Iterative Learning for Gravity Compensation

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Control goal

- regulation of arbitrary equilibrium configurations in the presence of gravity
  - without explicit knowledge of robot dynamic coefficients (nor of the structure of the gravity term)
  - without the need of "high" position gain
  - without complex conditions on the control gains

- based on an iterative control scheme that uses
  1. PD control on joint position error + constant feedforward term
  2. iterative update of the feedforward term at successive steady-state conditions

- derive sufficient conditions for the global convergence of the iterative scheme with zero final error
Preliminaries

- robot dynamic model
  \[ M(q)\ddot{q} + c(q, \dot{q}) + g(q) = u \]
- available bound on the gradient of the gravity term
  \[ \left\| \frac{\partial g(q)}{\partial q} \right\| \leq \alpha \]
- regulation attempted with a joint-based PD law (without gravity cancellation nor compensation)
  \[ u = K_P(q_d - q) - K_D \dot{q} \quad K_P > 0, K_D > 0 \]
- at steady state, there is a non-zero error left
  \[ q = \bar{q}, \dot{q} = 0 \quad g(\bar{q}) = K_P(q_d - \bar{q}) \quad \bar{e} = q_d - \bar{q} \neq 0 \]
Iterative control scheme

- control law at the $i$-th iteration (for $i = 1, 2, ...$)
  \[ u = \gamma K_P (q_d - q) - K_D \dot{q} + u_{i-1} \quad \gamma > 0 \]
  with a constant compensation term $u_{i-1}$ (feedforward)
  - $K_P > 0, K_D > 0$ are chosen diagonal for simplicity
  - $q_0$ is the initial robot configuration
  - $u_0 = 0$ is the ‘easiest’ initialization of the feedforward term
  - at the steady state of the $i$-th iteration ($q = q_i, \dot{q} = 0$), one has
  \[ g(q_i) = \gamma K_P (q_d - q_i) + u_{i-1} \]
- update law of the compensation term (for next iteration)
  \[ u_i = \gamma K_P (q_d - q_i) + u_{i-1} \quad [= g(q_i)] \]
  ← for implementation → [ for analysis ]
Convergence analysis

Theorem

(a) $\lambda_{\min}(K_P) > \alpha$

(b) $\gamma \geq 2$

guarantee that the sequence $\{q_0, q_1, q_2, \ldots\}$ converges to $q_d$ (and $\dot{q} = 0$) from any initial value $q_0$ (and $\dot{q}_0$), i.e., globally

- condition (a) is sufficient for the global asymptotic stability of the desired equilibrium state when using
  
  \[ u = K_P(q_d - q) - K_D\dot{q} + g(q_d) \]

  with a known gravity term and diagonal gain matrices

- the additional sufficient condition (b) guarantees the convergence of the iterative scheme, yielding

  \[ \lim_{i \to \infty} ||u_i|| = g(q_d) \]
Proof

- Let $e_i = q_d - q_i$ be the error at the end of the $i$-th iteration; based on the update law, it is $u_i = g(q_i)$ and thus

$$
\|u_i - u_{i-1}\| = \|g(q_i) - g(q_{i-1})\| \leq \alpha \|q_i - q_{i-1}\|
$$

$$
\leq \alpha (\|e_i\| + \|e_{i-1}\|)
$$

- On the other hand, from the update law it is

$$
\|u_i - u_{i-1}\| = \gamma \|K_pe_i\|
$$

- Combining the two above relations under (a), we have

$$
\gamma \alpha \|e_i\| < \gamma \lambda_{\text{min}}(K_P) \|e_i\| \leq \gamma \|K_pe_i\| \leq \alpha (\|e_i\| + \|e_{i-1}\|)
$$

or

$$
\|e_i\| < \frac{1}{\gamma} (\|e_i\| + \|e_{i-1}\|)
$$
Proof (cont)

- condition (b) guarantees that the error sequence \( \{e_0, e_1, e_2, \ldots \} \)

\[
\|e_i\| < \frac{1}{1 - \frac{1}{\gamma}} \|e_{i-1}\| = \frac{1}{\gamma - 1} \|e_{i-1}\|
\]

is a contraction mapping, so that

\[
\lim_{i \to \infty} \|e_i\| = 0
\]

with asymptotic convergence from any initial state

\[ \Rightarrow \text{the robot progressively approaches the desired configuration through successive steady-state conditions} \]

