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Transition Systems and Bisimulation

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Service Integration
A.A. 2008/09

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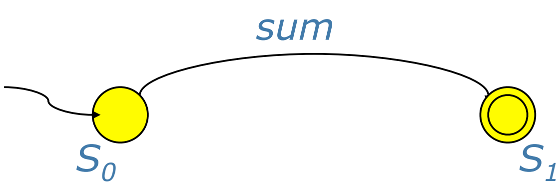
Transition Systems

Concentrating on behaviors: SUM two integers

- Consider a program for computing the sum of two integers.
- Such a program has essentially two states
  - the state S0 of the memory before the computation: including the two number to sum
  - the state S1 of the memory after the computation: including the result of the computation
- Only one action, i.e. "sum", can be performed

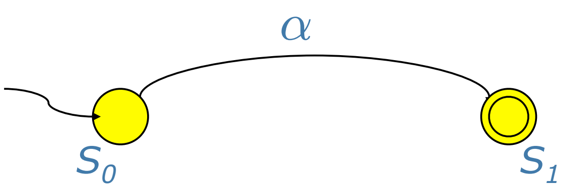


Concentrating on behaviors: CheckValidity

- Consider a program for computing the validity of a FOL formula:
- Also such a program has essentially two states
  - the state S_1 of the memory before the computation: including the formula to be checked
  - the state S_2 of the memory after the computation: including "yes", "no", "time-out"
- Only one action, i.e. "*checkValidity*", can be performed

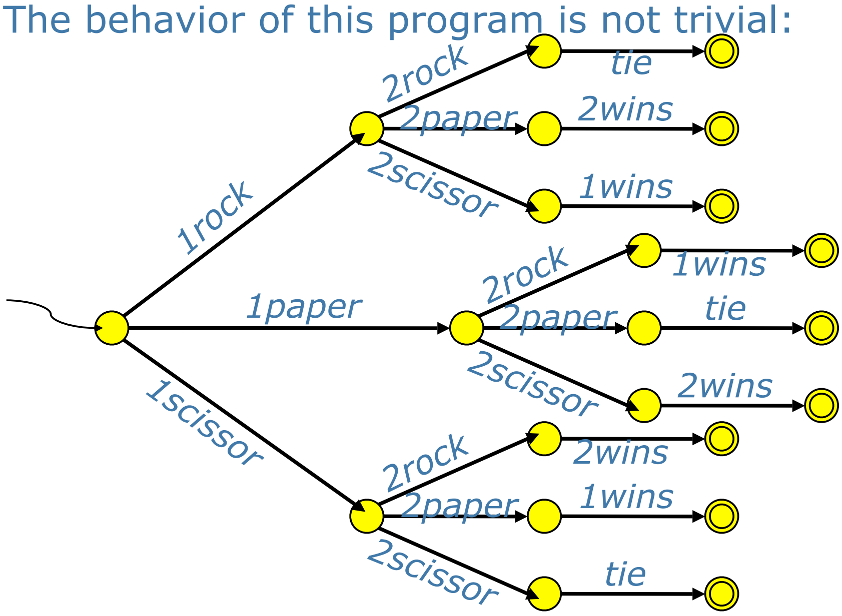
Concentrating on behaviors

- The programs SUM and CheckValidity are very different from a computational point of view.
  - SUM is trivial
  - CheckValidity is a theorem prover hence very complex
- However they are equally trivial from a behavioral point of view:
  - two states S_1 and S_2
  - a single action α causing the transition



Concentrating on behaviors: RockPaperScissor

- Consider the program RockPaperScissor that allows to play two players the the well-known game.
- The behavior of this program is not trivial:



Concentrating on behaviors: RockPaperScissor (automatic)

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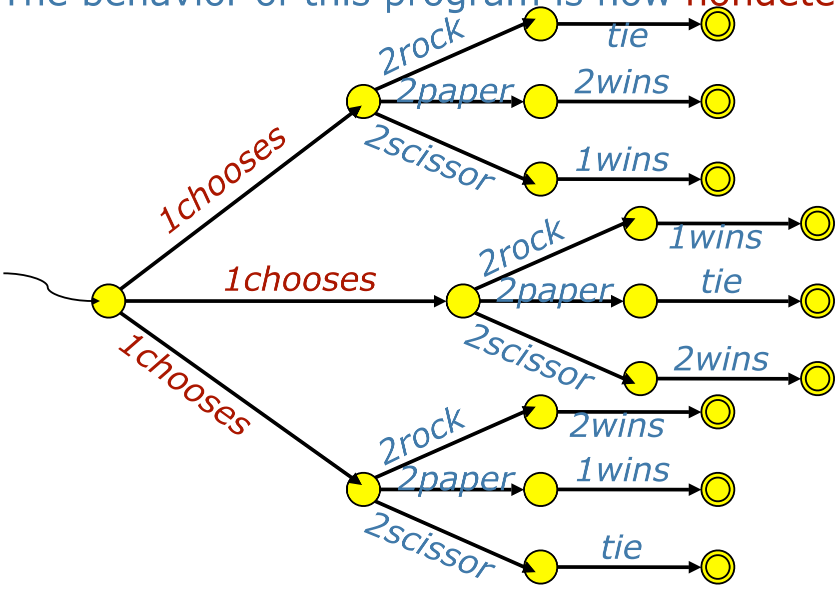
- Consider a variant of the program RockPaperScissor that allows one players to play against the computer.
- The behavior of this program is now nondeterministic:



Concentrating on behaviors: WebPage

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http://www.informatik.uni-trier.de/~ley/db/

A web page can have a complex behavior!

dblp.uni-trier.de

COMPUTER SCIENCE BIBLIOGRAPHY

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- Computer Science Organizations: ACM (DL / SIGMOD / SIGIR), IEEE Computer Society (DL), IEEE Xplore, IFIP, ...
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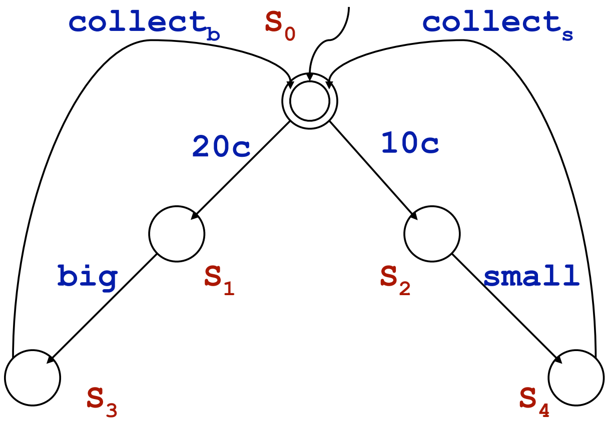
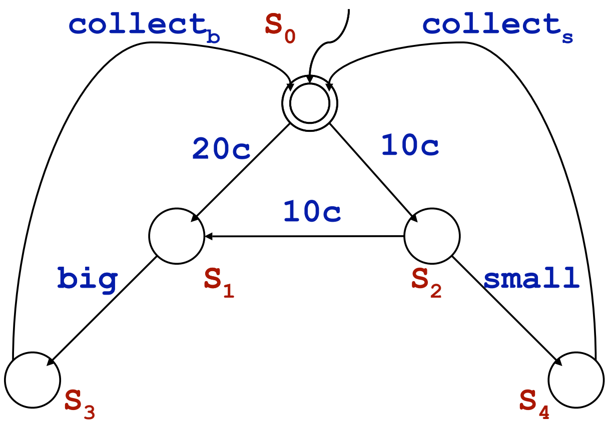
Concentrating on behaviors: Vending Machine



