Computational Ontologies

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Many names for the same notion

- Semantic networks
- Conceptual model
- Domain model
- Type network
- Type hierarchy
- Class hierarchy
- Concept base
- Knowledge graph
- Database schema
- Conceptual graph
- RDF graph
- ....
- Ontology

1. Introduction to (computational) ontologies

2. Ontology languages

3. Reasoning

4. Conclusion

The notion of ontology

- Ontology as “the metaphysical study of the nature of being and existence” is as old as the discipline of philosophy.
- More recently, ontologies have been studied in fields such as artificial intelligence, knowledge representation, because of the need to categorize and structure entities and concepts of interest.
- Computational ontology: a conceptualisation of a domain of interest, expressed in a computational format, i.e. in such a way that it can be manipulated by the computer to aid human and machine agents in their performance of tasks within that domain.

The structure of a computational ontology

- An ontology is specified at different levels:
  - Meta-level: specifies a set of modeling categories
  - Intensional level: specifies a set of elements (instances of categories) and constraints used to structure the description of the domain
  - Extensional level: specifies an actual world description (instances of elements) that is coherent with respect to the intensional level

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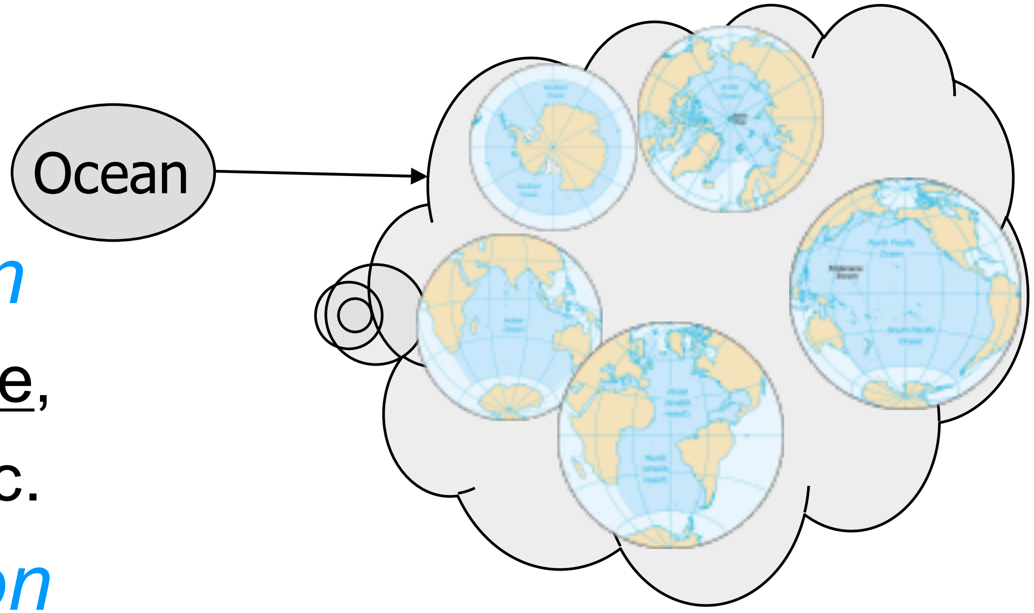
Ontology languages

- An ontology language for expressing the intensional level usually includes constructs for:
  - Concepts
  - Properties of concepts
  - Relationships between concepts, and their properties
  - Axioms
  - Individuals and facts about individuals
  - Queries
- Ontologies are typically rendered as diagrams (e.g., Semantic Networks, Entity-Relationship schemas, UML class diagrams)

Concepts

- A *concept* is an element of the ontology that denotes a collection of instances (e.g., the set of “oceans”)



- *Intensional definition*
  - Specification of name, relations, axioms, etc.
- *Extensional definition*
  - Specification of the instances

Properties

- A *property* qualifies an element (e.g., a concept) of an ontology
- Property definition (intensional and extensional)
  - Name
  - Type
    - Atomic (integer, real, string, …)

      e.g., “eye-color” → {blu, brown, green, grey}
    - Structured (date,sets,lists…)

      e.g., “date” → day/month/year
  - Default value

Relationships

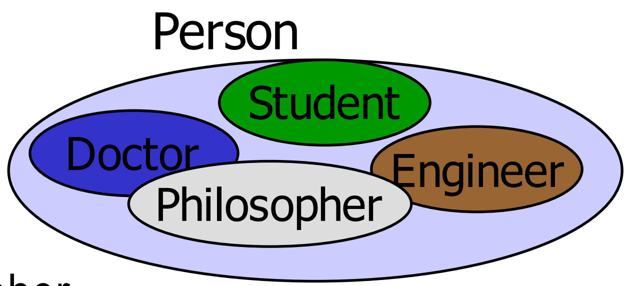
- A *relationship* expresses an association among concepts

- *Intensional definition*
  - Specification of involved concepts (example: workFor is defined on Employee and Company)
- *Extensional definition*
  - Specification of the occurrences, called facts (worksFor(Fulvio,IASI))

Axioms

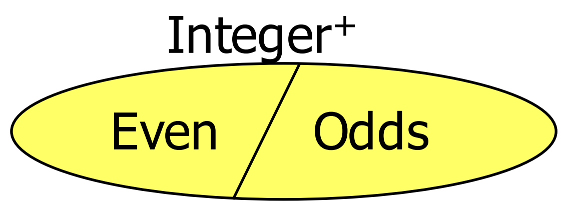
- An *axiom* is a logical formula that expresses at the intensional level a condition that must be satisified by the elements at the extensional level

- Examples:

