Robotics 2

September 19, 2025

Exercise 1

Consider the flexible robot arm in Fig. 1, with the generalized coordinates given therein. The robot is actuated by the torque τ provided by a motor at the base. There are two springs with torsional stiffness $k_1 > 0$ and $k_2 > 0$ along the structure, separating the link in three segments, respectively of length l_i and mass m_i, for $i = 1, 2, 3$, where each segment has uniform mass. This is a basic finite-dimensional approximation of distributed flexibility along a robot link.

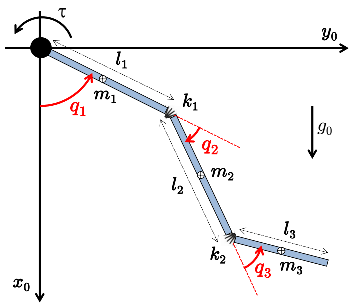

Figure 1: A planar robot arm with link flexibility in the vertical plane.

a) Compute the total potential energy of the robot arm, assuming elasticity in the linear domain.

b) Derive the dynamic model terms that depend only on the potential energy.

c) Factorize these terms as $\boldsymbol{Y}(\boldsymbol{q})\boldsymbol{a}$, with a $3 \times p$ regressor matrix $\boldsymbol{Y}$ and a minimal number of dynamic coefficients $\boldsymbol{a} \in \mathbb{R}^p$.

d) Linearize the potential terms in the dynamic model around a generic configuration $\boldsymbol{q} = \bar{\boldsymbol{q}}$, assuming small deformations of the two springs.

e) For a generic assigned value q_{1e} to q_1, discuss how to obtain the complete forced equilibrium configuration $\boldsymbol{q}_e = (q_{1e}, q_{2e}, q_{3e})$, with the associated equilibrium torque τ_e.

Exercise 2

The single link in Fig. 2 moves under gravity. It is actuated by a motor with inertia I_m through a transmission with reduction ratio $n_r > 1$, delivering a torque τ on the link side with $|\tau(t)| \le \tau_{\max}$. The link has length l, uniformly distributed mass m, and barycentric inertia I_c. The reduction ratio has been chosen so as to optimize the torque transfer from motor to link. The link should perform a rest-to-rest swing-up maneuver from $q_i = 0$ to $q_f = \pi$, counterclockwise and in time T.

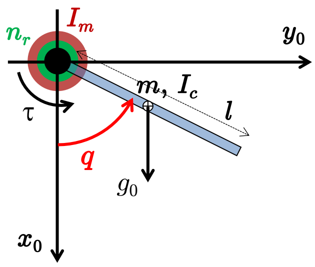

Figure 2: Swing-up maneuver of a single link.

a) Derive the dynamic model and determine the minimum value of $\tau_{\max}$ that allows to perform the swing-up maneuver for *any* sufficiently large time T. Discuss whether or not this maneuver is possible when the available motor torque is smaller than such a value, motivating your answer.

b) Let the link data be $m = 1$ kg, $l = 0.5$ m, and $I_c = 0.02$ kgm^2 and the motor torque limit $\tau_{\max} = 3$ Nm. A cubic polynomial trajectory is defined for the swing-up maneuver, with motion time $T = 2$ s. The corresponding torque profile is shown in Fig. 3, with the maximum absolute value of the torque reaching 2.47 Nm at the time instant $t = 0.94$ s. Using uniform scaling of the original trajectory, determine the minimum scaled time $T_s < T$ that will preserve feasibility of the torque profile, illustrating the steps in the procedure.

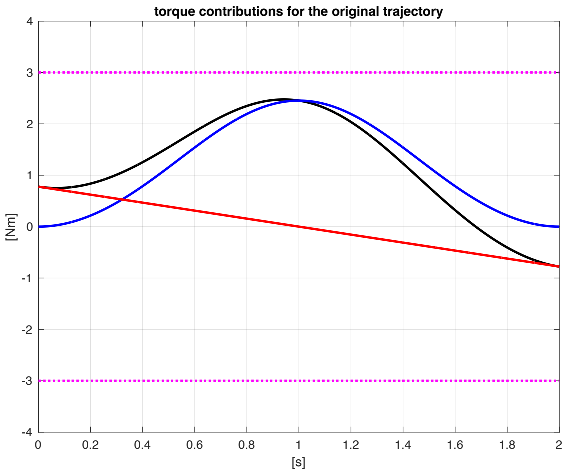

Figure 3: Original swing-up trajectory: inverse dynamics torque (black), with its inertial (red) and gravity (blue) contributions.

Exercise 3

It is well known that the dynamic model of a frictionless rigid robot with n joints can be written as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{c}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{Y}_{\boldsymbol{\pi}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\,\boldsymbol{\pi} = \boldsymbol{\tau}, \tag{1}$$

i.e., linearly in terms of a set of $10n$ standard dynamic parameters of the links, collected in vector $\boldsymbol{\pi} \in \mathbb{R}^{10n}$.

a) How will the regressor matrix $\boldsymbol{Y}_{\boldsymbol{\pi}}$ in (1) change if we model also dissipative terms that include viscous and Coulomb friction at the joints? And what about the addition of motor inertias?

b) If the linear factorization in (1) is used for identification purposes in robot experiments, some problems arise. Explain which ones and why, and how they could be mitigated or solved.

c) The factorization in (1) can be used also in adaptive control for trajectory tracking. Discuss advantages and disadvantages of such an implementation.

d) Suppose that all dynamic parameters of the robot are known, except for a single one —say, the last π_{10n}— that appears in the equations of motion. Write the simplest dynamic controller that achieves global asymptotic tracking of a desired trajectory despite of this missing knowledge.

[180 minutes, open books]