Concentrating on behaviors: Another Vending Machine



Concentrating on behaviors: Vending Machine with Tilt

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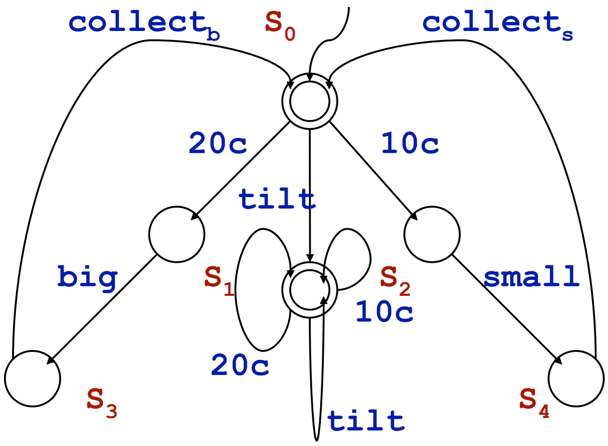

Transition Systems

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- A transition system TS is a tuple $T = < A, S, S^0, \delta, F>$ where:
  - A is the set of actions
  - S is the set of states
  - $S^0 \subseteq S$ is the set of initial states
  - $\delta \subseteq S \times A \times S$ is the transition relation
  - $F \subseteq S$ is the set of final states

- Variants:
  - No initial states
  - Single initial state
  - Deterministic actions
  - States labeled by propositions other than Final/¬Final

(c.f. Kripke Structure)

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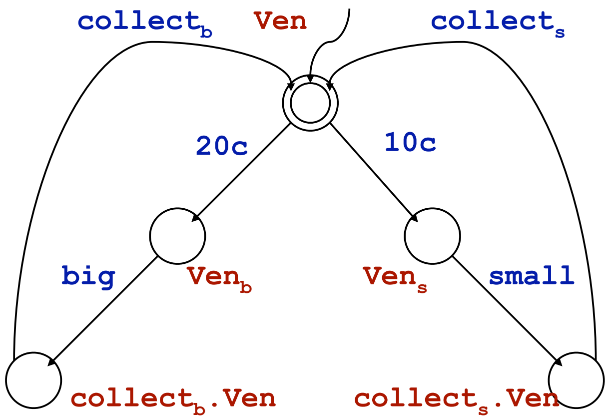
Process Algebras are Formalisms for Describing TS

- Trans (a la CCS)
  - $Ven = 20c.Ven_b + 10c.Ven_s$
  - $Ven_b = big.collect_b.Ven$
  - $Ven_l = small.collect_s.Ven$
- Final
  - $\surd$ Ven



- *TS may have infinite states -* *e.g., this happens when generated by process algebras involving iterated concurrency*
- *However we have good formal tools to deal only with finite states TS*

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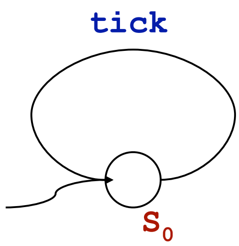
Example (Clock)

TS may describe (legal) nonterminating processes



Example (Slot Machine)

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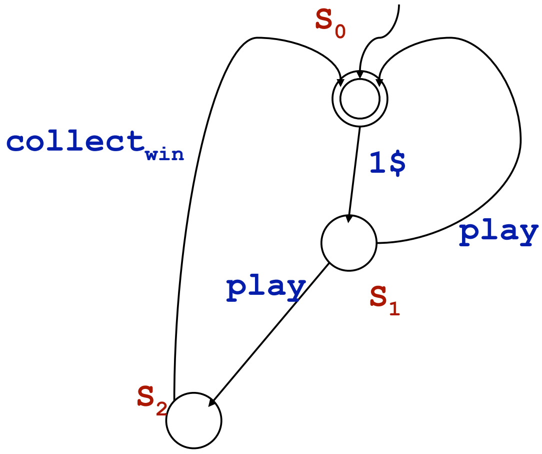
Nondereminisic transitions express choice that is not under the control of clients



Example (Vending Machine - Variant 1)

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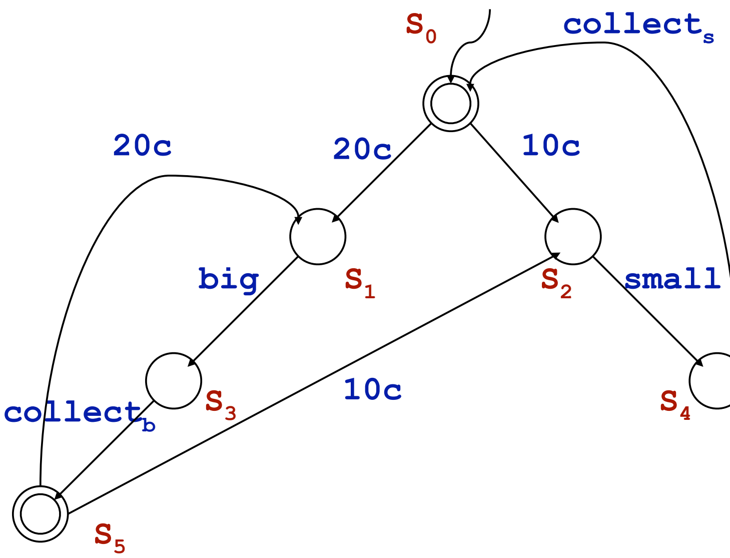
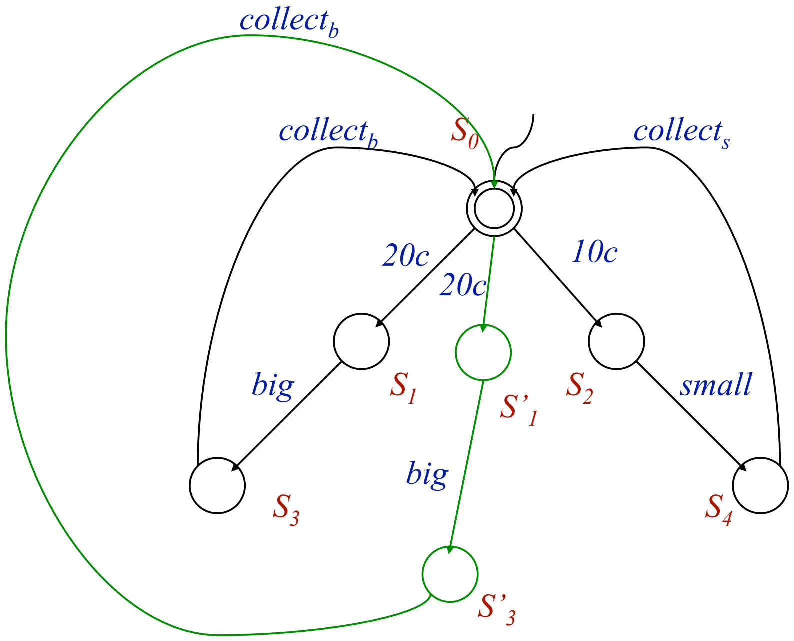