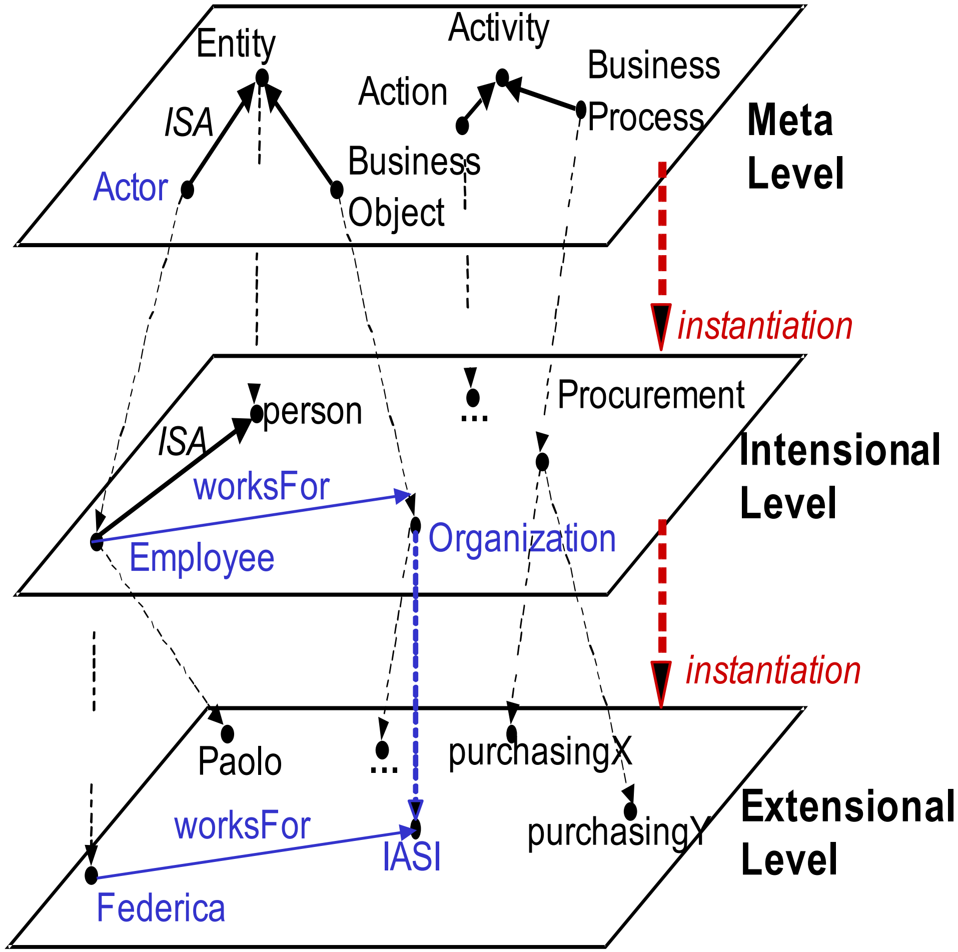


$\text{Person} \supseteq \text{Student} \cup \text{Doctor} \cup \text{Engineer} \cup \text{Philosopher}$

$\text{Integer}^{+} = \text{Even} \cup \text{Odds}, \text{Even} \cap \text{Odds} = \varnothing$



