that the following specifications are met:

a) the output asymptotically tracks the reference signal $r(t) = t\delta_{-1}(t)$, with a maximum allowed error in magnitude equal to 1;

b) no steady-state influence of a constant disturbance acting on the plant's output;

c) phase margin of at least 30°;

d) crossover frequency equal to $\omega_c^* = 0.1$ rad/sec.