Example (Vending Machine - Variant 2)



Inductive vs Coinductive Definitions: Reachability, Bisimilarity, …

Reachability

- A binary relation R is a **reachability-like relation** iff:
  - $(s,s) \in R$
  - if $\exists$ a, s'. $s \rightarrow_a s' \wedge (s',s'') \in R$ then $(s,s'') \in R$
- A state s_0 of transition system S **reaches** a state s_f iff for **all** a **reachability-like relations** R we have $(s_0, s_f) \in R$.
- Notably that
  - **reaches** is a reachability-like relation itself
  - **reaches** is the smallest reachability-like relation

Note it is a inductive definition!

Computing Reachability on Finite Transition Systems

Algorithm ComputingReachability

Input: transition system TS
Output: the **reachable-from** relation (the smallest reachability-like relation)

Body
$R = \emptyset$
$R' = \{(s,s) \mid s \in S\}$
while $(R \neq R')$ {
  $R := R'$
  $R' := R' \cup \{(s,s'') \mid \exists s',a.\ s \rightarrow_a s' \wedge (s',s'') \in R\}$
}
return R'
YdoB

Bisimulation

- A binary relation R is a **bisimulation** iff:

  $(s,t) \in R$ implies that
  - s is *final* iff t is *final*
  - for all actions a
    - if $s \rightarrow_a s'$ then $\exists t' . t \rightarrow_a t'$ and $(s',t') \in R$
    - if $t \rightarrow_a t'$ then $\exists s' . s \rightarrow_a s'$ and $(s',t') \in R$

- A state s_0 of transition system S is **bisimilar**, or simply **equivalent,** to a state t_0 of transition system T iff there **exists** a **bisimulation** between the initial states s_0 and t_0.

- Notably
  - **bisimilarity** is a bisimulation
  - **bisimilarity** is the largest bisimulation

Note it is a co-inductive definition!

Computing Bisimilarity on Finite Transition Systems

Algorithm ComputingBisimulation
Input: transition system $TS_S = < A, S, S^0, \delta_S, F_S>$ and
transition system $TS_T = < A, T, T^0, \delta_T, F_T>$
Output: the **bisimilarity** relation (the largest bisimulation)

Body
$R = S \times T$
$R' = S \times T - \{(s,t) \mid \neg(s \in F_S \equiv t \in F_T)\}$
while $(R \neq R')$ {
$R := R'$
$R' := R' - (\{(s,t) \mid \exists s',a.\ s \rightarrow_a s' \wedge \neg\exists t' .\ t \rightarrow_a t' \wedge (s',t') \in R' \}$
$\{(s,t) \mid \exists t',a.\ t \rightarrow_a t' \wedge \neg\exists s' .\ s \rightarrow_a s' \wedge (s',t') \in R' \})$
}
return R'
Ydob

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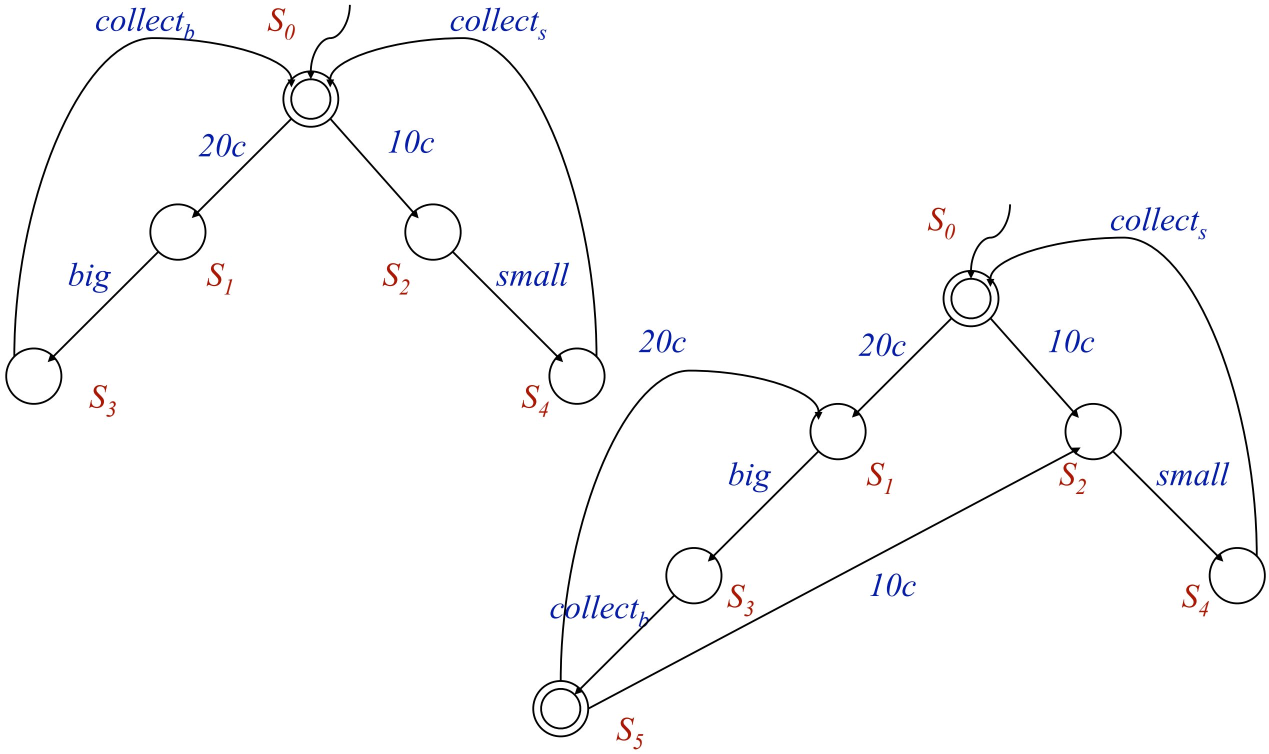
Example of Bisimulation