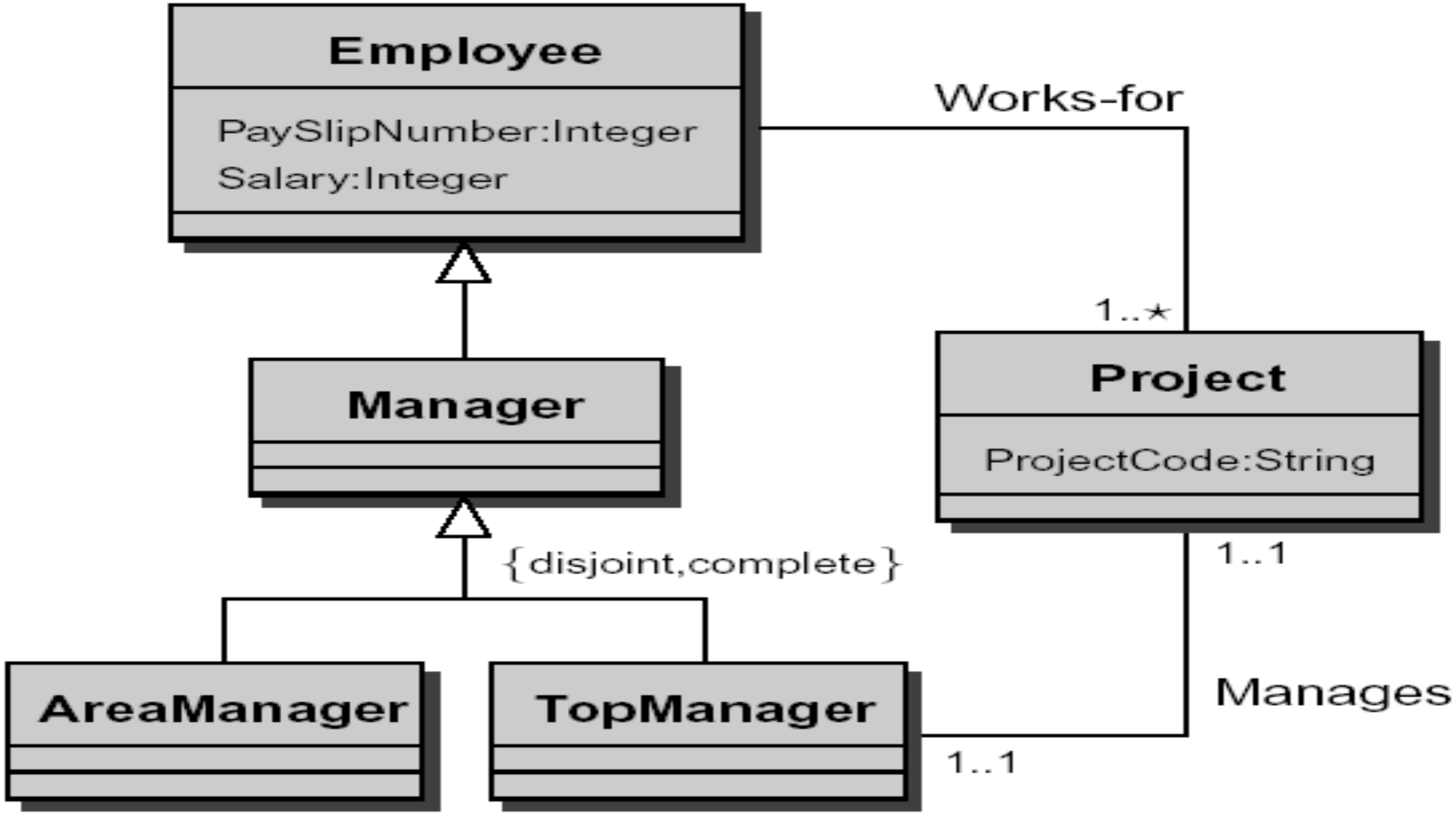
Instantiation



Queries

- An ontology language may also include constructs for expressing queries
  - Queries: expressions at the intensional level denoting collections of individuals satisfying a given condition
  - Meta-queries: expressions at the meta level denoting collections of elements satisfying a given condition
- The constructs for queries may be different from the constructs forming concepts and relationships

Example of query



$\{\ (x.Salary,\ y.ProjectCode)\ \mid\ Manages(x,y)\ \wedge\ \neg Works\text{-}for(x,y)\ \}$

A family of ontology languages: Description Logics

We start with alphabets for concepts, roles, and individuals. Syntactically, concepts and roles are either atomic (i.e., denoted by a name), or non-atomic, i.e. built out using the constructors of a given description language $\mathcal{L}$.

An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of

- a nonempty set $\Delta^{\mathcal{I}}$, the domain of $\mathcal{I}$
- a function $\cdot^{\mathcal{I}}$, the interpretation function of $\mathcal{I}$, that maps
  - every individual to an element of $\Delta^{\mathcal{I}}$
  - every concept to a subset of $\Delta^{\mathcal{I}}$
  - every role to a subset of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

  in such a way that suitable equations are satisfied.

Concept constructors

- atomic concept: $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ $\quad (\bot^{\mathcal{I}} = \emptyset,\ \top^{\mathcal{I}} = \Delta^{\mathcal{I}})$
- conjunction: $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
- disjunction: $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- negation: $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
- universal quantification: $(\forall R.C)^{\mathcal{I}} = \{a \mid \forall b.\, (a,b) \in R^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\}$
- existential quantification: $(\exists R.C)^{\mathcal{I}} = \{a \mid \exists (a,b) \in R^{\mathcal{I}} .\, b \in C^{\mathcal{I}}\}$
- unqualified existential quantification: $\exists R$ equivalent to $\exists R.\top$
- qualified number restrictions

  $(\geq n R.C)^{\mathcal{I}} = \{a : |\{b \in C^{\mathcal{I}} : (a,b) \in R^{\mathcal{I}}\}| \geq n\}$

  $(\leq n R.C)^{\mathcal{I}} = \{a : |\{b \in C^{\mathcal{I}} : (a,b) \in R^{\mathcal{I}}\}| \leq n\}$
- unqualified number restrictions: $(\geq n\ R), (\leq n\ R)$ eq. to $(\geq nR.\top), (\leq nR.\top)$
- individual: $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$

Examples

Atomic concepts: person, lawyer, doctor, male

Atomic roles: child, son, daughter, friend, colleague

$$\text{person} \sqcap (\exists \text{child}) \sqcap (\forall \text{son.lawyer}) \sqcap (\forall \text{daughter.doctor})$$

$$\text{person} \sqcap (\exists \text{child.male}) \sqcap (\leq 2\ \text{child.}(\text{lawyer} \sqcup \text{doctor}))$$

$$\text{person} \sqcap (\geq 5\ \text{friend}) \sqcap (\forall \text{colleague.male})$$

Role constructors

- atomic roles: $P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
- atomic transitive roles: $H^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
- conjunction: $(Q \sqcap R)^{\mathcal{I}} = Q^{\mathcal{I}} \cap R^{\mathcal{I}}$
- disjunction: $(Q \sqcup R)^{\mathcal{I}} = Q^{\mathcal{I}} \cup R^{\mathcal{I}}$
- difference: $(Q \setminus R)^{\mathcal{I}} = Q^{\mathcal{I}} \setminus R^{\mathcal{I}}$
- inverse: $(R^{-1})^{\mathcal{I}} = \{(a,b) \mid (b,a) \in R^{\mathcal{I}}\}$
- chaining: $(R \circ Q)^{\mathcal{I}} = \{(a,b) \mid \exists c.\, (a,c) \in R^{\mathcal{I}}, (c,b) \in Q^{\mathcal{I}}\}$
- self: $id(C)^{\mathcal{I}} = \{(a,a) \mid a \in C^{\mathcal{I}}\}$
- reflexive-transitive closure: $(R^{*})^{\mathcal{I}} = (R^{\mathcal{I}})^{*}$

Examples

Atomic concepts: person, doctor, lawyer, male

Atomic roles: child, son, daughter, friend, colleague

$$\text{person} \sqcap (\exists(\text{colleague} \setminus \text{friend})) \sqcap (\forall \text{colleague}.\text{male})$$

$$(\geq 2\ (\text{son} \sqcup \text{daughter})) \sqcap (\forall \text{son}.\text{lawyer}) \sqcap (\forall \text{daughter}.\text{doctor})$$

$$(\exists(\text{son} \sqcup \text{daughter})^*.\text{doctor}) \sqcap \forall((\text{son} \sqcup \text{daughter}) \circ \text{son}).(\text{lawyer} \sqcup \text{doctor})$$

