Autonomous and Mobile Robotics
Midterm Class Test, 2025/26

Problem 1

For a differential-drive robot, we want to extend the configuration by including the right and left wheel angles (ϕ_R and ϕ_L, respectively).

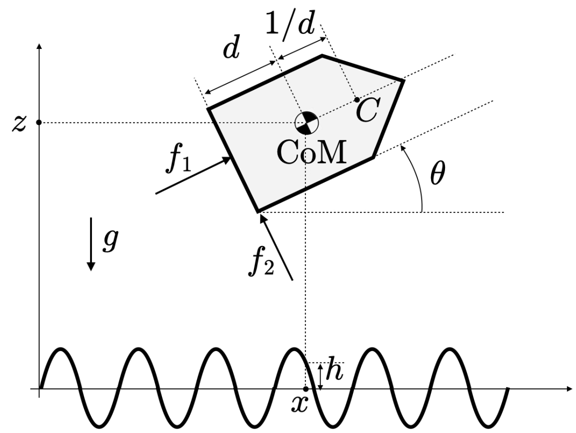
(a) Define the extended configuration space and write the associated kinematic model of the vehicle, having the angular velocities of the two wheels (ω_R and ω_l) as control inputs. *(Hint: start from the unicycle model.)*

(b) Is it possible to drive the system from any initial to any final configuration?

(c) Assume we only include ϕ_R in the configuration, while ϕ_L is ignored. Is it now possible to drive the system from any initial to any final configuration?

Problem 2

Consider the single-body flying robot shown in figure. The robot moves in the vertical plane (x, z) under the action of the force inputs f_1 and f_2, which are generated by two thrusters.



Denote by g the gravity acceleration, and assume unit mass and inertia. The dynamic equations of the robot are

$$\begin{aligned}\ddot{x} &= f_1 \cos\theta - f_2 \sin\theta \\ \ddot{z} &= -g + f_1 \sin\theta + f_2 \cos\theta \\ \ddot{\theta} &= -d\, f_2\end{aligned}$$

a) The system is differentially flat, with flat outputs $\eta_1 = x + (\cos\theta)/d$, $\eta_2 = z + (\sin\theta)/d$ (these are the coordinates of point C, the so-called *center of percussion*). Knowing that the reconstruction formula for θ is

$$\theta = \text{atan2}\,(\ddot{\eta}_2 + g, \ddot{\eta}_1) + k\pi$$

derive the other reconstruction formulas (including those for the inputs). Then, design an algorithm for planning a trajectory between two generic configurations (x_s, z_s, θ_s) and (x_g, z_g, θ_g).

b) Design a feedback control law allowing the robot to track a reference Cartesian trajectory specified as $(x_d(t), z_d(t))$. Provide a block scheme and discuss which measurements are needed.

c) The robot is equipped with (1) a range finder that measures the height of the CoM over the undulated ground, whose profile is described by $h = \sin x$, and (2) an Inertial Measurement Unit which provides $\ddot{x}$, $\ddot{z}$ and θ. Build a localization system for estimating the quantities needed to implement the above control law. Provide equations and a block scheme including all signals involved and showing how each sensor is used.

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Problem 3

Are the following claims *true* or *false*? Answer and provide a short explanation.

(a) Consider the robot of Problem 2. In the particular case $g = 0$, one can appy Chow Theorem (i.e., the Lie Algebra Rank Condition) to check whether the system is controllable or not.

(b) If $g = 0$, the robot of Problem 2 is subject to a second-order kinematic constraint of the form $\boldsymbol{a}^T(\boldsymbol{q})\ddot{\boldsymbol{q}} = 0$.

(c) If a robot is differentially flat, then a feasible trajectory exists between two arbitrary configurations, even in the presence of actuator limits (bounds on the maximum value of each input).

(d) In a unicycle robot, the contact point with the ground cannot move along arbitrary trajectories, due to the pure rolling constraint. However, any other point of the robot along the sagittal axis can follow an arbitrary trajectory.

(e) Consider a unicycle robot tracking a certain trajectory under the action of the controller based on dynamic feedback linearization. Then, the evolution of the robot orientation at steady state will depend on the initial configuration of the unicycle.

[3 h]