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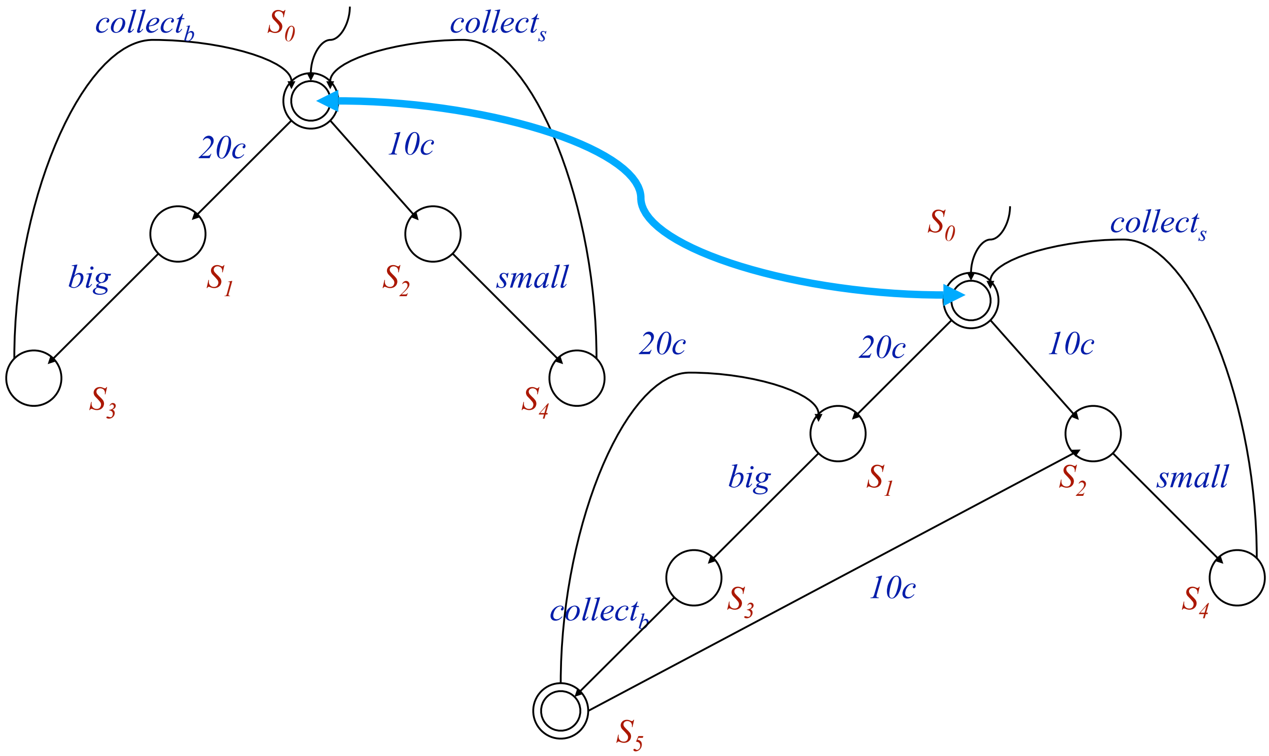

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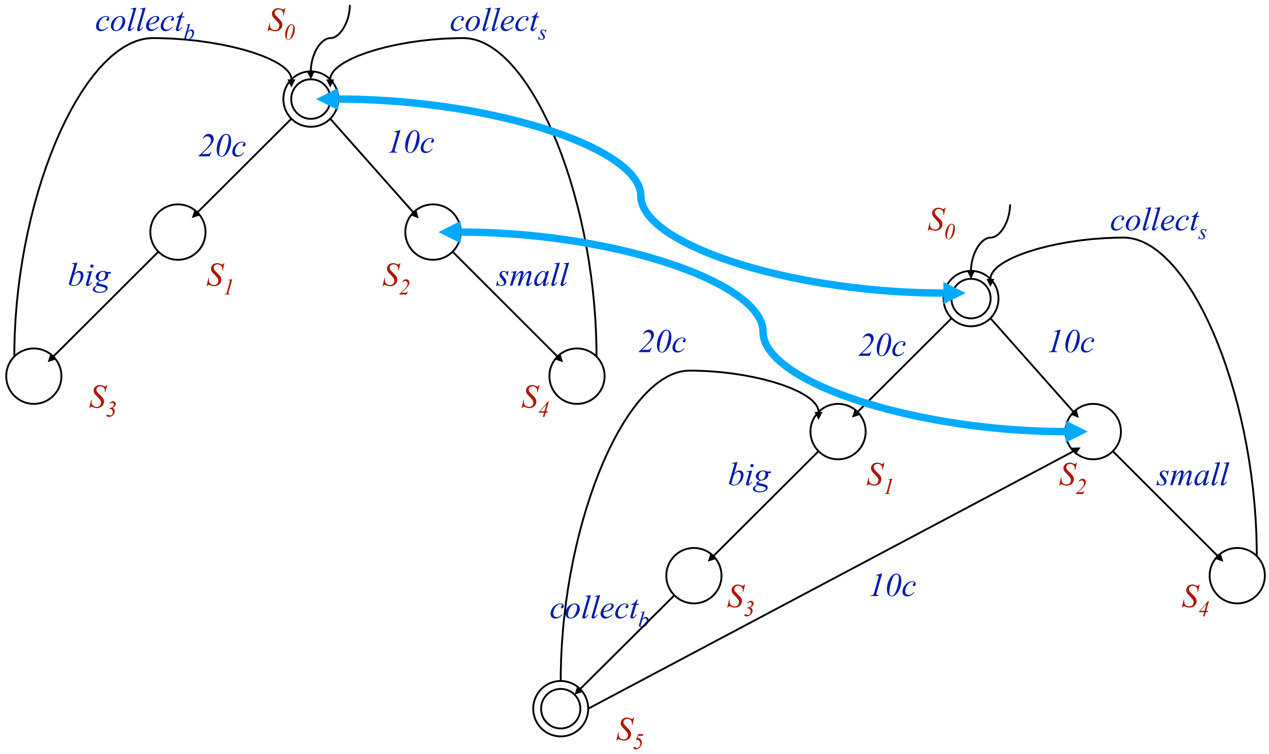

