Autonomous and Mobile Robotics
Midterm Class Test, 2024/25

Problem 1

Consider a bicycle robot of length ℓ with front-wheel drive. Define the configuration vector as $\boldsymbol{q} = (x_f, y_f, \theta, \phi)$, where (x_f, y_f) are the Cartesian coordinates of the (contact point of the) front wheel, θ is the vehicle orientation, and ϕ is the steering angle of the front wheel.

(a) Starting from the kinematic constraints acting on the system, derive a kinematic model of the vehicle with the front wheel driving and steering velocities as control inputs, and prove its controllability.

(b) Are x_f, y_f flat outputs for this model? Motivate your answer and give an interpretation in terms of state/input reconstructability.

(c) Design a feedback controller for driving x_f, y_f along a desired trajectory $x_f^*(t), y_f^*(t)$ and provide the corresponding block scheme. *Hint: find an invertible mapping between the time derivative (of a suitable order) of the outputs and the dynamically extended control inputs.*

Problem 2

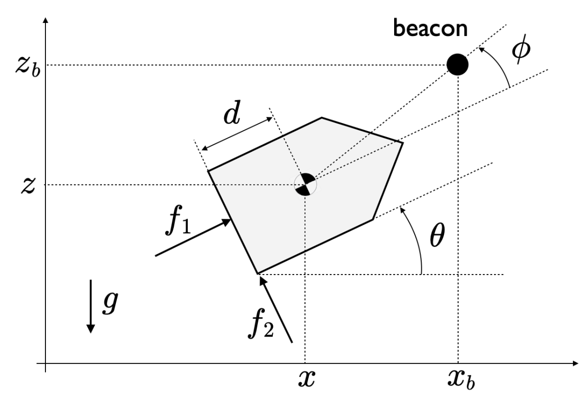
Consider a (2,4) chained form with state $\boldsymbol{z} = (z_1, z_2, z_3, z_4)$ and input $\boldsymbol{v} = (v_1, v_2)$.

(a) Prove that z_1 and z_4 are flat outputs by deriving the corresponding reconstruction formulas.

(b) Design an algorithm for planning a geometric path that leads the chained form from $\boldsymbol{z}_i$ to $\boldsymbol{z}_f$.

Problem 3

Consider the single-body flying robot shown in figure. The robot moves in the vertical plane under the action of the force inputs f_1 and f_2, which are generated by two thrusters.



Denoting by g the gravity acceleration, and assuming unit mass and inertia, the dynamic equations of this robot are

$$\begin{aligned} \ddot{x} &= f_1 \cos\theta - f_2 \sin\theta \\ \ddot{z} &= -g + f_1 \sin\theta + f_2 \cos\theta \\ \ddot{\theta} &= -d\, f_2 \end{aligned}$$

A digital control scheme is used, with f_1 and f_2 constant within each sampling interval of duration T. The sensing equipment includes (1) a radio sensor mounted at the robot Center of Mass (CoM), which measures the bearing ϕ of a known beacon placed at (x_b, z_b) (2) an Inertial Measurement Unit which provides the CoM velocities $\dot{x}$ and $\dot{z}$ (by integration) as well as the robot angular velocity $\dot{\theta}$.

Build a localization system for estimating the whole *state* of the robot. Provide the filter equations (define all symbols), together with a block scheme including all signals involved and showing how each sensor is used.

[3 h]