TBox e ABox

An $\mathcal{L}$-Tbox $\mathcal{T}$ is a set of statements (**inclusion assertions**) of the form:

$$\boxed{C \sqsubseteq D} \qquad \boxed{R \sqsubseteq Q}$$

An $\mathcal{L}$-ABox Σ is a set of statements (**membership assertions**) of the forms (a, b are individuals, and we have $a^\mathcal{I} \neq b^\mathcal{I}$ if $a \neq b$):

$$\boxed{C(a)} \qquad \boxed{R(a,b)}$$

- $C \sqsubseteq D$ is satisfied by $\mathcal{I}$ if $C^\mathcal{I} \subseteq D^\mathcal{I}$
- $R \sqsubseteq Q$ is satisfied by $\mathcal{I}$ if $R^\mathcal{I} \subseteq Q^\mathcal{I}$
- $C(a)$ is satisfied by $\mathcal{I}$ if $a^\mathcal{I} \in C^\mathcal{I}$
- $R(a,b)$ is satisfied by $\mathcal{I}$ if $(a^\mathcal{I}, b^\mathcal{I}) \in R^\mathcal{I}$

Knowledge base (Ontology)

An $\mathcal{L}$-knowledge base is a pair $\langle T, \Sigma \rangle$, where T is an $\mathcal{L}$-Tbox, and Σ is an $\mathcal{L}$-ABox.

An interpretation $\mathcal{I}$ is a model of $K = \langle T, \Sigma \rangle$ if it satisfies all assertions of T and all assertions of Σ. K is said to be satisfiable if it admits a model.

K logically implies an assertion α (written $K \models \alpha$) if α is satisfied by every model of K. C is subsumed by D in K, if $K \models C \sqsubseteq D$.

open world assumption

Example

Note: $\{ C \sqsubseteq D,\ D \sqsubseteq C \}$ is written simply as $C = D$

TBox $\mathcal{T}$:

$$\exists(\text{child}^-)^*.\exists\text{live.SouthOfPo} \sqsubseteq \neg\,\text{RealPadano}$$

$$\text{RealPadano} = \text{Italian} \sqcap (\forall\text{child}^*.\text{RealPadano}) \sqcap (\forall\text{friend}^*.\text{RealPadano})$$

ABox Σ:

RealPadano(Umberto),

child(Umberto,Aldo),

$\neg$ RealPadano(Gianfranco)

OWL Ontology Web Language

OWL concept constructors:

Constructor	DL Syntax	Example	Modal Syntax
intersectionOf	$C_1 \sqcap \ldots \sqcap C_n$	Human $\sqcap$ Male	$C_1 \wedge \ldots \wedge C_n$
unionOf	$C_1 \sqcup \ldots \sqcup C_n$	Doctor $\sqcup$ Lawyer	$C_1 \vee \ldots \vee C_n$
complementOf	$\neg C$	$\neg$Male	$\neg C$
oneOf	$\{x_1\} \sqcup \ldots \sqcup \{x_n\}$	{john} $\sqcup$ {mary}	$x_1 \vee \ldots \vee x_n$
allValuesFrom	$\forall P.C$	$\forall$hasChild.Doctor	$[P]C$
someValuesFrom	$\exists P.C$	$\exists$hasChild.Lawyer	$\langle P \rangle C$
maxCardinality	$\leqslant nP$	$\leqslant$1hasChild	$[P]_{n+1}$
minCardinality	$\geqslant nP$	$\geqslant$2hasChild	$\langle P \rangle_n$

OWL Ontology Web Language

Types of axioms:

Axiom	DL Syntax	Example
subClassOf	$C_1 \sqsubseteq C_2$	Human $\sqsubseteq$ Animal $\sqcap$ Biped
equivalentClass	$C_1 \equiv C_2$	Man $\equiv$ Human $\sqcap$ Male
disjointWith	$C_1 \sqsubseteq \neg C_2$	Male $\sqsubseteq \neg$Female
sameIndividualAs	$\{x_1\} \equiv \{x_2\}$	{President_Bush} $\equiv$ {G_W_Bush}
differentFrom	$\{x_1\} \sqsubseteq \neg\{x_2\}$	{john} $\sqsubseteq \neg${peter}
subPropertyOf	$P_1 \sqsubseteq P_2$	hasDaughter $\sqsubseteq$ hasChild
equivalentProperty	$P_1 \equiv P_2$	cost $\equiv$ price
inverseOf	$P_1 \equiv P_2^-$	hasChild $\equiv$ hasParent$^-$
transitiveProperty	$P^+ \sqsubseteq P$	ancestor$^+$ $\sqsubseteq$ ancestor
functionalProperty	$\top \sqsubseteq \leqslant 1 P$	$\top \sqsubseteq \leqslant$1hasMother
inverseFunctionalProperty	$\top \sqsubseteq \leqslant 1 P^-$	$\top \sqsubseteq \leqslant$1hasSSN$^-$

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Reasoning over ontologies

- Given an ontology, it is possible that *additional* properties can be inferred, by
  - Meta-querying
  - Logical reasoning
- Different goals of reasoning
  - Verification
  - Validation
  - Analysis
  - Synthesis

Logical reasoning

- Based on logic
- No logical reasoning without formal semantics: soundness and completeness
- Great interest in automated logical reasoning
- Feasibility/complexity of automated reasoning

Types of logical reasoning

- Based on semantic property
  - Classical
  - Non-classical (e.g., non-monotonic reasoning, common-sense reasoning, etc.)
- Based on the type of desired conclusions
  - Deduction
  - Induction
  - Abduction