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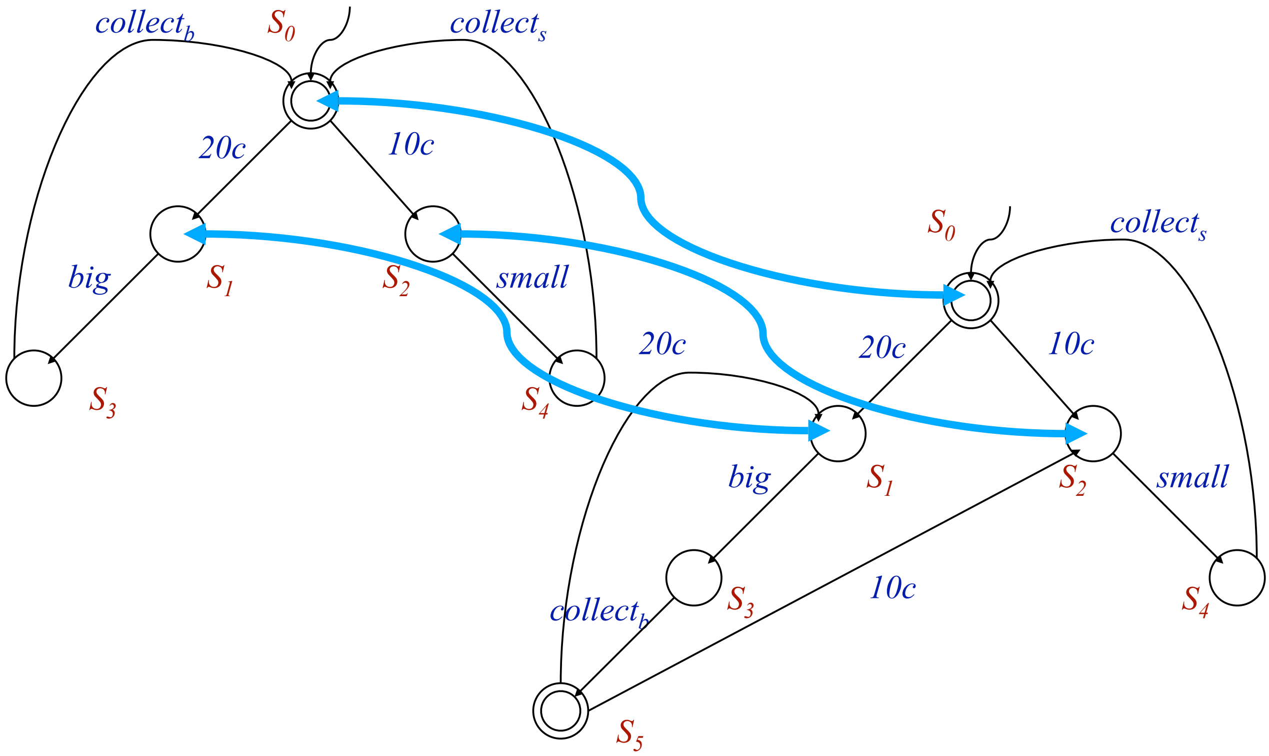

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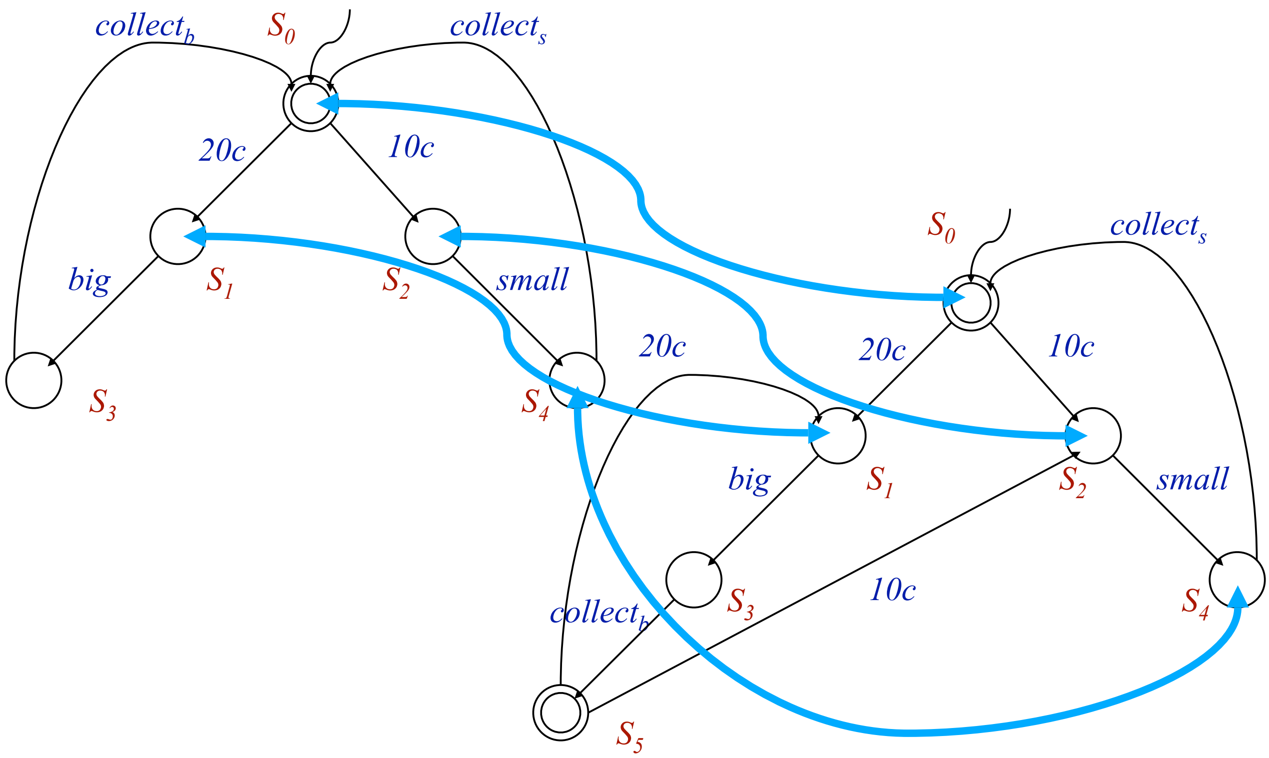