- \( K_P \) and \( K_D \) affect each transient phase
- coefficient \( \gamma \) drives the convergence rate of intermediate steady states to the final one
Remarks

- combining (a) and (b), the sufficient condition only requires the **doubling** of the proportional gain w.r.t. the known gravity case

\[ \widehat{K}_P = \gamma K_P \quad \Rightarrow \quad \lambda_{\text{min}}(\widehat{K}_P) > 2\alpha \]

- for a diagonal \( \widehat{K}_P \), this condition implies a (positive) lower bound on the single diagonal elements of the matrix

- again, it is only a **sufficient** condition
  - the scheme may converge even if this condition is violated ...

- the scheme can be interpreted as using an **integral term**
  - updated only in correspondence of a **discrete sequence of time instants**
  - with guaranteed **global** convergence (and implicit stability)
Numerical results

- 3R robot with uniform links, moving in the vertical plane
  \[ l_1 = l_2 = l_3 = 0.5 \text{ [m]} \]
  \[ m_1 = 30, m_2 = 20, m_3 = 10 \text{ [kg]} \]
  \[ \alpha \cong 400 \]
- with saturations of the actuating torques
  \[ U_{1,\text{max}} = 800, U_{2,\text{max}} = 400, U_{3,\text{max}} = 200 \text{ [Nm]} \]
- three cases, from the downward position \( q_0 = (0, 0, 0) \)
  
  **I:** \( q_d = (\pi/2, 0, 0) \)
  \[ \hat{K}_P = \text{diag}\{1000, 600, 280\} \]
  \[ K_D = \text{diag}\{200, 100, 20\} \]
  
  **II:** \( q_d = (3\pi/4, 0, 0) \)
  \[ \hat{K}_P = \text{diag}\{500, 500, 500\} \]
  \[ K_D = \text{as before} \]
  
  **III:** \( q_d = (3\pi/4, 0, 0) \)
  \[ \hat{K}_P = \text{diag}\{500, 500, 500\} \]
  \[ K_D = \text{as before} \]
Case I: \( q_d = (\pi/2, 0, 0) \)

joint position errors (zero after 3 updates)

control torques
Case II: \( q_d = (3\pi/4, 0, 0) \)

joint position errors (zero after 5 updates)

c Control torques
Case III: \( q_d = (3\pi/4, 0, 0) \), reduced gains

joint position errors (limit cycles, no convergence!)

control torques
Final comments

- only **few iterations** are needed for obtaining convergence, learning the correct gravity compensation at the desired \( q_d \)

- **sufficiency** of the condition on the \( P \) gain
  - even if violated, convergence can **still be obtained** (first two cases); otherwise, a limit motion cycle takes place between two equilibrium configurations that are both incorrect (as in the third case)
  - this shows how ‘distant’ is sufficiency from **necessity**

- analysis can be refined to get lower bounds on the \( K_{P_i} \) (diagonal case) that are smaller, but still sufficient for convergence
  - intuitively, lower values for \( K_{P_i} \) should be sufficient for distal joints

- in practice, update of the feedforward term occurs when the robot is **close enough to a steady state** (joint velocities and position variations are below **suitable thresholds**)

Robotics 2
Control experiments with flexible robots without gravity

rest-to-rest maneuver in given motion time for a single flexible link (PD + feedforward)

end-effector trajectory tracking for FlexArm—a planar 2R robot with flexible forearm
Extension to flexible robots

- the same iterative learning control approach has been extended to position regulation in robots with **flexible joints and/or links** under gravity
  - at the motor/joint level
  - at the Cartesian level (end-effector tip position, **beyond** flexibility), using a **double iterative** scheme

- experimentally validated on the **Two-link FlexArm @ DIS** (now DIAG!)

with supporting base tilted by approx $\Delta = 6^\circ$ (inclusion of gravity)
Experimental results for tip regulation

motion task: 
$$(0^\circ,0^\circ) \Rightarrow (90^\circ,0^\circ)$$

(factor $\gamma \rightarrow 1/\beta$ in the original paper)