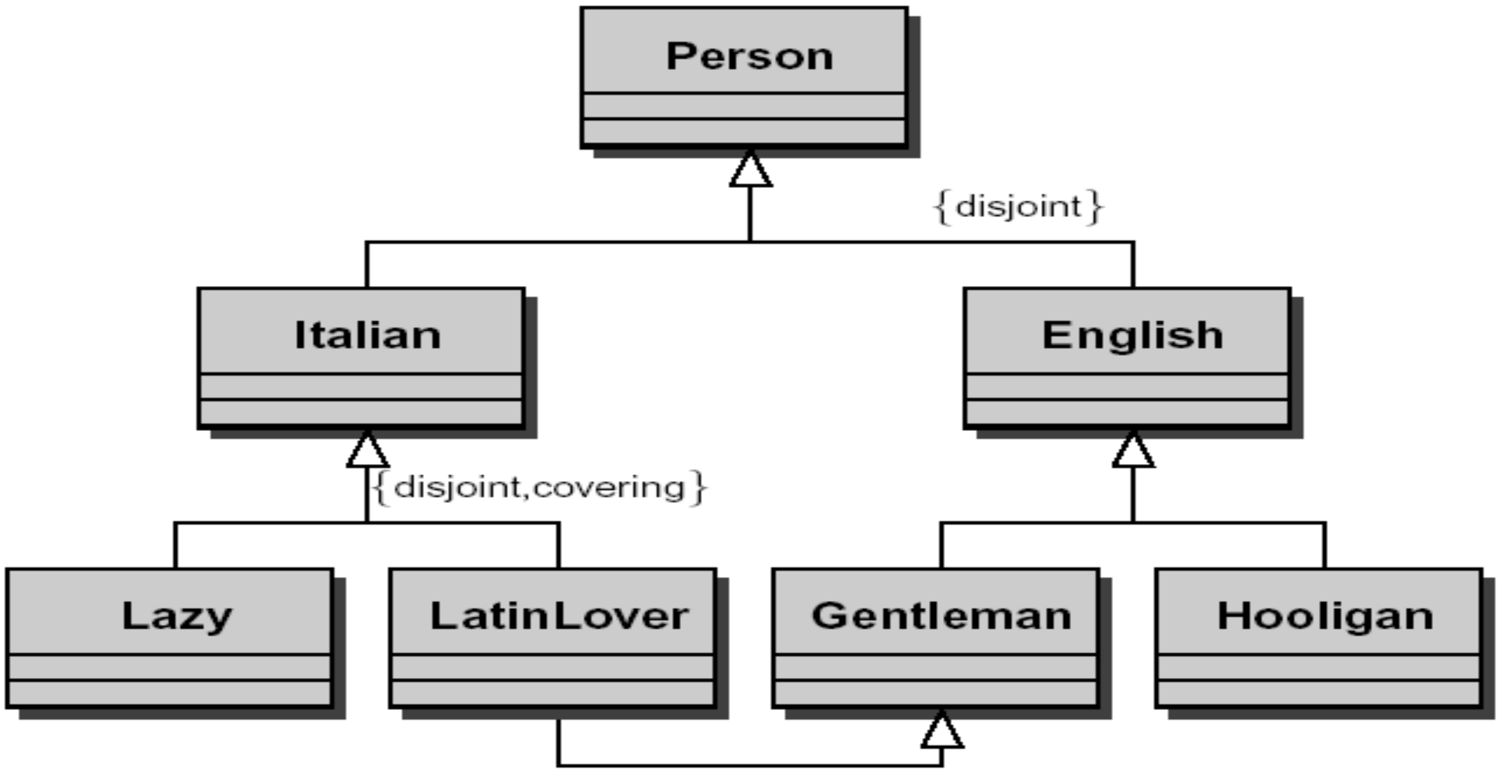
Classical reasoning: deduction

Let Ω and σ be the intensional level and the extensional level of an ontology, respectively.