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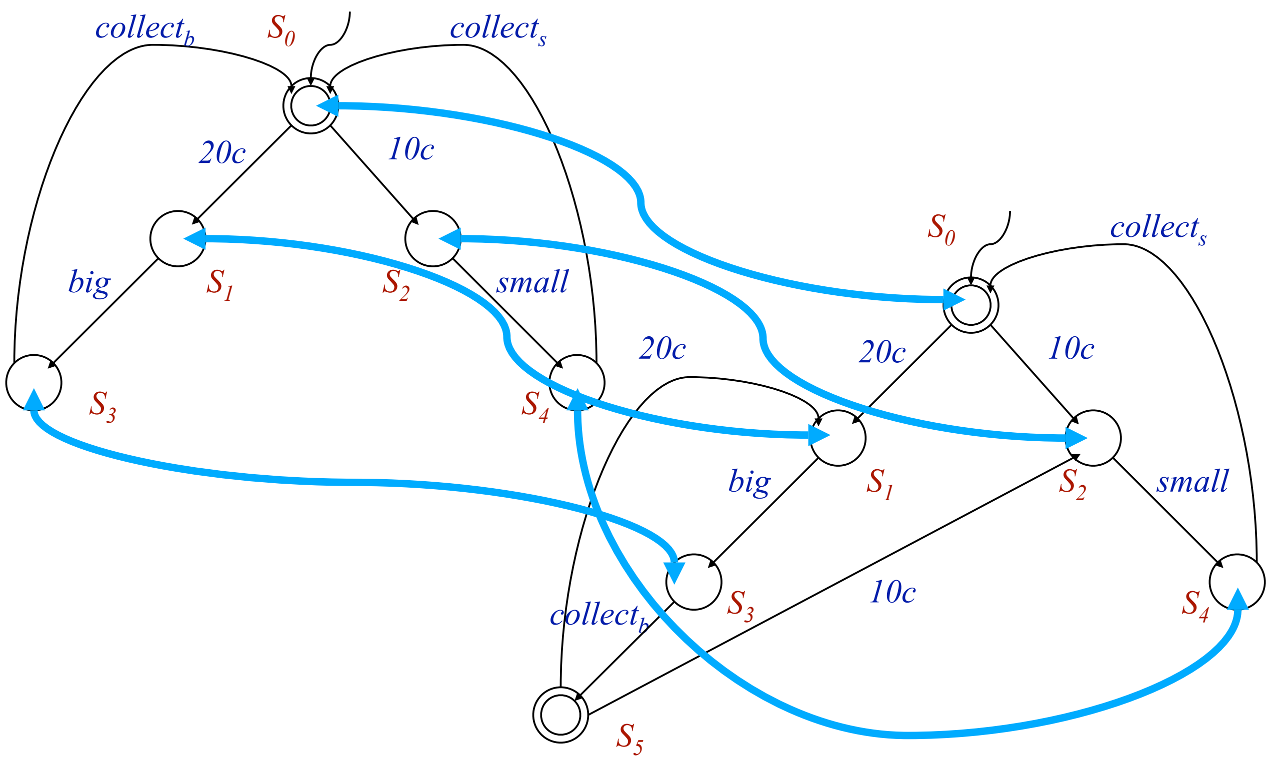

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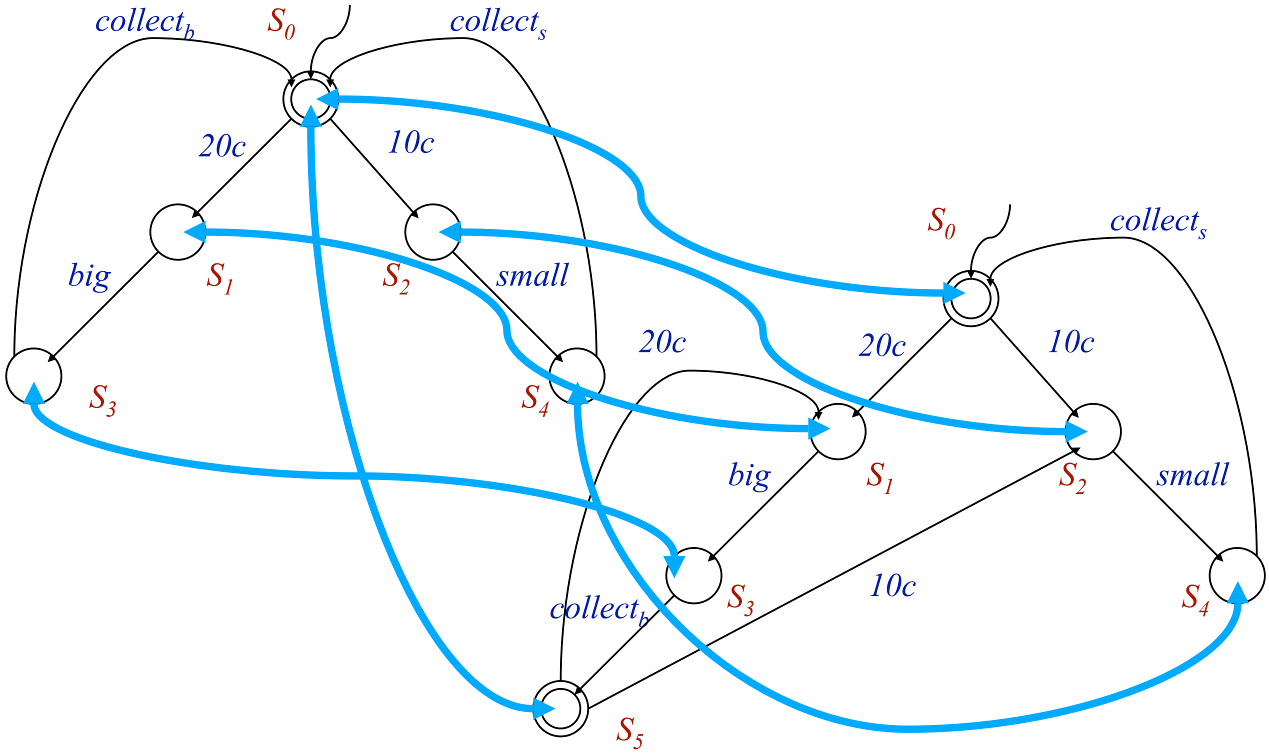