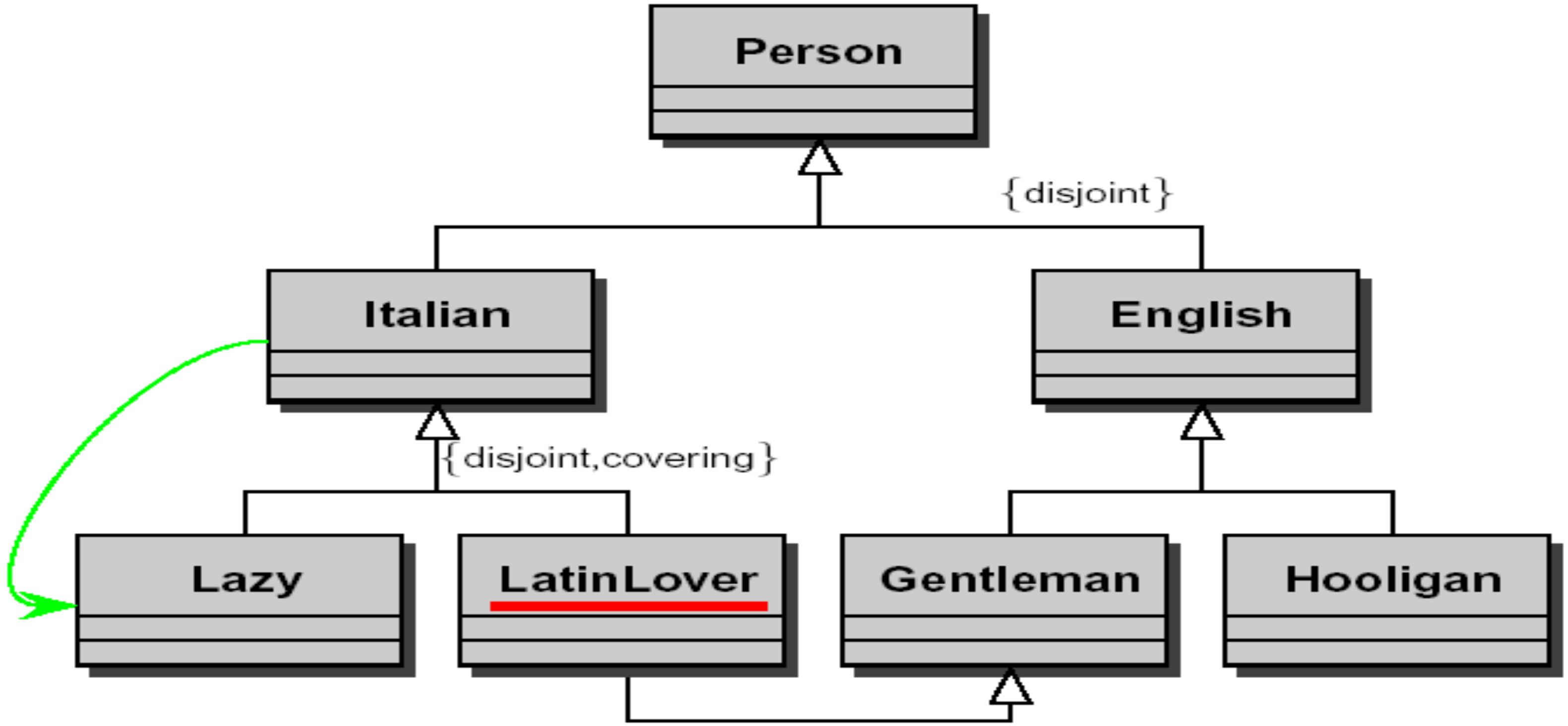
Deduction

P is a deductive conclusion from Ω ($\Omega,\sigma \models P$) if P holds in every situation coherent with Ω and σ, i.e., if P is true in every (logical) model of Ω and σ

Example of deduction



Example of deduction



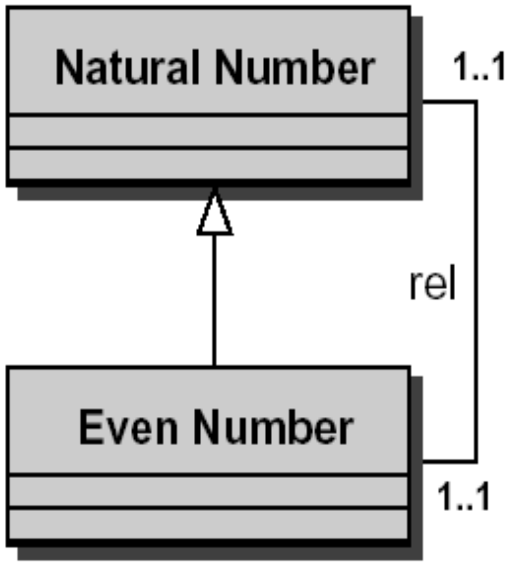
implies

$$\text{LatinLover} = \emptyset$$

$$\text{Italian} \subseteq \text{Lazy}$$

$$\text{Italian} \equiv \text{Lazy}$$

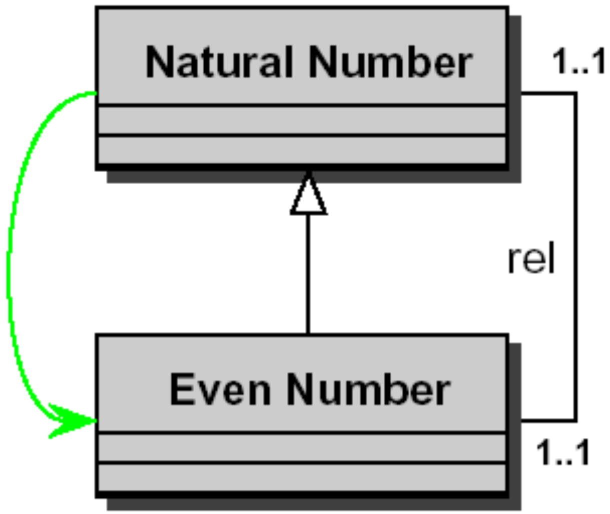
Example of deduction



implies

"the classes *'Natural Number'* and *'Even Number'* contain the same number of instances".

Example of logical reasoning



implies

"the classes *'Natural Number'* and *'Even Number'* contain the same number of instances".

If the domain is finite: Natural Number $\equiv$ Even Number

Logical reasoning: induction

Let T and A be the intensional level and the extensional level of an ontology, respectively.
Let α be a set of observations at the extensional level.

Induction

P is an inductive conclusion wrt T, A and α if P is an intensional level property such that

- $T,A \not\models \alpha$ (T,A do not already imply α)
- $T,\{A,\alpha\} \not\models \neg P$ (P is consistent with $T,\{A,\alpha\}$)
- $\{T,P\},A \models \alpha$ ($\{T,P\},A$ imply α)

Logical reasoning: abduction

Let T and A be the intensional level and the extensional level of an ontology, respectively.
Let α be a set of observations (facts at the extensional level).

Abduction

E is an abductive conclusion wrt T, A and α if E is an extensional level property such that

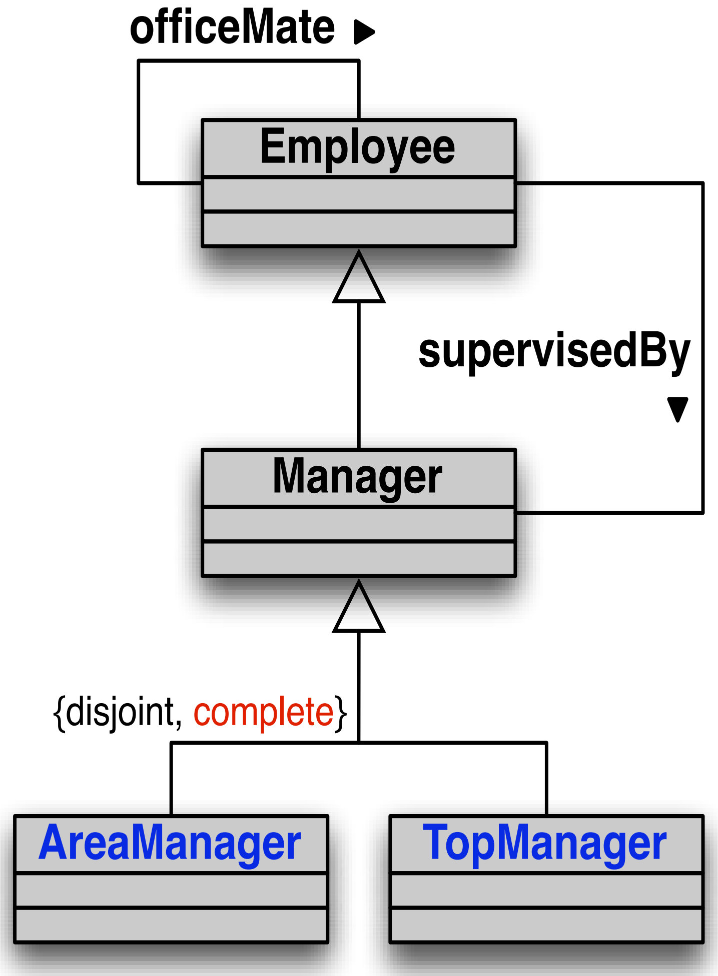
- $T,A \not\models \alpha$ (T,A do not already imply α)
- $T,\{A,\alpha\} \not\models \neg E$ (E is consistent with $T,\{A,\alpha\}$)
- $T,\{A,E\} \models \alpha$ ($T,\{A,E\}$ imply α)