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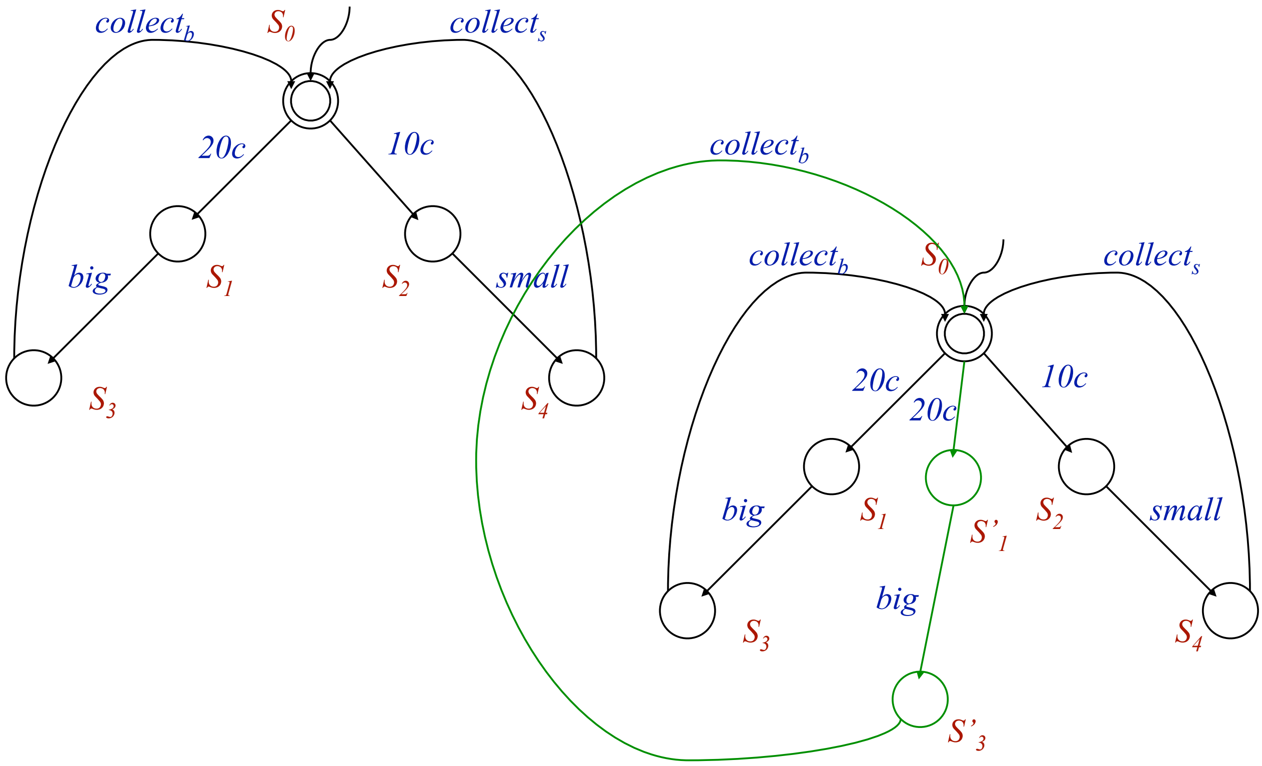

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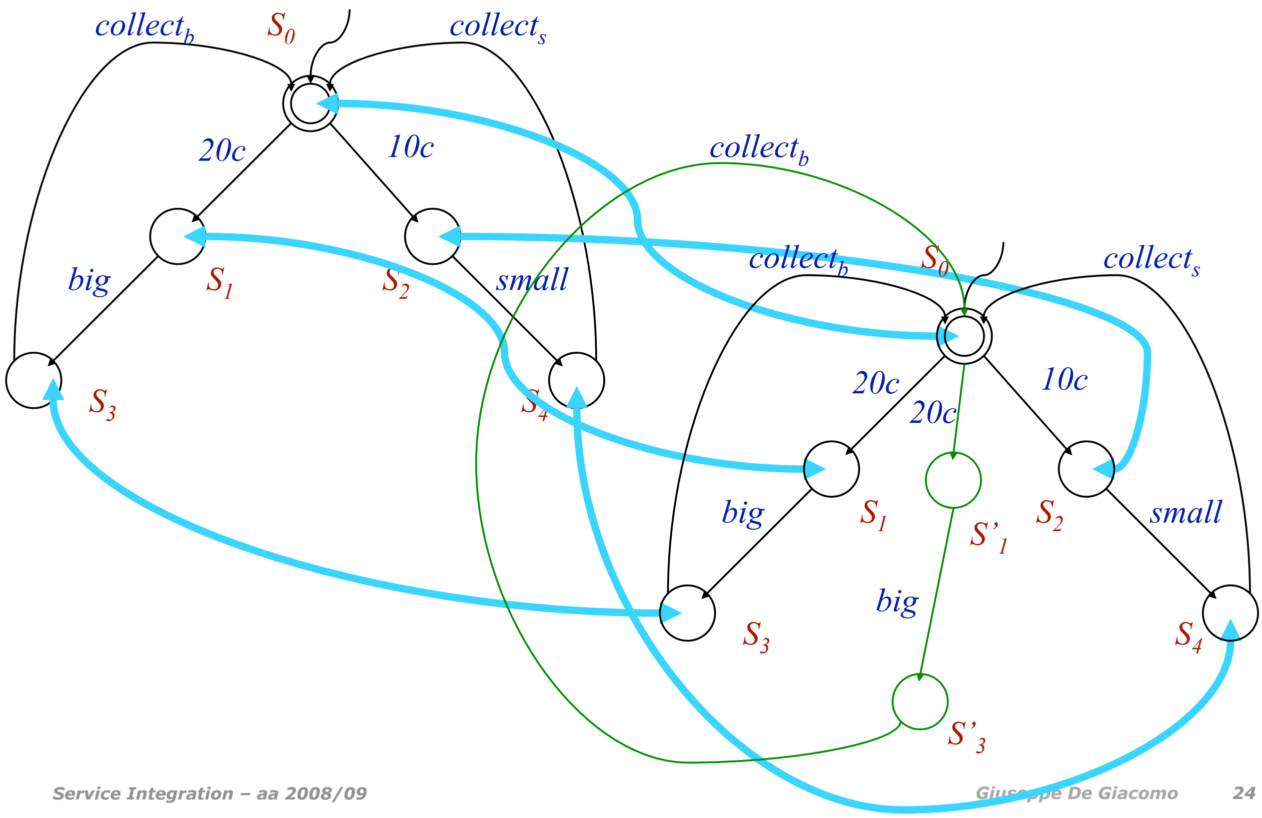