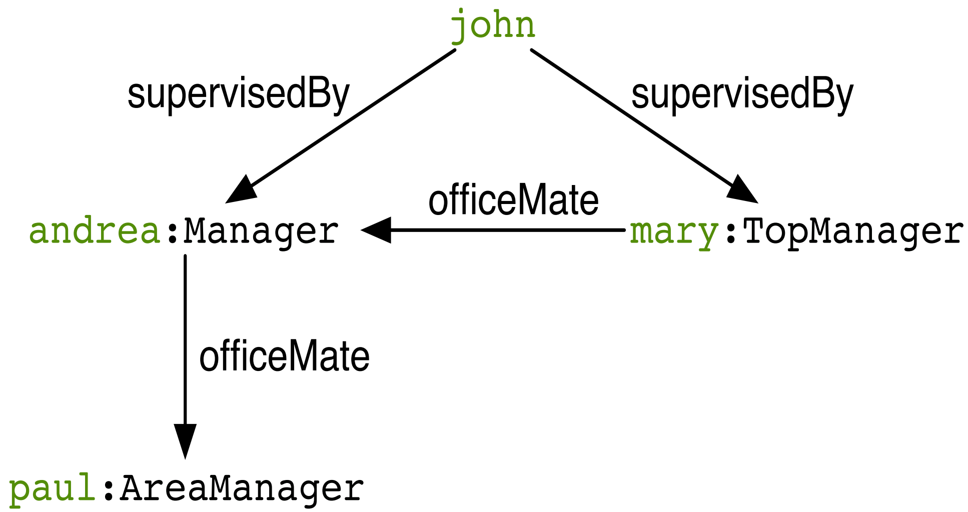
Query answering

- Query answering is a kind of deductive reasoning of special importance

- In general, query answering over ontologies is very different from and much more complex than query answering in databases, because an ontologies can be seen as an abstraction for a set of models (i.e., databases)

Example of query answering

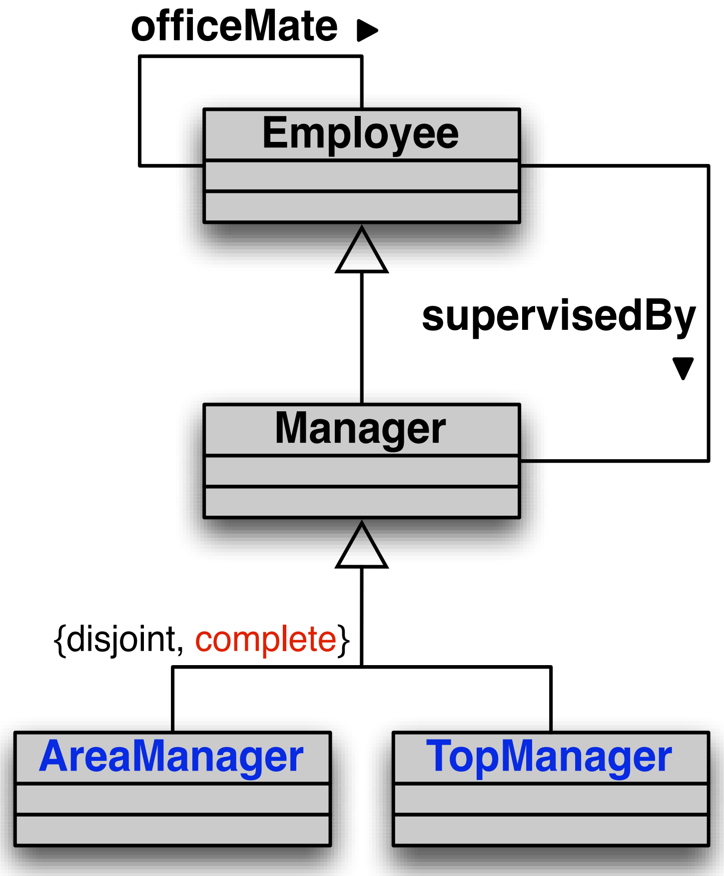


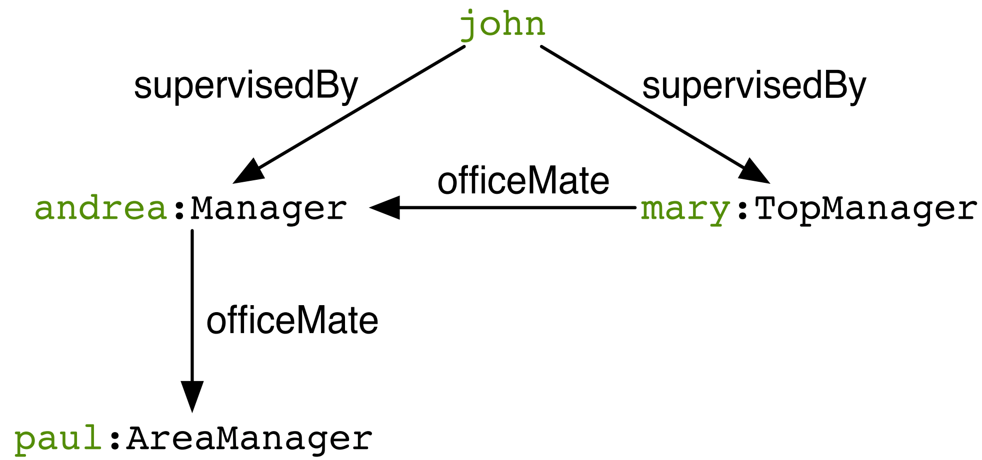


$$q(x) \leftarrow \exists y, z.\, \text{supervisedBy}(x, y),\ \text{TopManager}(y),\ \text{officeMate}(y, z),\ \text{AreaManager}(z)$$

Answer: ???