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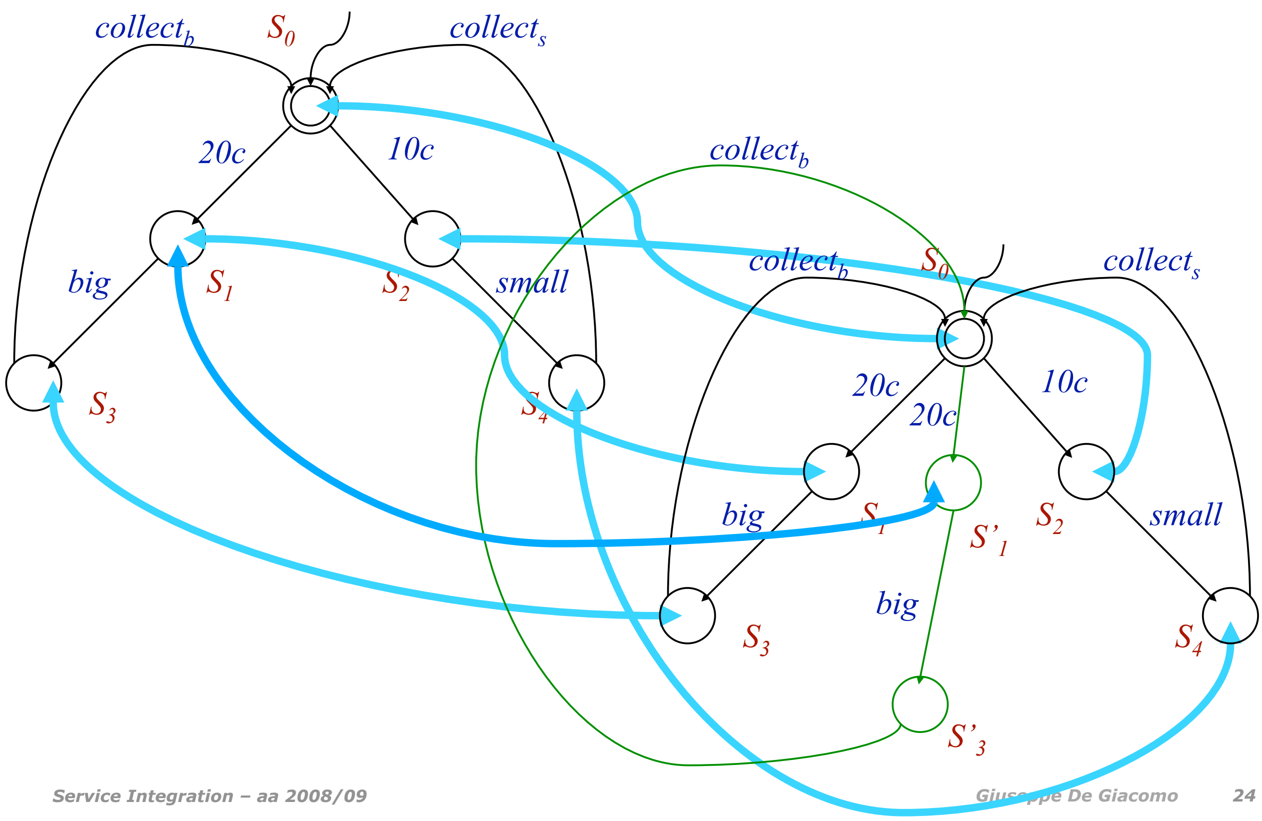

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Example of Bisimulation

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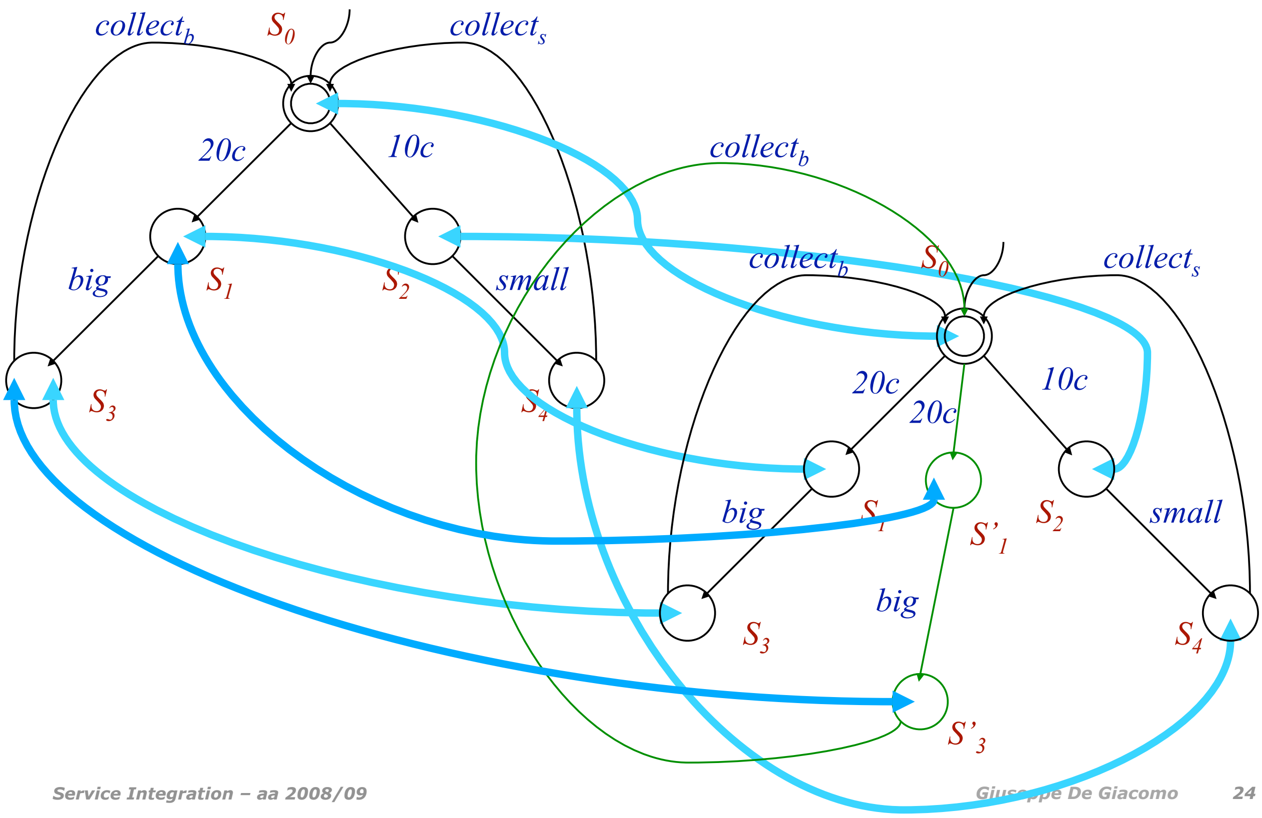

Automata vs.Transition Systems

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- Automata
  - define sets of runs (or traces or strings): (finite) length sequences of actions
- TSs
  - … but I can be interested also in the alternatives "encountered" during runs, as they represent client's "choice points"