Example of query answering





$$q(x) \leftarrow \exists y, z.\, \mathsf{supervisedBy}(x, y),\ \mathsf{TopManager}(y),\ \mathsf{officeMate}(y, z),\ \mathsf{AreaManager}(z)$$

Answer: { john }

To determine this answer, we need to resort to **reasoning by cases**.

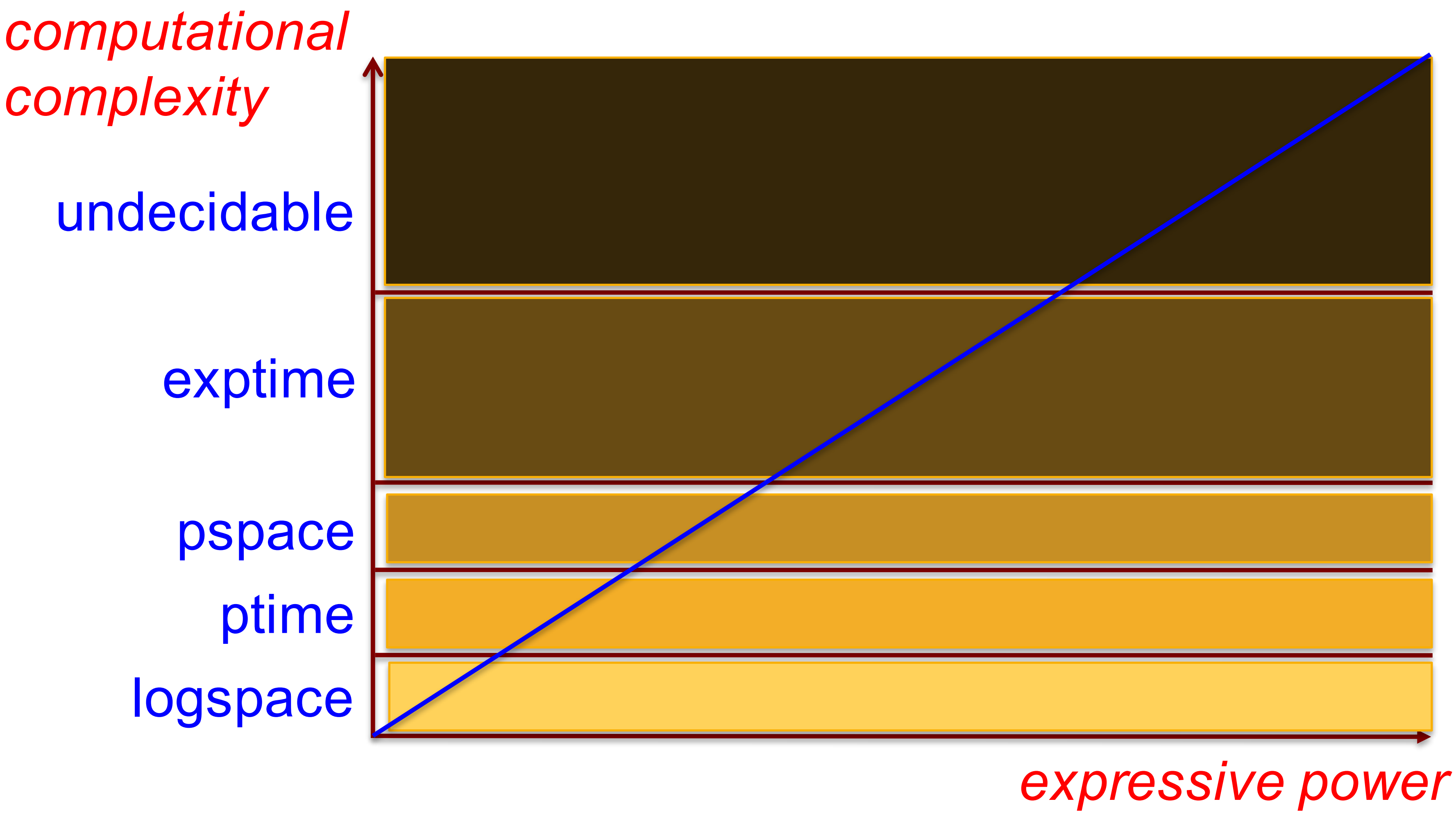
Example of complexity analysis

$\mathcal{AL}$ +	P		CoNP		NP			PSPACE							
$C \sqcup D$			×	×				×	×		×	×	×		×
$(\geq n\, R)$ $(\leq n\, R)$		×		×						×	×		×	×	×
$\exists R.C$					×		×	×			×	×		×	×
$R \sqcap R'$						×	×		×	×		×	×	×	×

polynomial time

exponential time

A fundamental trade-off



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(Some) Use of ontologies

- To make domain assumptions explicit and to share common understanding of a domain (bioinformatics, medicine, finance, …) among people or software agents
- To enable interoperability of different systems and data exchange
- To enable reuse of domain knowledge (Natural Language processing, Robotics, …)
- To separate domain knowledge from the operational knowledge
- To analyze domain knowledge
- Ontology-based information retrieval
- Ontology-based data management (See later)

Conclusion

- The notion of computational ontology is gaining attention in several fields
- Automated reasoning is one crucial aspects of computational ontologies
- We will investigate one particular aspect of computational ontologies in what follows